

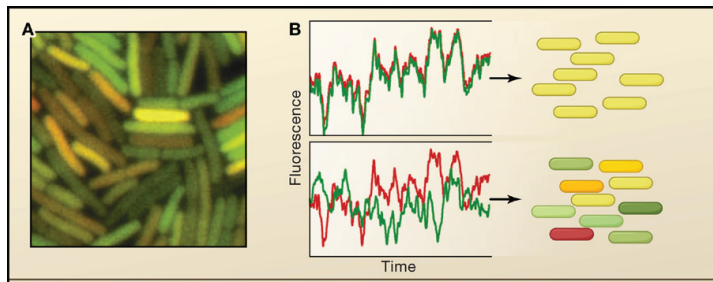
# Mathematical Modelling of Hormonal Regulation

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# Stochasticity in biological systems



*Fluorescence imaging of individual E. coli reveals marked variability in protein expression*  
[Raj et al. 2008. DOI 10.1016/j.cell.2008.09.050]

## **extrinsic noise:**

fluctuations in cellular environmental factors

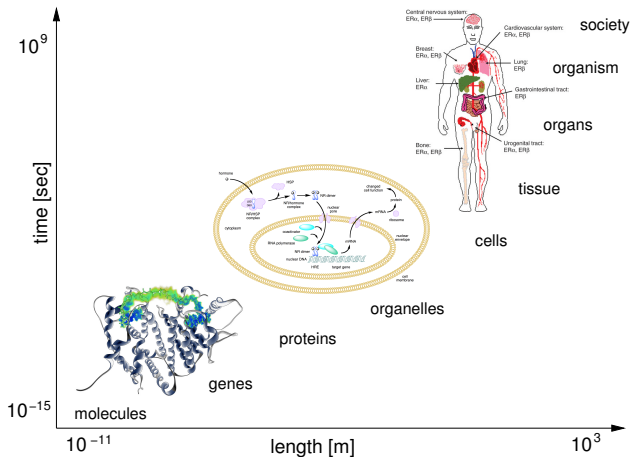
## **intrinsic noise:**

fluctuations arising from low copy numbers (*E.coli*: 10 mRNA molecules per cell)

**This presentation focuses on modelling of extrinsic noise!**

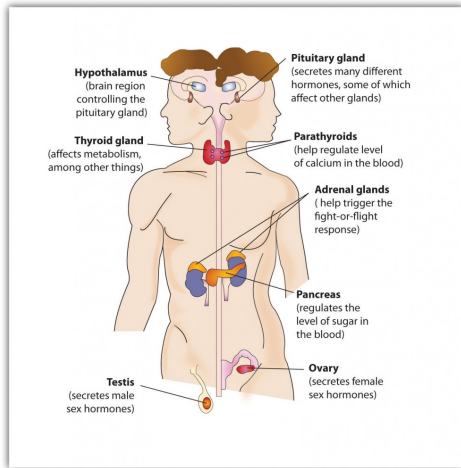
# Challenge

Integration of experimental data (in vitro and in vivo) into a system's understanding at **different scales in space and time**



- Why modelling hormonal regulation?
- Physiological background
- Experimental data *in vivo*
- Model development for the human menstrual cycle
- Conclusion and outlook

# Hormones

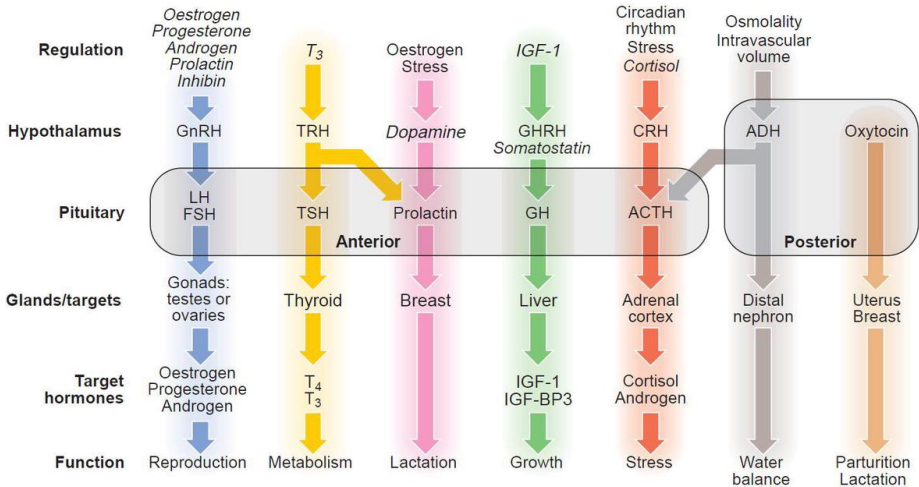


- hormones control a lot of functions: sexual reproduction and development, whole-body metabolism, blood glucose levels, plasma calcium concentration, growth,...
- hormones are produced in, and released from, diverse places
- they are carried in the bloodstream and capable of acting on target cells throughout the body

[<https://opentextbc.ca/introductiontopsychology/chapter/>

[3-4-putting-it-all-together-the-nervous-system-and-the-endocrine-system/](#)]

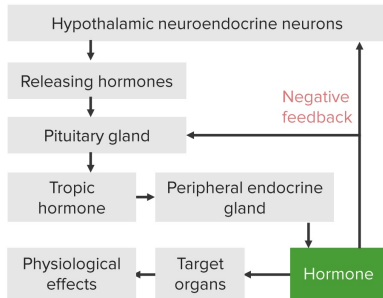
# Hormonal axes



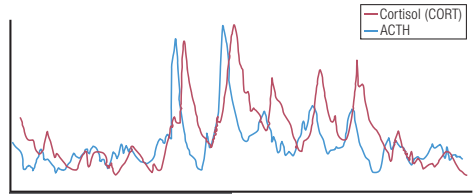
Source: Davidson's Essentials of Medicine, 2nd ed.

# Hormonal regulation

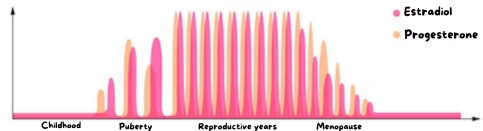
Feedback mechanisms lead to **oscillatory behavior** (milliseconds to days)  
→ significant implications for treatment



[<https://www.lecturio.com/concepts/hypothalamic-and-pituitary-hormones/>]



[Lightman et al. (2020), doi: 10.1210/endrev/bnaa002]



# Why modelling?

- endocrine axes are largely studied in isolation (reductive approach)
- dynamic changes in hormone levels are not captured well by single-point measurements
- challenging to collect longitudinal data on long-time processes like puberty and menopausal transition
- under-representation of women in clinical studies due to (i) potential maternal-fetal liability and (ii) the menstrual cycle as confounding variable



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- allows to formulate and test **hypothesis** *in-silico* (answer various “what if?” scenarios), including those that are costly, challenging or not feasible in-vitro or in-vivo

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## Mathematical modelling...

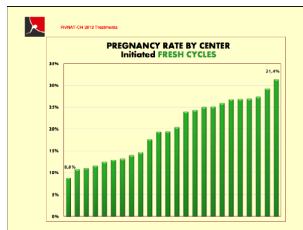
- allows to formulate and test **hypothesis** *in-silico* (answer various “what if?” scenarios), including those that are costly, challenging or not feasible in-vitro or in-vivo
- provides new information about potential **mechanisms** and can identify areas of deficient knowledge
- helps in quantifying inter- and intra-individual **variability**

Increased chance for successful pregnancy by modern techniques:

- In-vitro fertilization (IVF)
- Intracytoplasmic sperm injection (ICSI)

**Success rates:** 8 - 35%

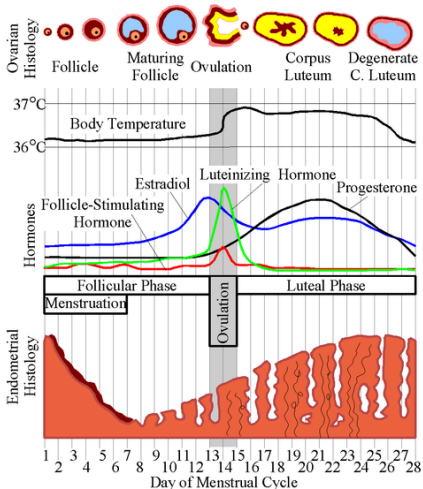
Depending on the clinic due to different treatment strategies!



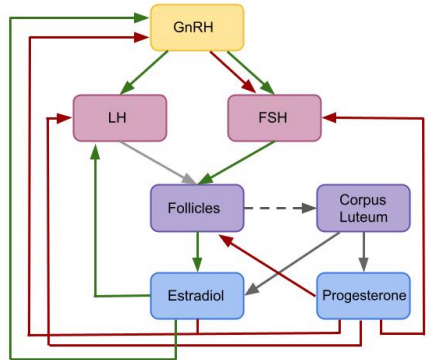
**Aim:** supply of model-based *clinical decision support system* for reproductive endocrinologists

- better **understanding** of complex processes
- simulation and optimization of **treatment strategies** *in silico* (cost-saving and efficient)

# The human menstrual cycle



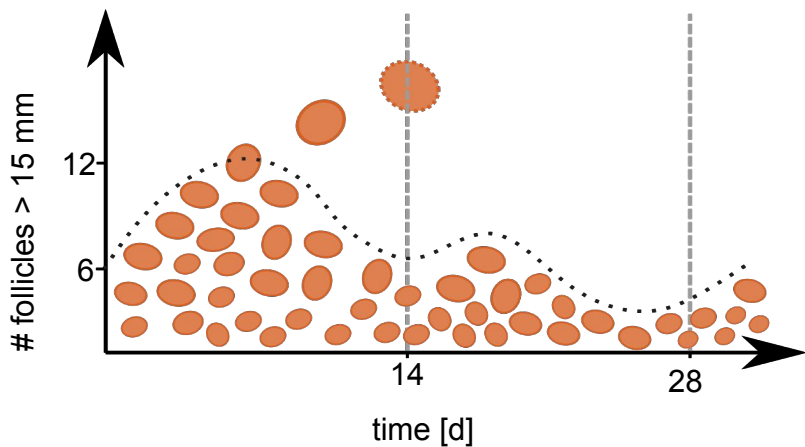
Orchestrated interplay of hormones along the HPG-axis:



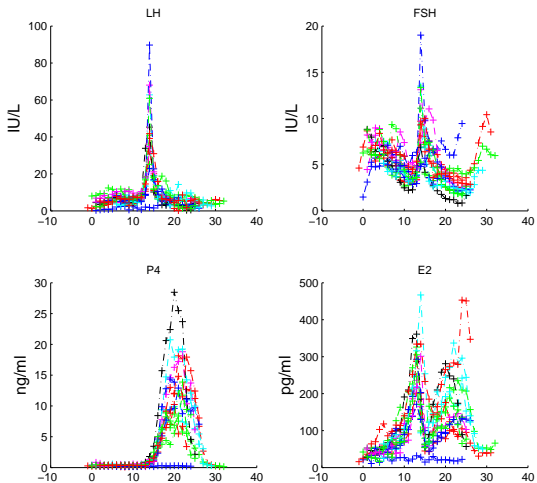
(Average values. Durations and values may differ between different females or different cycles.)

[Chris 73 / Wikimedia Commons]

# Follicular development



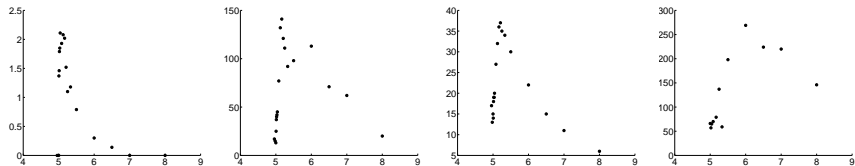
# Clinical trials: Hormone blood data from healthy women



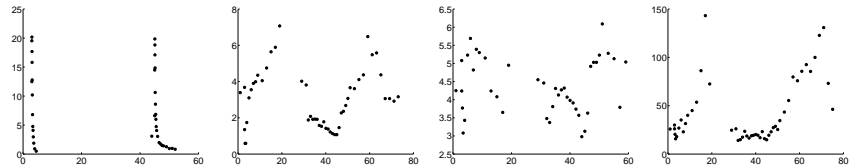


# Clinical trials: Pharmacokinetik (PK) data

Single dose Nafarelin (GnRH agonist):

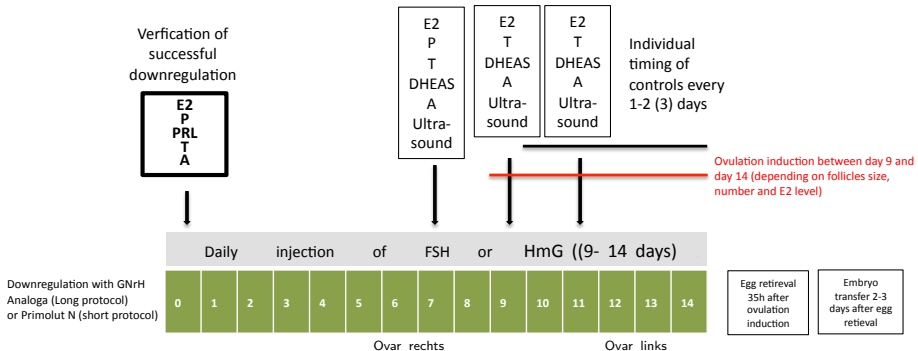


Single and multiple dose Cetorelix (GnRH antagonist):



measurements: drug, LH, FSH, E2

# Ovarian stimulation: Treatment protocol data



Downregulation with GnRH Analoga (Long protocol) or Primolut N (short protocol)

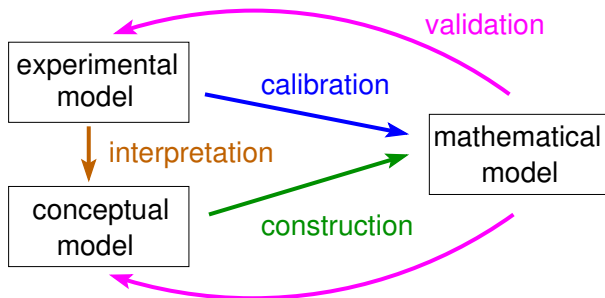
Ovulation induction between day 9 and day 14 (depending on follicles size, number and E2 level)

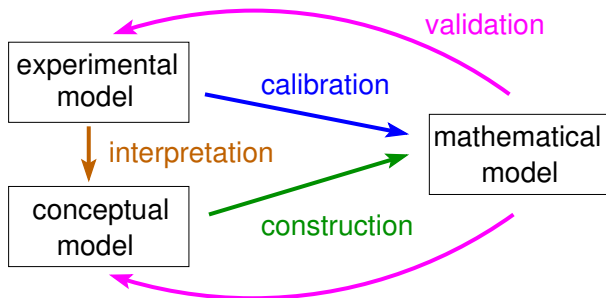
GnRHa	hMG/FSH	Tag	Datum	BT	E2 pmol/L	P4 nmol/L	Ovar rechts										Ovar links							
							< 10	10-11	12-13	14-15	16-17	18-19	≥ 20	< 10	10-11	12-13	14-15	16-17	18-19	≥ 20				
1	225	Fr	07.06.13	8	2841		4	1	1								5	1	1					
1	225	Sa	08.06.13	9																				
1	225	So	09.06.13	10																				
1	225	Mo	10.06.13	11	6062		2		1				1	1			2	3	5		3	3		

# Typical problems

- time series data of **a few** components
- **many** patients, but **few** data points per patient, mostly under treatment
- high inter- and intra-individual **variability**
- different physical units, sometimes **not even convertible**
- missing **measurement errors**
- missing information about the **cycle day**
- averaged data for women with different **cycle length** or in different stages of the cycle

# Modelling in biology and medicine





**“Essentially, all models are wrong, but some are useful”**

(George Box)

“fitness for purpose” rather than being “right or true”

**“A (mathematical) model should be as simple as possible, but not any simpler”**

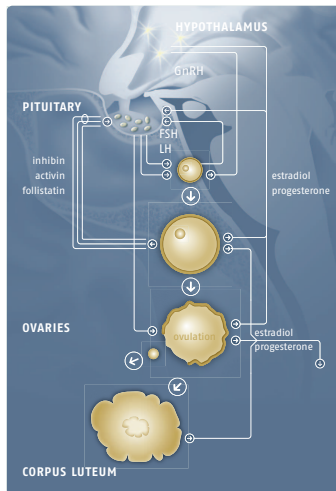
(A. Einstein)

# Conceptual model

**Compartments:** blood, ovaries, uterus, pituitary, hypothalamus

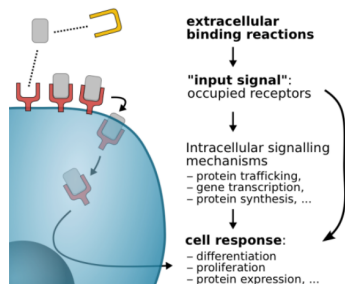
**Components:**

- Estradiol
- Progesterone
- Inhibin A and B
- LH + receptor binding
- FSH + receptor binding
- GnRH + receptor binding
- 6 follicular stages
- 6 luteal stages (corpus luteum)



# Mathematical model

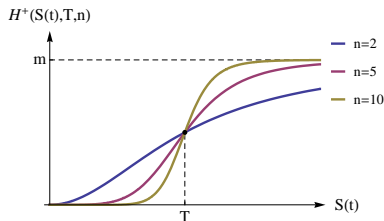
## Mass action kinetics:



[Pivonka et al., InTech 2012]

## Hill kinetics: stimulatory/inhibitory effects

$$H^+(S(t), T, n) := \frac{S(t)^n}{S(t)^n + T^n}$$



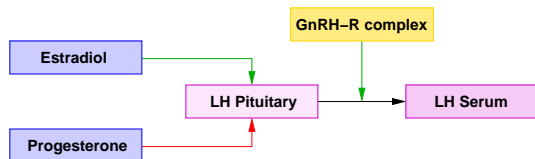
The model: ordinary differential equations (ODEs)

$$x'(t) = f(x(t), u(t), \theta), \quad x(0) = x_0$$

Experimental measurements (with additive Gaussian noise):

$$y(t) = h(x) + \epsilon(t), \quad \epsilon(t) \sim \mathcal{N}(0, \sigma^2(t))$$

# Example: Modelling of LH



$$Syn_{LH}(t) = (b_{Syn_{LH}} + m_{E2} \cdot H^+(E2, T_{E2}; n_{E2})) \cdot H^-(P4, T_{P4}; n_{P4})$$

$$Rel_{LH}(t) = (b_{Rel_{LH}} + m_{GnRH-R} \cdot H^+(GnRH-R, T_{GnRH-R}, n_{GnRH-R})) \cdot LH_{Pit}(t)$$

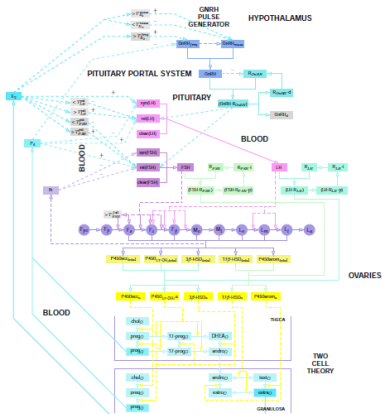
$$\frac{d}{dt} LH_{Pit}(t) = Syn_{LH}(t) - Rel_{LH}(t)$$

$$\frac{d}{dt} LH_{blood}(t) = \frac{1}{V_{blood}} Rel_{LH}(t) - k_{on} \cdot LH_{blood} \cdot R_{LH} - c \cdot LH_{blood}$$

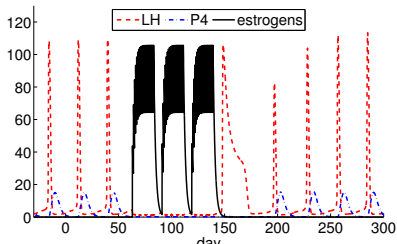


# The FemCyc model

**Purpose:** simulate the effect of birth control pill on hormone blood concentrations



49 DDEs, 208 parameters



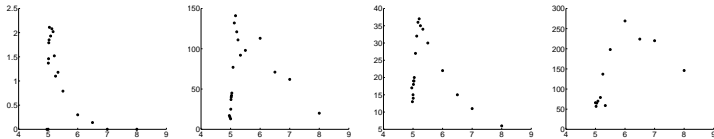
[I. Reinecke, P. Deufhard. A complex mathematical model of the human menstrual cycle. *J Theor Biol.* 2007; 247(2):303-30. doi: 10.1016/j.jtbi.2007.03.011]

# The GynCycle model

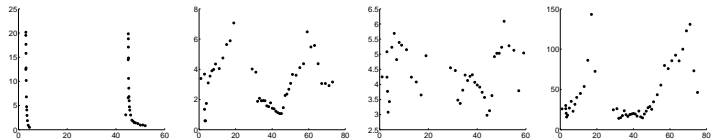
**Purpose:** development of a **pharmacokinetic/pharmacodynamic (PKPD)** model for single and multiple dose administration of **GnRH analogues** (in collaboration with Pfizer UK)

**Blood measurements** (drug, LH, FSH, E2):

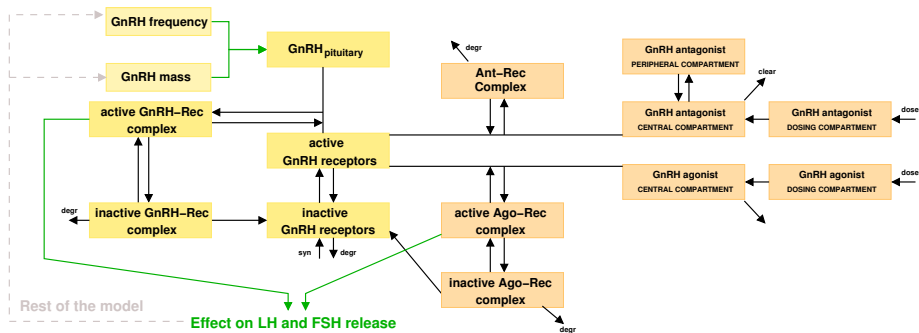
single dose Nafarelin (GnRH agonist)



single and multiple dose Cetorelix (GnRH antagonist)



# PKPD Modelling: GnRH agonists



G protein-coupled receptor (GPCR) model coupled to PK model for GnRH agonists and antagonists

PK description as unique parametrizations of important drugs:

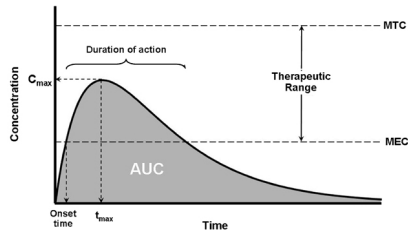
Administered compound	dimension	dose	beta	clearance rate
Triptorelin (Decapeptyl 0.1 mg)	mg	2.5	250.0	6.00
FSH (Merional 75 I.E.)	I.E.	5.347	4.271	0.488
FSH (Menopur 600 I.E.)	I.E.	13.378	9.871	0.417
FSH (Puregon/Gonal-f 600 I.E.)	I.E.	21.387	4.271	0.488
LH (Merional 75 I.E.)	I.E.	0.594	6.041	3.199
LH (Menopur 600 I.E.)	I.E.	5.669	6.041	3.199
LH (Ovitrelle 250)	I.E.	39.632	6.041	3.199
Norethisterone (Primolut N)	mg	27.291	52.324	11.090

<http://www.kompodium.ch/home/de>

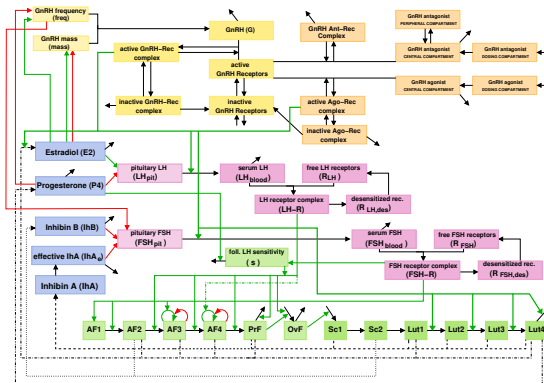
$$\frac{dc_{\text{drug}}(t)}{dt} = D\beta^2 t \exp(-\beta t) - c_L c_{\text{drug}}(t)$$

parameters can be determined via nonlinear equations from PK parameters

$t_{\text{max}}$ ,  $c_{\text{max}}$ ,  $AUC_{0-\infty}$



# The GynCycle model



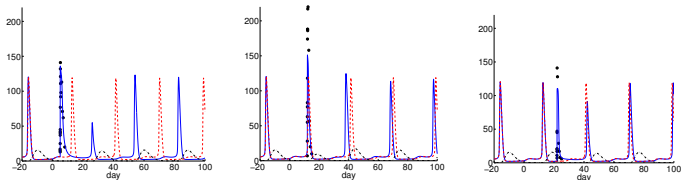
GynCycle: 33(+8) ODEs, 114 parameters

BioModels database: <http://biomodels.caltech.edu/BIOMD0000000494>

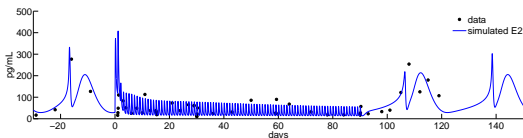
[S. Röblitz et al. A mathematical model of the human menstrual cycle for the administration of GnRH analogues. *J. Theoret. Biol.* 321:8–27, 2013. DOI: 10.1016/j.jtbi.2012.11.020]

The model **allows to**

- systematically study the influence of **drug, dose and timing of administration** on hormone profiles in a “normal” menstrual cycle



- study **long-time effect** of drug administration under different **compliance** behaviors

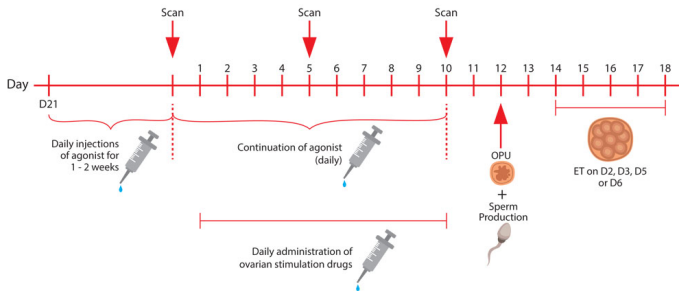


# The PAEON model

**Purpose:** Simulation of female fertility treatments

**downregulation + stimulation + oocyte retrieval**

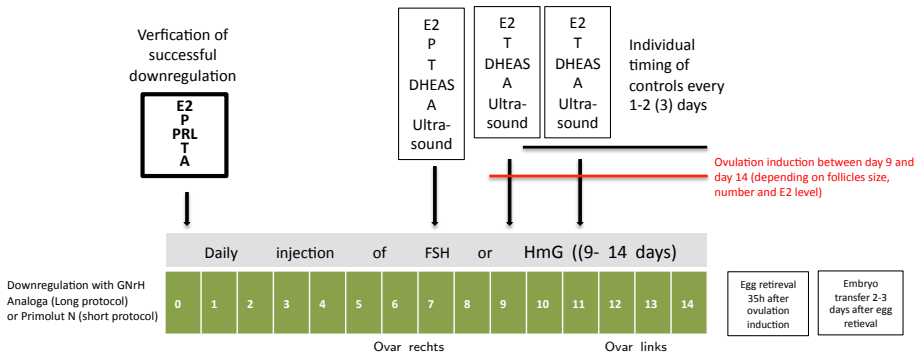
LONG DOWN REGULATION PROTOCOL



**Aim:** between 11 and 15 mature oocytes

**Risk:** Ovarian hyperstimulation syndrome (OHSS)

# Treatment protocol data

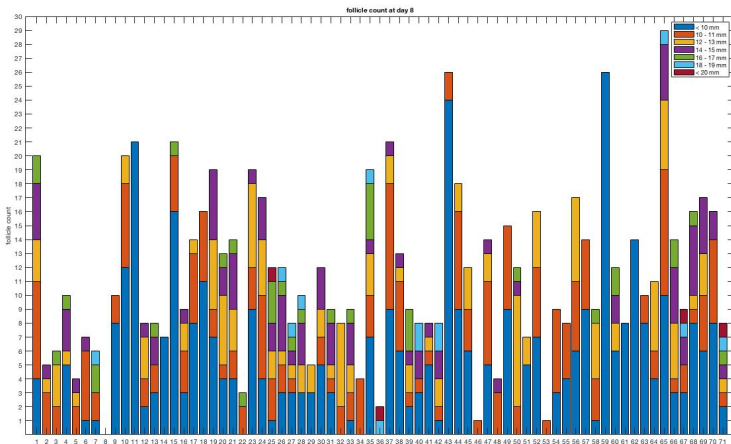


Downregulation with GnRH Analoga (Long protocol) or Primolut N (short protocol)

GnRHa	hMG/FSH	Tag	Datum	BT	E2 pmol/L	P4 nmol/L	Ovar rechts										Ovar links								
							< 10	10-11	12-13	14-15	16-17	18-19	≥ 20	< 10	10-11	12-13	14-15	16-17	18-19	≥ 20					
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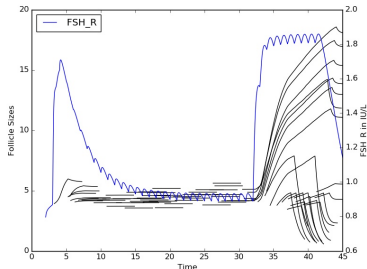
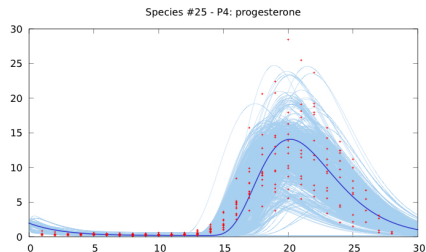
# Treatment cycle: ultrasound measurements



# The PAEON model

## Tasks:

- create a virtual patient population
- develop a new model for follicular development to simulate the stimulation phase



PAEON project: Model-Driven Computation of Treatments for Infertility Related Endocrinological Diseases (02/13-01/16)

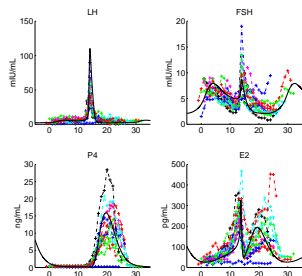
Partners: ZIB, La Sapienza U Rome, U Lucerne, U Hospital Zürich, Hannover Medical School

## Frequentists's approach:

$$\|F(\theta)\|_2^2 \xrightarrow{\theta} \min$$

with sum of least squares errors

$$\|F(\theta)\|_2^2 = \sum_{k=1}^n \sum_{l=1}^{m_k} \frac{(z_{kl} - y_k(t_l, \theta))^2}{2\sigma_{kl}^2}$$



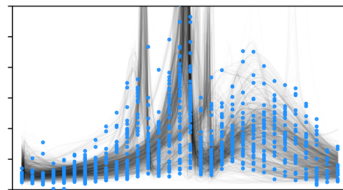
## Bayesian approach:

computation of probability distributions according to **Bayes' theorem**

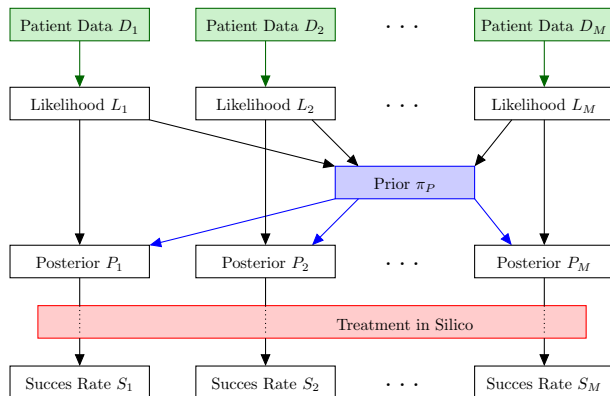
$$P(\theta|z) \propto P(z|\theta)P(\theta)$$

with likelihood

$$P(z|\theta) \propto \exp(-\|F(\theta)\|_2^2)$$



# Prior estimation

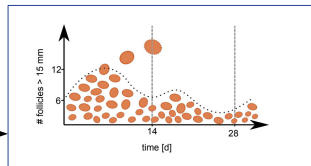
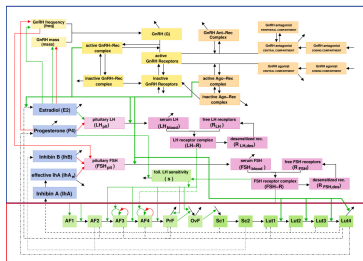


We introduced a nonparametric, transformation-invariant estimator for the prior distribution defined in terms of the missing information.

[Klebanov et al. Objective priors in the empirical Bayes framework. *Scand J Statistics* 48(4), 2021.]

# A new follicle model

Replacing the “old” follicle model with a new one...



# A new follicle model

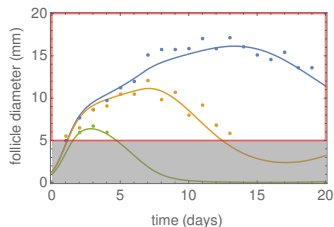
**Purpose:** simulate the competitive growth of multiple follicles

$$x_i' = x_i(\xi - x_i)(\gamma - \kappa(\sum_j x_j^\nu - \mu x_i^\nu)), \quad x_i(0) \in (0, \xi)$$

- $x_i(t)$ [mm]: diameter of follicle  $i = 1, \dots, n$
- $\xi$ [mm]: upper limit for the size of a follicle (usually 20 mm)
- $\gamma$ [1/(mm · d)]: individual growth rate
- $\kappa$ [1/(mm<sup>3</sup> · d)]: strength of competition
- $\mu \in (0, 1)$ : proportion of self-harm
- $\nu$ : fractal dimension

Fit to bovine ultrasound data:

[S. Cummins et al. *J Dairy Science* 95(7), 2012]



Number of dominant follicles:

$$d = \left\lceil \mu + \frac{\gamma}{\kappa \xi^\nu} \right\rceil$$

[Lange et al. (2019). doi: 10.1007/s00285-018-1284-0]

# A hormone-dependent follicle model

## Biological knowledge:

- FSH concentrations need to surpass a distinct level to stimulate ovarian follicle growth (**FSH threshold concept**).
- A limited duration of elevated FSH levels above the threshold is needed for single dominant follicle selection (**FSH window concept**).
- **Progesterone** exerts an inhibitory action on follicular development.
- There is **follicle-to-follicle variability** in the response to FSH.

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- **Progesterone** exerts an inhibitory action on follicular development.
- There is **follicle-to-follicle variability** in the response to FSH.

## Model assumptions:

- The **FSH-receptor complex** level regulates growth and selection.
- High system level **FSH** inhibits competition.
- **Progesterone** inhibits the follicular growth rate.
- Follicles have **individual FSH sensitivity thresholds**.

$$\frac{d}{dt}x_i = H^+(\text{FSHR}, T_{\text{FSHR}}(i), n_{\text{FSHR}}) \cdot (\xi - x_i)x_i \left( \gamma - \kappa \left( \sum x^2 - x_i^2 \right) \right)$$

$$\kappa = \kappa_0 \cdot H^-(\text{FSH}, T_{\text{FSH}}^\kappa, n_{\text{FSH}}^\kappa)$$

$$\gamma = \gamma_0 \cdot H^-(P_4, T_{P_4}^\gamma, n_{P_4}^\gamma) \cdot H^+(\text{FSHR}, T_{\text{FSHR}}^\gamma, n_{\text{FSHR}}^\gamma).$$



# A stochastic follicle model

## Biological knowledge:

- The higher the FSH blood level, the more follicles are recruited.
- Follicular atresia is irreversible.

# A stochastic follicle model

## Biological knowledge:

- The higher the **FSH** blood level, the more follicles are **recruited**.
- **Follicular atresia** is irreversible.

## Model assumption:

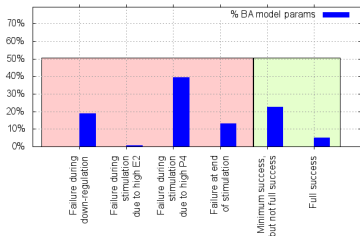
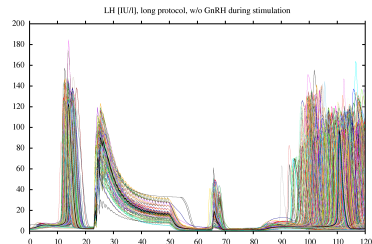
- The **recruitment** of follicles follows a Poisson process. The Poisson parameter  $\lambda$  (expected number of follicles that start growing within a certain time interval) is modulated by the **FSH** concentration.

$$\lambda = \lambda_0 \cdot (1 + s_{FSH}^{Pois} \cdot H^+(FSH(T), T_{FSH}^{Pois}, n_{FSH}^{Pois}))$$

- Four possible follicular destinies:
  - (i) growth ( $x_i'(t) \geq 0$ )
  - (ii) ovulation ( $x_i > 18\text{mm}$ ,  $LH \geq T_{LH} = 25 \text{ mIU/mL}$ )
  - (iii) decay ( $x_i'(t) < 0$ )
  - (iv) large ( $x_i > 18\text{mm}$ ) for 2 days, but not ovulating ( $LH < T_{LH}$ )

Follicles (ii)-(iv) are removed from the simulation.

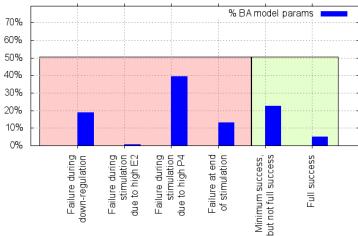
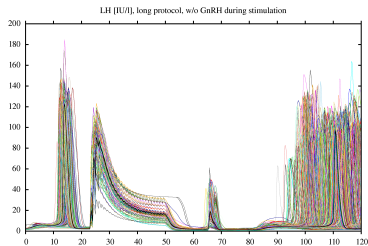
## Treatment Verification



[Tronci et al. (2014). doi: 10.1109/FMCAD.2014.6987615] [Mancini et al. (2018). doi: 10.29007/g864]

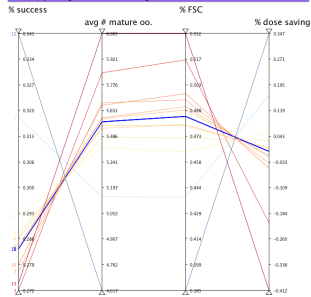
# The PAEON model: Results

## Treatment Verification



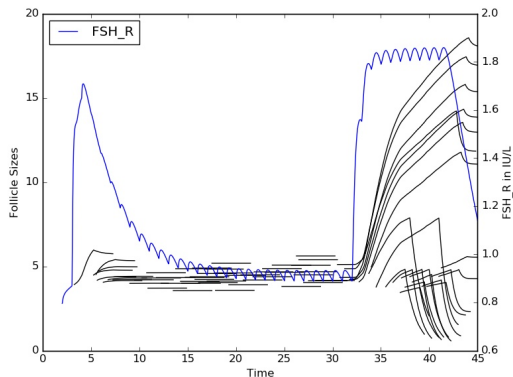
## Design & Optimization

w.r.t. **efficacy, cost, safety**



[Tronci et al. (2014). doi: 10.1109/FMCAD.2014.6987615] [Mancini et al. (2018). doi: 10.29007/g864]

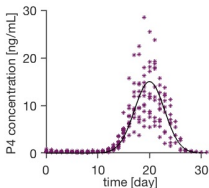
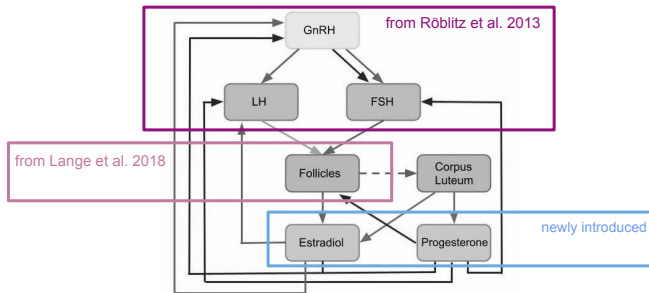
# Follicular dynamics under treatment



**Note!** The PAEON model is not fully coupled and can only be used to simulate follicular growth dynamics under GnRH agonist treatment when the feedback loop is interrupted due to GnRH-receptor downregulation in the pituitary.

# A fully coupled model

**Purpose:** Simulation of hormone dynamics and follicular growth throughout consecutive cycles

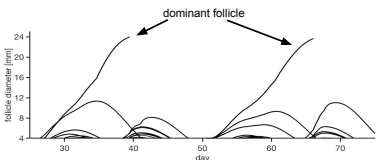
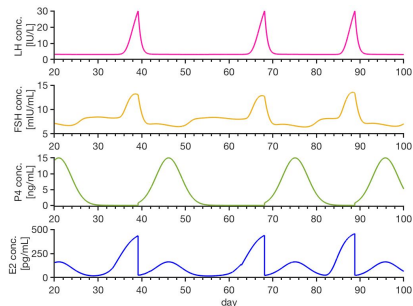


$$FS = \pi \cdot \sum H^+(x_i, T_{FS}, n_{FS}) \cdot (x_i)^2$$

$$E2(t) = b_{syn}^{E2} + s_{FS} \cdot FS + h_{E2} \cdot \exp\left(-w_{E2}(t - (T_{Ovu} + \tau))^2\right)$$

$$P4(t) = b_{syn}^{P4} + h_{P4} \cdot \exp\left(-w_{P4}(t - (T_{Ovu} + \tau))^2\right)$$

# Unstimulated cycle

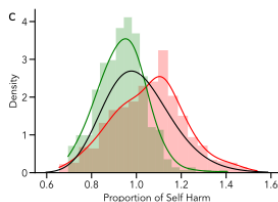
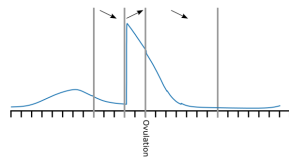


- the model generates **quasi-periodic** solutions for all four hormones
- **wave-like growth** behavior of the follicles (not enforced by implementation!)
- ovulation of a dominant follicle 12 h after the LH peak
- **variability** in cycle length ( $30.56 \pm 7.00$ ) and number of follicles per cycle ( $16.19 \pm 3.08$ ):
- no correlation between cycle length and follicular count

[S. Fischer et al. (2021). DOI: 10.3389/fendo.2021.613048]

# Parameter space exploration

- the likelihood  $P(z|\theta)$  is difficult to compute for a **stochastic model**
- → Approximate Bayesian Computation (ABC) rejection algorithm with summary statistics and log-normal prior
- acceptance criteria:
  - characteristic FSH profile in  $> 80\%$  of the cycles
  - mean cycle length and standard deviation within physiological ranges



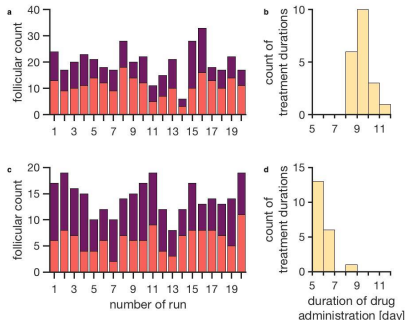
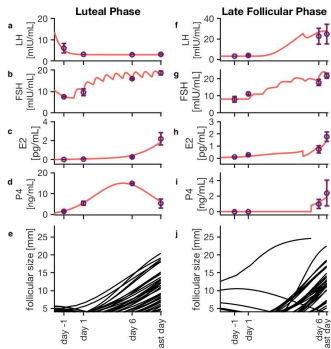
## Results:

- $\mu$  (proportion of self-harm) is lower in normal cycles and negatively correlated with cycle length

[S. Fischer et al. (2022). DOI: 10. 1016/j.jtbi.2022.111150]



# Random start ovarian stimulation



	Luteal Phase Stimulation [Kuang et al.]	Simulation	Late Follicular Phase Stimulation [Zhu et al.]	Simulation
# follicles 10 - 14 mm	13.9 ± 7.8	11.1 ± 3.5		6.3 ± 2.2
# follicles > 14 mm	11.1 ± 5.5	8.9 ± 3.7	11.7 ± 6.2	8.0 ± 2.2
treatment duration	10.2 ± 1.6	9.4 ± 0.7	10.93 ± 1.66	6.0 ± 0.7

[X. Zhu, Y. Fu. (2019). doi: 10.3389/fendo.2019.00448; Y. Kuang et al. (2014). doi: 10.1016/j.fertnstert.2013.09.007; S. Fischer et al. (2021). DOI: 10.3389/fendo.2021.613048]

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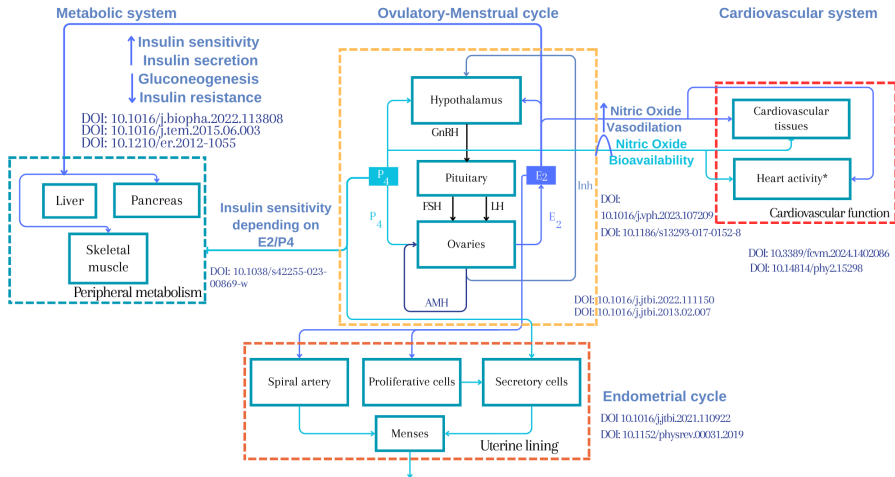
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- Model simulations confirm that **ovarian hyperstimulation** can be started at random time points in the cycle.



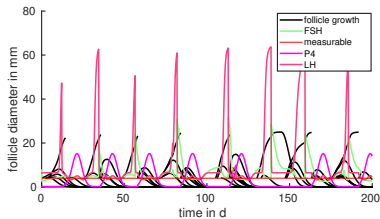
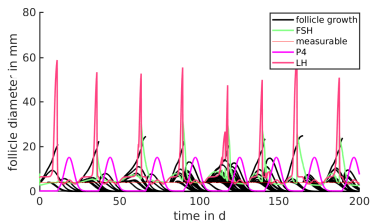
The hypothalamic-pituitary-ovarian axis interacts with other endocrine sub-systems!



## Hormonal rhythms change with age and during pregnancy!



## Simulation of menopausal transition by reducing the follicular recruitment rate:



## Key publications

- S. Fischer-Holzhausen, S. Röblitz. Hormonal regulation of ovarian follicle growth in humans: Model-based exploration of cycle variability and parameter sensitivities. *Journal of Theoretical Biology* 547:111150, 2022. <https://doi.org/10.1016/j.jtbi.2022.111150>
- S. Fischer, R. Ehrig, S. Schäfer, E. Tronci, T. Mancini, M. Egli, F. Ille, T. H. C. Krüger, B. Leeners, S. Röblitz. Mathematical Modeling and Simulation Provides Evidence for New Strategies of Ovarian Stimulation. *Frontiers in Endocrinology* 12:613048, 2021. <https://doi.org/10.3389/fendo.2021.613048>
- A. Lange, R. Schwieger, J. Plöntzke, S. Schäfer and S. Röblitz. Follicular competition in cows: The selection of dominant follicles as a synergistic effect. *Journal of Mathematical Biology* 78(3):579–606, 2019. <https://doi.org/10.1007/s00285-018-1284-0>
- S. Röblitz, C. Stötzel, P. Deuflhard, H. M. Jones, D.-O. Azulay, P. van der Graaf, and S. W. Martin. A mathematical model of the human menstrual cycle for the administration of GnRH analogues. *J. Theoret. Biol.* 321:8–27, 2013. <https://doi.org/10.1016/j.jtbi.2012.11.020>.

# Acknowledgment



Sophie Fischer

## Collaborators:



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Enrico Tronci (La Sapienza U Rome)



Carolina Ramirez Mazo (UNAL/CBU)

