Reduced Order Model for PDE Eigenvalue Problems

Abdul Halim[1], Daniele Boffi [2], Fleurianne Bertrand[1]

[1] Department of Applied Mathematics, University of Twente

[2] Department of Mathematical and Computer Sciences, KAUST

Introduction

Eigenvalue problems (EVPs) occurs naturally for modeling physical phenomena. The PDE eigenvalue problems arising from fluid dynamics, structural mechanics etc.

Challanges

- Not straight forward form source problem
- Multiple Eigenmodes
- Medium size device may exhaust
- May Depend on parameters
- Parameters can be deterministic or stochastic.

Strategies

- Design & Analyze Reduced Order Models
- Study in Three Category
  - I. EVPs without any parameter but introduce a fictitious parameter
  - II. EVPs with deterministic parameter
  - III. EVPs with stochastic parameter

Eigenvalue Problems with no parametric dependence

In spirit of [1], consider the Laplace eigenvalue problem with zero Dirichlet boundary condition. Introducing variable t

$$u_t - \Delta u(x,t) = \lambda(t,u)u(x,t) \qquad (1)$$

Where formula for λ is

$$\lambda(t,u) = \frac{\int \nabla u \cdot n \, ds}{\int u \, dx} \qquad (2)$$

Applying the FEM we will get matrix vector form

$$Au^{k+1} = (\lambda^k + \frac{1}{\Delta t})Mu^k$$

Let K be the matrix of n_s snapshots.

$K_{i,j} = K_{i,j} - \phi_i$ *where* $\phi_i = 1/n_s \sum_j K\_\{i,j\}$.

Write $K = U\Sigma V^t$, *the first* n_p *columns are of* U *are POD basis. The solution of (1) can be written as*

$$u^k = \phi + U\psi^k$$

The

$$U^t A U \psi^{k+1} = -U^t\phi + (\lambda^k + \frac{1}{\Delta t})(U^t M\phi + U^t M U\psi^k)$$

Rewrite the formula (2) in terms of ψ^k.

Eigenvalue Problems with Deterministic Parameter (approach)

- Design Efficient acheme and convergence analysis
- Selection of parameters to draw snapshots
- Determination of which eigenpairs have to compute in offline stage
- Optimize offline stage using technique of machine learning and data science

Eigenvalue Problems With Stochastic Parameter (approach)

- Self-adjoint PDES with random data
- Use stochastic Galerkin Approach
- Use synthetic data driven algorithm to determine snapshots following [2]

References

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