Lecture 1: Freeform design

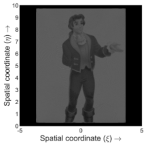
48th Woudschoten Conference, 25–27 September 2024

Martijn Anthonissen

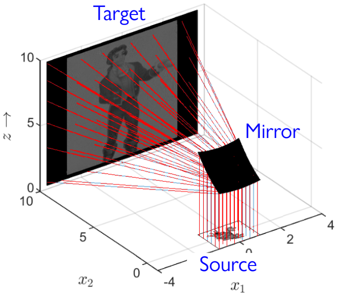
Computational Illumination Optics Group

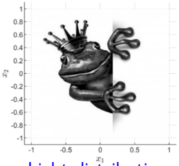
EINDHOVEN UNIVERSITY OF TECHNOLOGY

Can we turn a frog into a prince?



Light distribution target





Light distribution source

- *"In the original Grimm version of the story, the frog's spell was broken when the princess threw the frog against the wall, at which he transformed back into a prince, while in modern versions the transformation is triggered by the princess kissing the frog."*

  https://en.wikipedia.org/wiki/The_Frog_Prince

- Can we do it with light?

Romijn, L. B. (2021)
Generated Jacobian Equations in Freeform Optical Design: Mathematical Theory and Numerics
PhD thesis, Eindhoven University of Technology

Outline

Computational illumination optics at TU/e

Nonimaging freeform optics

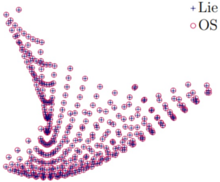
One mathematical framework for basic systems

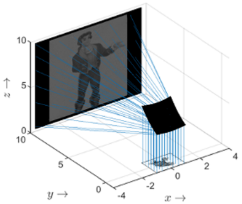
Iterative least-squares solver for 3D systems

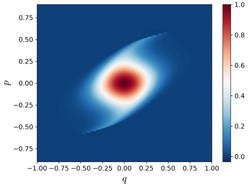
Numerical results

Beyond the basic systems

Conclusions and future work







Computational Illumination Optics Group at TU/e


Martijn Anthonissen


Wilbert IJzerman
Signify Research


Lisa Kusch

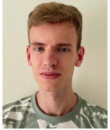

Koondi Mitra


Jan ten Thije Boonkkamp

OPTIC
AWAVE
Math & CS
OTP


Antonio Barion


Pieter Braam


Roel Hacking


Willem Jansen


René Köhle


Sanjana Verma


2024
Teun van Roosmalen

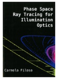

2024
Vì Kronberg


2023
Maikel Bertens


2023
Robert van Gestel

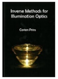

2021
Lotte Romijn


2018
Nitin Yadav



2018
Carmela Filosa


2017
Bart van Lith

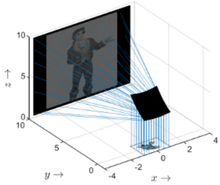


2014
Corien Prins

Lines of research

Line A

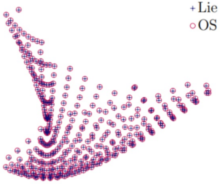
Nonimaging freeform optics



- Luminaires, street lights, ...
- Compute optical surfaces that convert given source into desired target distribution
- Freeform surfaces
- Fully nonlinear PDE of Monge-Ampère type

Line B

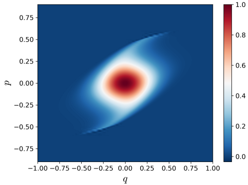
Imaging optics



- Cameras, telescopes, ...
- Make a very precise image of an object, minimizing aberrations
- Description with Lie transformations

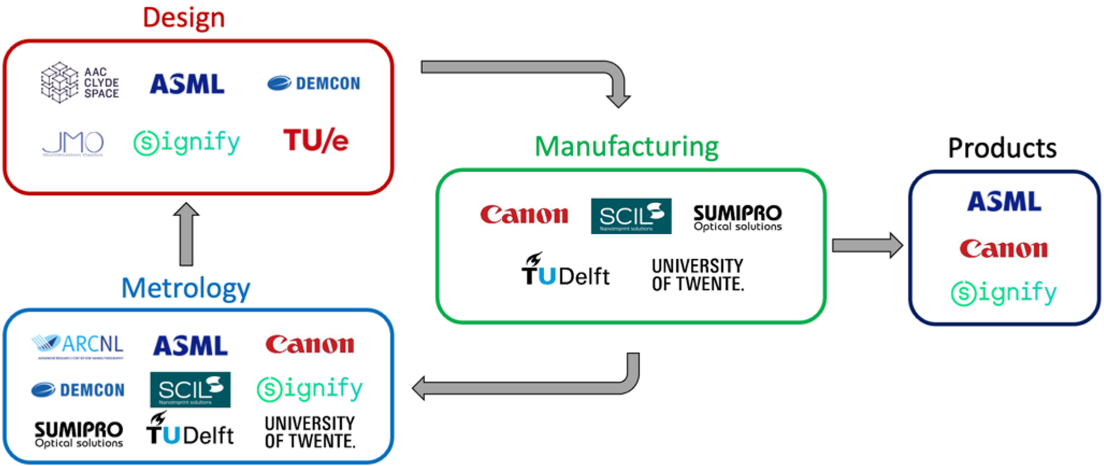
Line C

Improved direct methods



- Ray tracing: iterative procedure to compute final design. Slow convergence
- Advanced numerical schemes for Hamiltonian systems and Liouville's equation

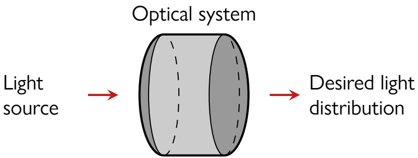
Academic cooperation and industrial embedding



TU/e

Nonimaging freeform optics

Design of optical systems for illumination purposes



Industry standard: ray tracing

- Easy to implement
- Slow convergence
- Manual adjustments

Inverse methods

- Directly compute required optical system
- Need solving PDE of Monge-Ampère type
- Avoid iterations and manual optimization

Sixteen basic systems

Parallel target | Point target | Near-field target | Far-field target

Reflector systems

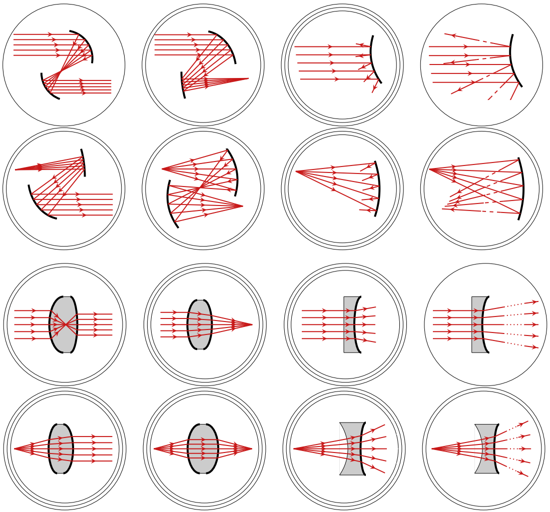
Parallel source

Point source

Lens systems

Parallel source

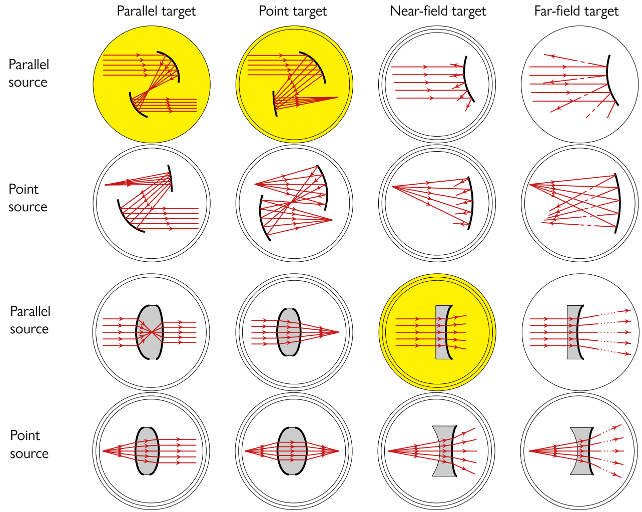
Point source



Sixteen basic systems

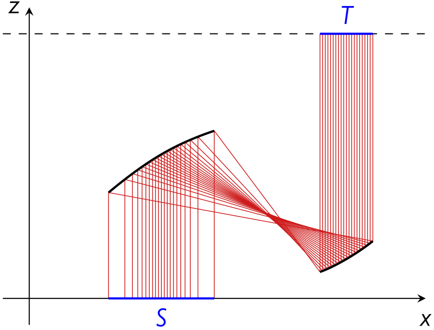
Reflector systems

Lens systems



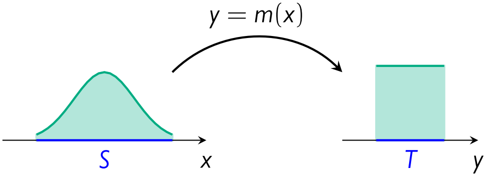
Parallel-to-parallel reflector 2D

SMA



- Source: parallel beam with light distribution $f(x)$, $x \in S$
- Target: parallel beam with light distribution $g(y)$, $y \in T$
- **Goal:** Find the two freeform reflector surfaces

- For the figure we used



TU/e

Mathematical model

- Path of a ray
  - Leaves source S at $P = (x, 0)$
  - Hits first reflector at $A = (x, u(x))$
  - Hits second reflector at $B = (y, L - w(y))$
  - Arrives at target T at $Q = (y, L)$
- **Optical path length**

$$V = u(x) + d(A, B) + w(y)$$

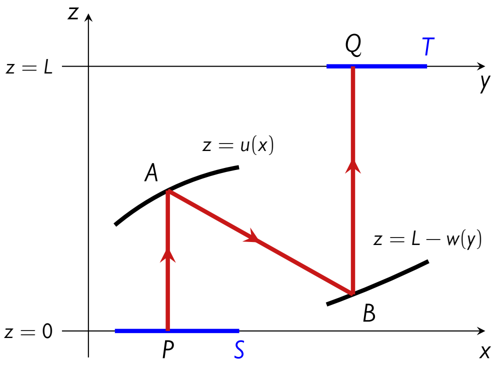
where $d(A, B)^2 = (y - x)^2 + (L - w(y) - u(x))^2$

- Eliminate $d(A, B)$ and rewrite as

$$u(x) + w(y) = \tfrac{1}{2}(V - L) + L - \frac{(y - x)^2}{2(V - L)}$$

$$=: c(x, y)$$

$c(x, y)$ is called cost function
Here: c is quadratic function



Optimal transport formulation

- Cost function relation

$$u(x) + w(y) = c(x, y)$$

Many solution pairs $(u(x), w(y))$

- Special choice: c-convex pair

$$u(x) = \max_{y \in T} \big(c(x, y) - w(y)\big)$$

$$w(y) = \max_{x \in S} \big(c(x, y) - u(x)\big)$$

- Necessary condition:

$$\frac{\partial c}{\partial x}(x, y) - u'(x) = 0$$

- For the current optical system, we have

$$c(x, y) = \tfrac{1}{2}(V - L) + L - \frac{(y - x)^2}{2(V - L)}$$

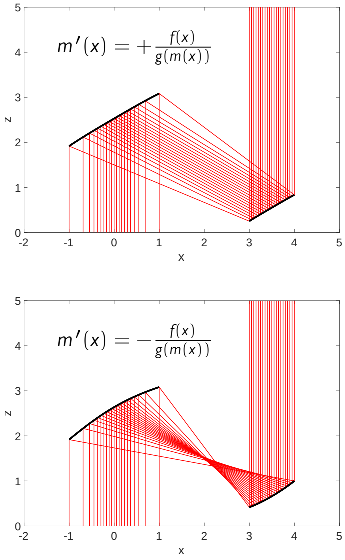
so

$$y = m(x) = x + (V - L)u'(x)$$

This is called the **optical mapping** $y = m(x)$

Energy conservation

SMA



- Source light distribution: $f = f(x)$, $x \in S$
  Target light distribution: $g = g(y)$, $y \in T$
  In the figure: $S = (-1, 1)$, $T = (3, 4)$
- Energy conservation:

$$\int_{-1}^{x} f(\xi)\, d\xi = \pm \int_{m(-1)}^{m(x)} g(y)\, dy = \pm \int_{-1}^{x} g(m(\xi))\, m'(\xi)\, d\xi$$

- Differentiate to x:

$$m'(x) = \pm \frac{f(x)}{g(m(x))}$$

  Solve ODE for m
- Differentiate $u(x) + w(y) = c(x, y)$ to x, substitute $y = m(x)$:

$$u'(x) = \frac{\partial c}{\partial x}(x, m(x))$$

  Solve ODE for u
- Second reflector: $w(m(x)) = c(x, m(x)) - u(x)$

TU/e

Parallel-to-parallel reflector, 2D and 3D

SMA

2D

- Optical mapping:

$$y = m(x) = x + (V - L)u'(x) = \phi'(x)$$

- Optimal transport formulation:

$$u(x) + w(y) = c(x, y)$$
$$c(x, y) = \tfrac{1}{2}(V - L) + L - \frac{(y - x)^2}{2(V - L)}$$

- Energy conservation:

$$m'(x) = \phi''(x) = \pm \frac{f(x)}{g(m(x))}$$

3D: $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad \mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$

- Optical mapping:

$$\mathbf{y} = \mathbf{m}(\mathbf{x}) = \mathbf{x} + (V - L)\nabla u(\mathbf{x}) = \nabla\phi(\mathbf{x})$$

- Optimal transport formulation:

$$u(\mathbf{x}) + w(\mathbf{y}) = c(\mathbf{x}, \mathbf{y})$$
$$c(\mathbf{x}, \mathbf{y}) = \tfrac{1}{2}(V - L) + L - \frac{\|\mathbf{y} - \mathbf{x}\|^2}{2(V - L)}$$

- Energy conservation:

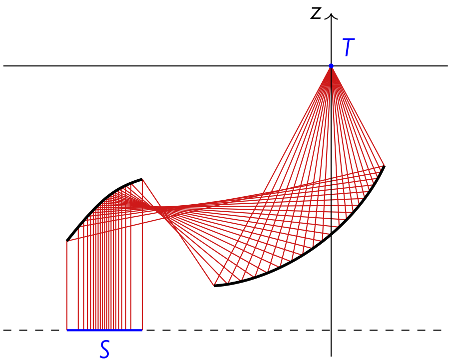
$$\det(\mathrm{D}\mathbf{m}(\mathbf{x})) = \frac{\partial^2\phi}{\partial x_1^2}\frac{\partial^2\phi}{\partial x_2^2} - \left(\frac{\partial^2\phi}{\partial x_1\,\partial x_2}\right)^2 = \pm \frac{f(\mathbf{x})}{g(\mathbf{m}(\mathbf{x}))}$$

Standard Monge-Ampère equation

Second-order nonlinear PDE

Parallel-to-point reflector 2D

GMA



- Source: parallel beam with light distribution $f(x)$
- Target: point with light distribution $g(y)$
- **Goal:** Find the two freeform reflector surfaces

TU/e

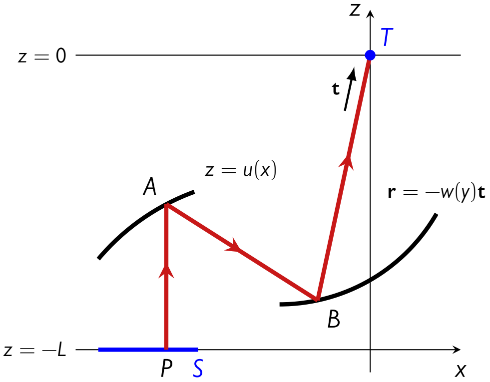
Mathematical model

GMA

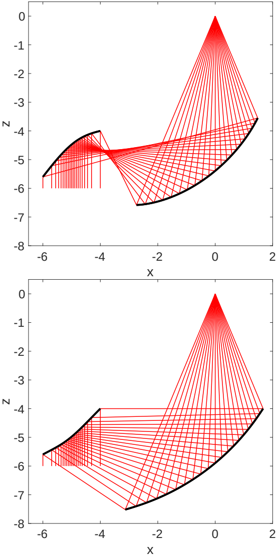
- Path of a ray
  - Leaves source S at $P = (x, -L)$
  - Hits first reflector at $A = (x, -L + u(x))$
  - Hits second reflector at $B = (-w(y)t_1, -w(y)t_2)$
  - Arrives at target $T = (0, 0)$
- Optical path length: $V = u(x) + d(A, B) + w(y)$
- Geometric relation ($\beta := V - L$)

$$\left(-\frac{u}{\beta} - \frac{x^2}{2\beta^2} + \frac{V+L}{2\beta}\right) \cdot \left(\frac{\beta}{w}(1+y^2) - 2y^2\right) = \left(1 + \frac{xy}{\beta}\right)^2$$

- Take logarithms: $u_1(x) + u_2(y) = c(x, y)$
  Cost function is non-quadratic



Numerical method



- Energy conservation:

$$m'(x) = \pm \frac{f(x)}{g(m(x))} \frac{1 + (m(x))^2}{2}$$

Solve ODE for m

- Differentiate $u_1(x) + u_2(y) = c(x, y)$ to x, substitute $y = m(x)$:

$$u_1'(x) = \frac{\partial c}{\partial x}(x, m(x))$$

Solve ODE for u_1
From u_1 compute u

- Second reflector:

$$u_2(m(x)) = c(x, m(x)) - u_1(x)$$

From u_2 compute w

Parallel-to-point reflector, 2D and 3D

GMA

2D

- Energy conservation:

$$m'(x) = \pm \frac{f(x)}{g(m(x))} \frac{1 + (m(x))^2}{2}$$

- Optimal transport formulation:

$$u_1(x) + u_2(y) = c(x, y)$$

$$c(x, y) = \log\left(\left(1 + \frac{xy}{\beta}\right)^2\right)$$

3D: $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad \mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$

- Energy conservation:

$$\det(\mathrm{D}\mathbf{m}(\mathbf{x})) = \pm \frac{f(\mathbf{x})}{g(\mathbf{m}(\mathbf{x}))} \frac{\left(1 + \|\mathbf{m}(\mathbf{x})\|^2\right)^2}{4}$$

- Optimal transport formulation:

$$u_1(\mathbf{x}) + u_2(\mathbf{y}) = c(\mathbf{x}, \mathbf{y})$$

$$c(\mathbf{x}, \mathbf{y}) = \log\left(\left(1 + \frac{\mathbf{x} \cdot \mathbf{y}}{\beta}\right)^2\right)$$

- Differentiate, substitute $\mathbf{y} = \mathbf{m}(\mathbf{x})$, differentiate

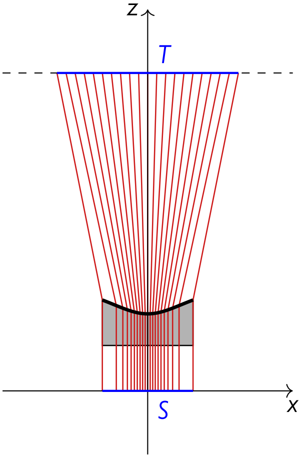
$$\underbrace{\mathrm{D}_{\mathbf{xy}} c(\mathbf{x}, \mathbf{m}(\mathbf{x}))}_{\mathbf{C}} \, \mathrm{D}\mathbf{m}(\mathbf{x}) = \underbrace{\mathrm{D}^2 u_1(\mathbf{x}) - \mathrm{D}_{\mathbf{xx}} c(\mathbf{x}, \mathbf{m}(\mathbf{x}))}_{\mathbf{P}}$$

- **Generalized Monge-Ampère equation**

$$\det(\mathrm{D}\mathbf{m}(\mathbf{x})) = \frac{\det(\mathbf{P})}{\det(\mathbf{C})} = \pm \frac{f(\mathbf{x})}{g(\mathbf{m}(\mathbf{x}))} \frac{\left(1 + \|\mathbf{m}(\mathbf{x})\|^2\right)^2}{4}$$

Parallel-to-near-field lens

GJE



- Source: parallel beam with light distribution $f(x)$
- Target: near-field with light distribution $g(y)$
- **Goal:** Find the single freeform lens surface

TU/e

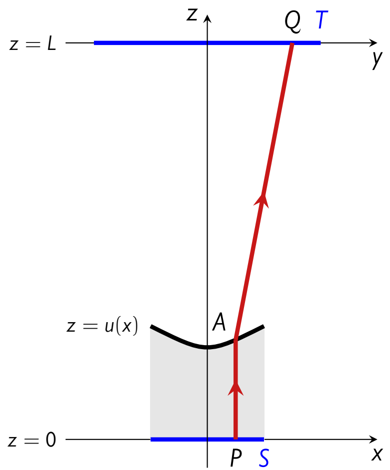
Mathematical model

GJE

- Path of a ray
  - Leaves source S at $P = (x, 0)$
  - Hits freeform lens surface at $A = (x, u(x))$
  - Arrives at target T at $Q = (y, L)$
- Optical path length:

$$\begin{aligned} V(y) &= n \cdot u(x) + d(A, Q) \\ &= n \cdot u(x) + \sqrt{(y - x)^2 + (L - u(x))^2} \end{aligned}$$

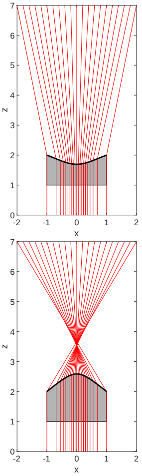
Cannot be formulated as optimal transport problem



TU/e

Numerical method

GJE



- Energy conservation:

$$m'(x) = \pm \frac{f(x)}{g(m(x))}$$

Solve ODE for m

- Geometric relation:

$$V(y) = n \cdot u(x) + \sqrt{(y-x)^2 + (L-u(x))^2}$$

differentiate to x, substitute $y = m(x)$, solve for $u'(x)$:

$$u'(x) = \frac{m(x) - x}{n \cdot \sqrt{(m(x)-x)^2 + (L-u(x))^2} + u(x) - L}$$

Solve ODE for u

TU/e

Parallel-to-near-field lens, 2D and 3D

GJE

2D

- Energy conservation:

$$m'(x) = \pm \frac{f(x)}{g(m(x))}$$

- Geometric relation:

$$V(y) = n \cdot u(x) + \sqrt{(y-x)^2 + (L-u(x))^2}$$

3D

- Energy conservation:

$$\det(\mathrm{D}\mathbf{m}(\mathbf{x})) = \pm \frac{f(\mathbf{x})}{g(\mathbf{m}(\mathbf{x}))}$$

- Geometric relation:

$$V(\mathbf{y}) = n \cdot u(\mathbf{x}) + \sqrt{\|\mathbf{y}-\mathbf{x}\|^2 + (L-u(\mathbf{x}))^2} =: H(\mathbf{x}, \mathbf{y}, u(\mathbf{x}))$$

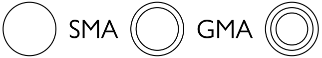
- Differentiate to $\mathbf{x}$: $\nabla_{\mathbf{x}} \big(H(\mathbf{x}, \mathbf{y}, u(\mathbf{x})) \big) = \mathbf{0}$
- Let $\widetilde{H}(\mathbf{x}, \mathbf{y}) := H(\mathbf{x}, \mathbf{y}, u(\mathbf{x}))$, substitute $\mathbf{y} = \mathbf{m}(\mathbf{x})$, differentiate to $\mathbf{x}$:

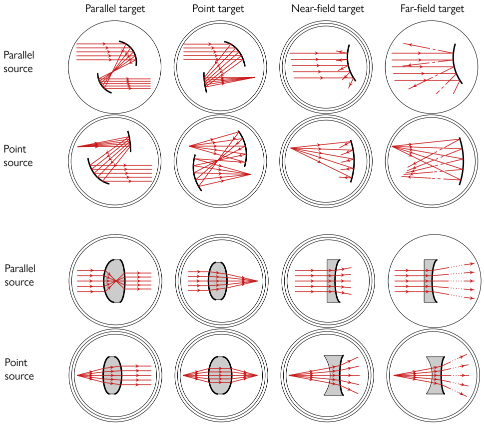
$$\mathrm{D}_{\mathbf{xx}} \widetilde{H}(\mathbf{x}, \mathbf{m}(\mathbf{x})) + \underbrace{\mathrm{D}_{\mathbf{xy}} \widetilde{H}(\mathbf{x}, \mathbf{m}(\mathbf{x}))}_{\mathbf{C}} (\mathrm{D}\mathbf{m})(\mathbf{x}) = \mathbf{O}$$

- **Generated Jacobian equation**

$$\det\big(\mathrm{D}_{\mathbf{xx}} \widetilde{H}(\mathbf{x}, \mathbf{m}(\mathbf{x}))\big) = \pm \det(\mathbf{C}) \frac{f(\mathbf{x})}{g(\mathbf{m}(\mathbf{x}))}$$

Unified mathematical framework





- **Energy conservation**

$$\det(\mathrm{D}\mathbf{m}) = F(\mathbf{x}, \mathbf{m}(\mathbf{x}))$$

- **Matrix equation optical map**

$$\mathbf{P} = \mathbf{C}\,\mathrm{D}\mathbf{m}$$

- **Model 1: Standard Monge-Ampère (SMA)**
  Optimal transport formulation, $c(\mathbf{x}, \mathbf{y})$ quadratic

$$\mathbf{C} = \mathbf{I} \qquad \mathbf{P} = \mathrm{D}^2 u$$

- **Model 2: Generalized Monge-Ampère (GMA)**
  Optimal transport formulation, $c(\mathbf{x}, \mathbf{y})$ non-quadratic

$$\mathbf{C} = \mathrm{D}_{\mathbf{xy}} c \qquad \mathbf{P} = \mathrm{D}^2 u_1 - \mathrm{D}_{\mathbf{xx}} c$$

- **Model 3: Generated Jacobian equation (GJE)**
  No optimal transport formulation

$$\mathbf{C} = \mathrm{D}_{\mathbf{xy}} \widetilde{H} \qquad \mathbf{P} = -\mathrm{D}_{\mathbf{xx}} \widetilde{H}$$

Anthonissen, M.J.H., Romijn, L.B., ten Thije Boonkkamp, J.H.M., IJzerman, W.L. (2021). *Unified mathematical framework for a class of fundamental freeform optical systems*, Optics Express, 29(20), 31650-31664, https://doi.org/10.1364/OE.438920

TU/e

Iterative least-squares solver for 3D systems

SMA GMA GJE

- Find mapping $\mathbf{m} : S \to T$ such that

$$\det(\mathrm{D}\mathbf{m}) = F(\mathbf{x}, \mathbf{m}(\mathbf{x}))$$
$$\mathbf{m}(\partial S) = \partial T$$

- Break down in substeps
  We compute $\mathbf{P}$, $\mathbf{b}$, $\mathbf{m}$ such that

$$\mathbf{P} = \mathbf{C}\,\mathrm{D}\mathbf{m}$$
$$\det(\mathbf{P}) = \det(\mathbf{C})\,F(\mathbf{x}, \mathbf{m}(\mathbf{x}))$$
$$\mathbf{b}(\mathbf{x}) = \mathbf{m}(\mathbf{x}) \qquad \mathbf{x} \in \partial S$$

  $\mathbf{b}$ maps ∂S to ∂T
- Find surface from $\nabla_{\mathbf{x}} c(\mathbf{x}, \mathbf{m}(\mathbf{x})) = \nabla u_1(\mathbf{x})$

- Iterative procedure:
  1. Choose an initial guess $\mathbf{m}^0$
     Let n = 0
  2. Let $J_I(\mathbf{m}, \mathbf{P}) = \frac{1}{2} \iint_S \|\mathbf{C}\,\mathrm{D}\mathbf{m} - \mathbf{P}\|^2 \, d\mathbf{x}$

     $$\mathbf{P}^{n+1} = \underset{\mathbf{P}}{\operatorname{argmin}}\; J_I(\mathbf{m}^n, \mathbf{P})$$

     Constrained minimization problem
  3. Let $J_B(\mathbf{m}, \mathbf{b}) = \frac{1}{2} \int_{\partial S} \|\mathbf{m} - \mathbf{b}\|^2 \, ds$

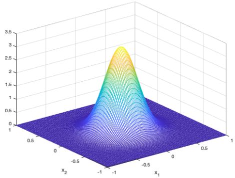
     $$\mathbf{b}^{n+1} = \underset{\mathbf{b}}{\operatorname{argmin}}\; J_B(\mathbf{m}^n, \mathbf{b})$$

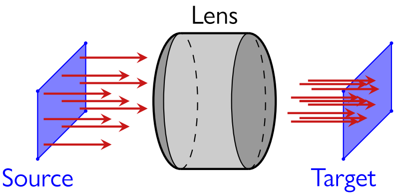
     Projection on boundary ∂T
  4. Let $J(\mathbf{m}, \mathbf{P}, \mathbf{b}) = \alpha J_I(\mathbf{m}, \mathbf{P}) + (1 - \alpha) J_B(\mathbf{m}, \mathbf{b})$

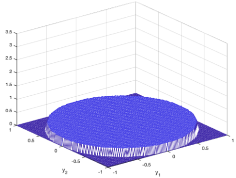
     $$\mathbf{m}^{n+1} = \underset{\mathbf{m}}{\operatorname{argmin}}\; J(\mathbf{m}, \mathbf{P}^{n+1}, \mathbf{b}^{n+1})$$

     Elliptic PDEs for m_1 and m_2 — FVM
  5. Let $n := n + 1$, go to Step 2

Laser beam shaping (parallel-to-parallel lens 3D)







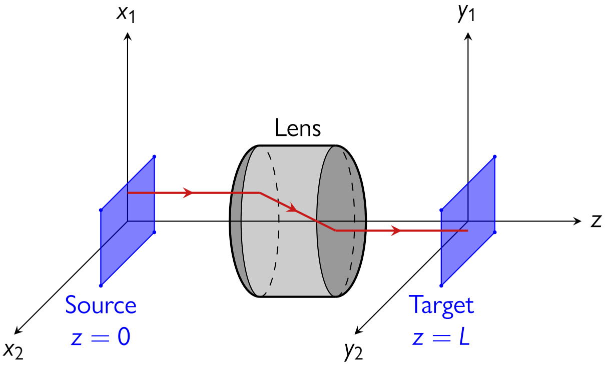
Source distribution:
Gaussian profile
Source emits parallel light rays

Optical system:
one lens with two
freeform surfaces

Desired target distribution:
circular top hat profile
Output beam: parallel light rays

Goal
Find the two freeform surfaces

Laser beam shaping



- Light rays travel from left to right
- Source plane: $z = 0$

  Cartesian coordinates $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

  Source S emits parallel light rays

  Light distribution: $f(\mathbf{x})$, $\mathbf{x} \in S$
- Target plane: $z = L$

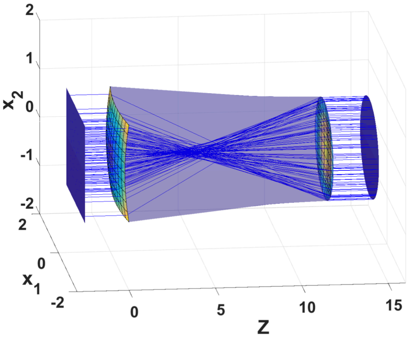
  Cartesian coordinates $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

  Light distribution: $g(\mathbf{y})$, $\mathbf{y} \in T$
- First lens surface: $z = u(\mathbf{x})$, $\mathbf{x} \in S$

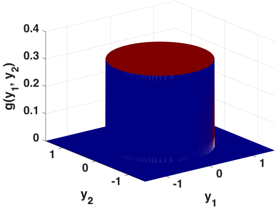
  Second lens surface: $L - z = w(\mathbf{y})$, $\mathbf{y} \in T$

TU/e

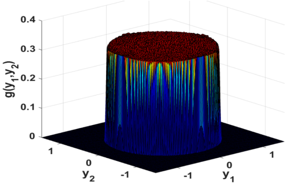
Numerical results for the laser beam shaping problem



Desired target distribution

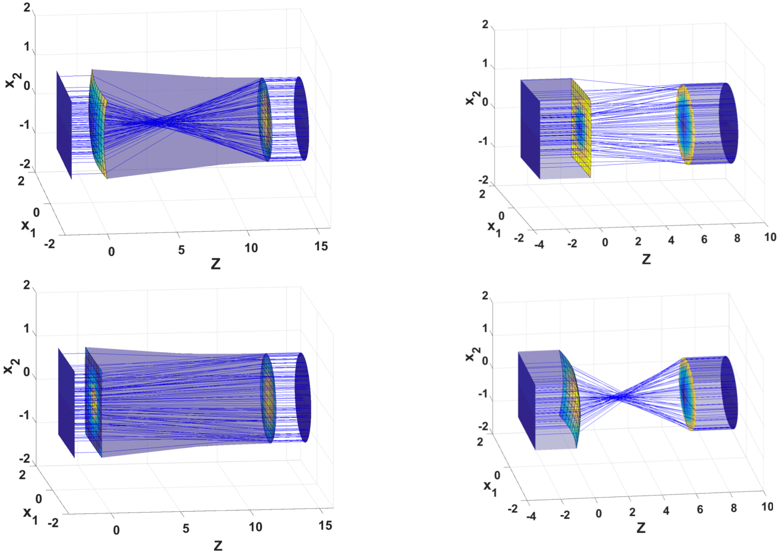


Achieved



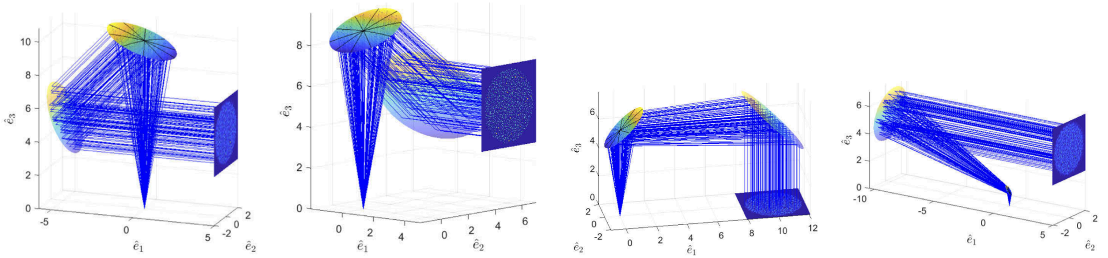
Yadav, N.K., ten Thije Boonkkamp, J.H.M., IJzerman, W.L. (2019)
Computation of double freeform optical surfaces using a Monge–Ampère solver: Application to beam shaping
Optics Communications, 439, 251-259
https://doi.org/10.1016/j.optcom.2019.01.069

Alternative optical systems



TU/e

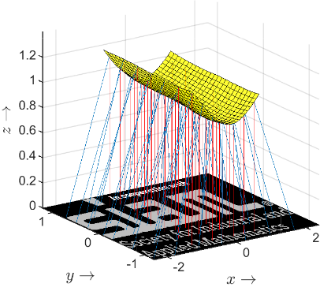
Point-to-parallel reflector system 3D



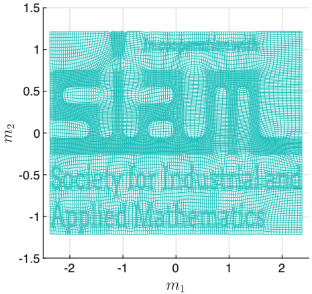
Van Roosmalen, A. H., Anthonissen, M. J. H., IJzerman, W. L., ten Thije Boonkkamp, J. H. M. (2021)
Design of a freeform two-reflector system to collimate and shape a point source distribution
Optics Express, 29(16), 25605-25625
https://doi.org/10.1364/OE.425289

TU/e

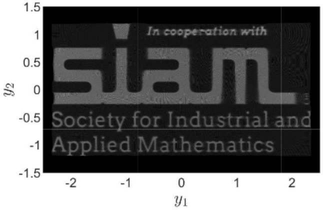
Parallel-to-near-field reflector 3D



Surface



Mapping



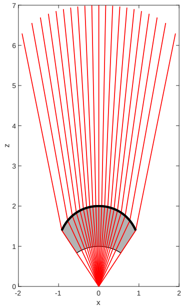
Ray trace

Romijn, L.B., Anthonissen, M.J.H., ten Thije Boonkkamp, J.H.M., IJzerman, W.L. (2021)
An iterative least-squares method for generated Jacobian equations in freeform optical design
SIAM Journal on Scientific Computing, 43(2), B298-B322
https://doi.org/10.1137/20M1338940

TU/e

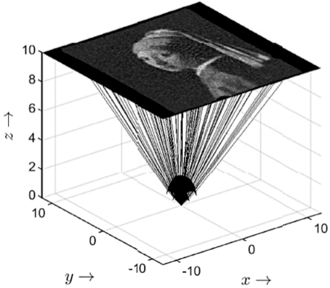
Beyond the basic systems: double freeform lens

Point-to-far-field lens requires only one freeform surface



Top surface: freeform
Bottom surface: spherical

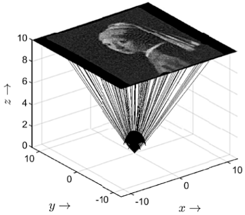
Idea: Use two freeform surfaces to distribute refractive power over lens surfaces



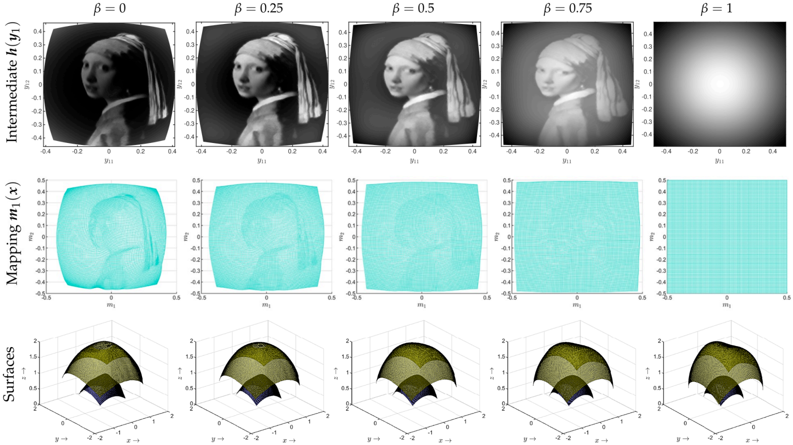
Point source with Lambertian light distribution
Far-field target with distribution derived from Vermeer's *Girl with a pearl earring*

TU/e

Beyond the basic systems: double freeform lens



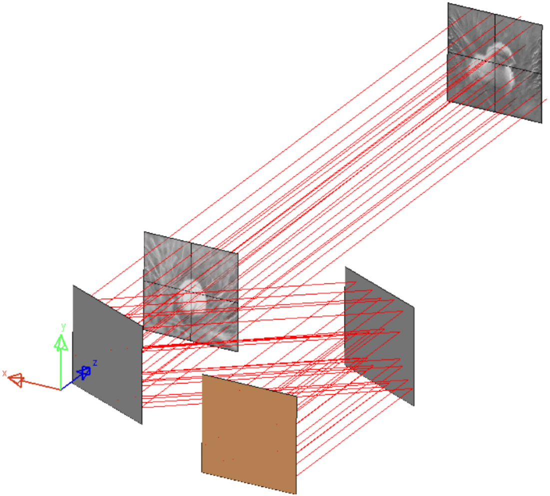
- Parameter $\beta \in [0, 1]$
- $\beta = 1$
  Bottom surface spherical
  Does not alter light rays
- β smaller
  Bottom surface gets more details



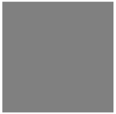
Romijn, L. B., ten Thije Boonkkamp, J. H. M., Anthonissen, M. J. H., IJzerman, W. L. (2021). *Generating-function approach for double freeform lens design*, Journal of the Optical Society of America A, Optics, Image Science and Vision, 38(3), 356-368. https://doi.org/10.1364/JOSAA.411883

TU/e

Two-target reflector system



- Source:



- Target 1:

- Target 2:

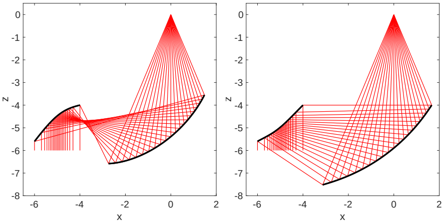
Braam, P.A., *Numerical Inverse Method for a Freeform Two-Target Reflector System*
In preparation

TU/e

Concave, convex and saddle surfaces

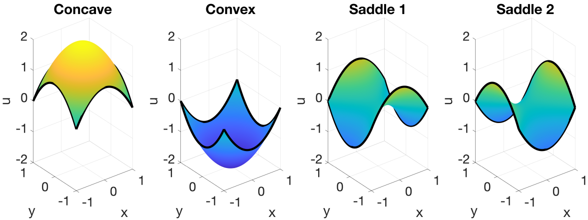
2D

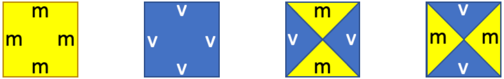
$$m'(x) = \pm \frac{f(x)}{g(m(x))} \frac{1 + (m(x))^2}{2}$$



3D

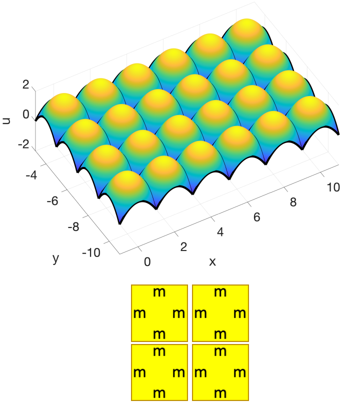
$$\det(D\mathbf{m}(\mathbf{x})) = \pm \frac{f(\mathbf{x})}{g(\mathbf{m}(\mathbf{x}))} \frac{\left(1 + \|\mathbf{m}(\mathbf{x})\|^2\right)^2}{4}$$

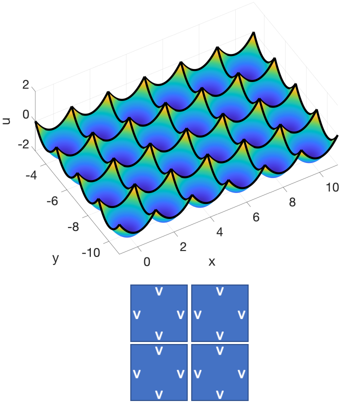


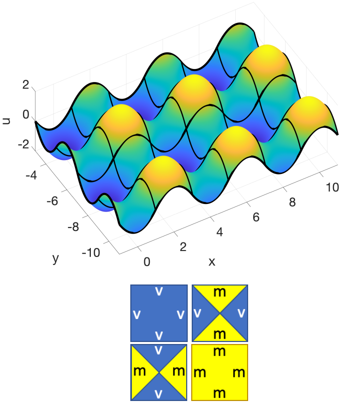


m : mountain   v : valley

Lens arrays







Concave elements only

Not smooth at interfaces

Convex elements only

Not smooth at interfaces

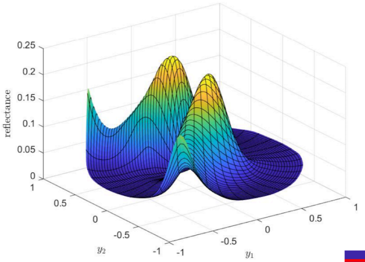
Concave, convex and saddle

Smooth everywhere

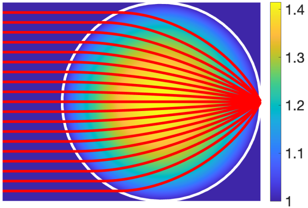
Conclusions and future work

OPTIC
AWAVE
Math & CS
OTP

- Mathematical models for freeform optical design
- One framework for basic systems
- Least-squares solver directly computes freeform optical surfaces
- Include Fresnel reflection and scattering
- Design lens arrays with smooth surfaces
- Use machine learning to speed up or replace the least-squares solver
- Model finite sources
- Model GRIN optics



Reflectance on a freeform lens for street illumination



Luneburg lens

Friday morning—**Lecture 2: Hamiltonian optics, Lie algebra and Liouville's equation**
More info: https://martijna.win.tue.nl/Optics/

TU/e