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Multilevel Markov Chain Monte Carlo for full-field data assimilation

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In a Nutshell

Goal Achieve a computational speed-up for the prohibitively expensive assimilation of full-field data in a high resolution setting.

Model Beam model in structural mechanics, slightly adapted from [1].

Method Multilevel Markov Chain Monte Carlo method as proposed by [2].

Result The method is significantly cheaper than single-level approaches. Scaling data between levels is work in progress.

Context

New observation techniques in structural mechanics allow for the acquisition of **high resolution full-field data**. Such data can be immensely useful in e.g. structural damage assessment, where it enables the construction of full-field maps of the variation in material stiffness. However, there is a **lack of efficient models** to deal with the computationally expensive case of full-field data.

Markov Chain Monte Carlo

- We want to infer information on a parameter θ based on noisy observations of some forward operator on θ

$$y_n = \mathcal{F}(\theta) + \varepsilon_n.$$

- In the Bayesian approach we construct a **posterior distribution** on θ conditional on prior and likelihood information given the data

$$p(\theta) \sim \mathcal{L}(\boldsymbol{y}|\theta)\pi(\theta).$$

- MCMC methods allow us to construct a chain that samples from the posterior by proposing moves

$$\theta' \sim q(\cdot|\theta^n)$$

and accepting or rejecting them based on some acceptance criterion

$$\theta^{n+1} = \begin{cases} \theta' & \text{with probability } \alpha \\ \theta^n & \text{with probability } 1-\alpha. \end{cases}$$

- Once sufficiently burnt in, we can then use this chain to obtain **MC estimators** for the moments of the posterior distribution of θ

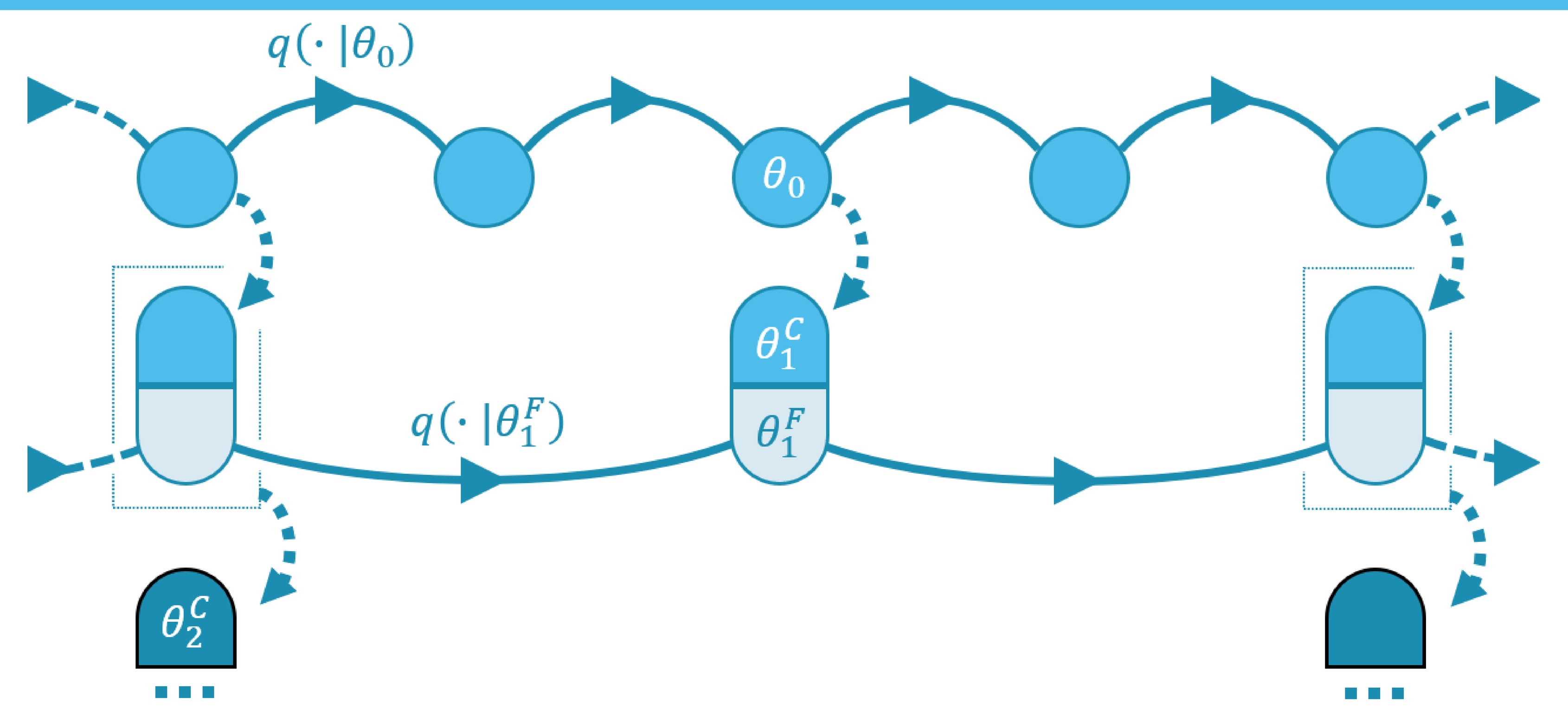
$$\widehat{Q}^{MC}(\theta) = \frac{1}{N} \sum_{j=n_0+1}^{N+n_0} Q(\theta^j).$$

- The multilevel approach consists of computing MC estimators on a **hierarchy of discretisation levels**, using correlated samples between levels. The estimators for the expensive finer levels are then used as **corrections** on the initial estimator for the coarsest level

$$\widehat{Q}^{ML}(\theta) = \widehat{Q}_0^{MC}(\theta) + \sum_{\ell=1}^{L} \left(\widehat{Q}_\ell^{MC}(\theta) - \widehat{Q}_{\ell-1}^{MC}(\theta) \right).$$

- Because the correction terms are decreasing, fewer samples are needed on each finer, more expensive level, leading to a **significant computational speed-up**.

Schematic representation



Beam model

- In the structural mechanics beam model, θ is a **full-field parameter** representing spatial variation of the material stiffness. Observations are displacements $\boldsymbol{u}$ along the beam edge, related to θ by solving the following PDE on a **finite element mesh**:

$$\nabla \cdot (\theta \nabla \boldsymbol{u}) = F_{\text{body}}.$$

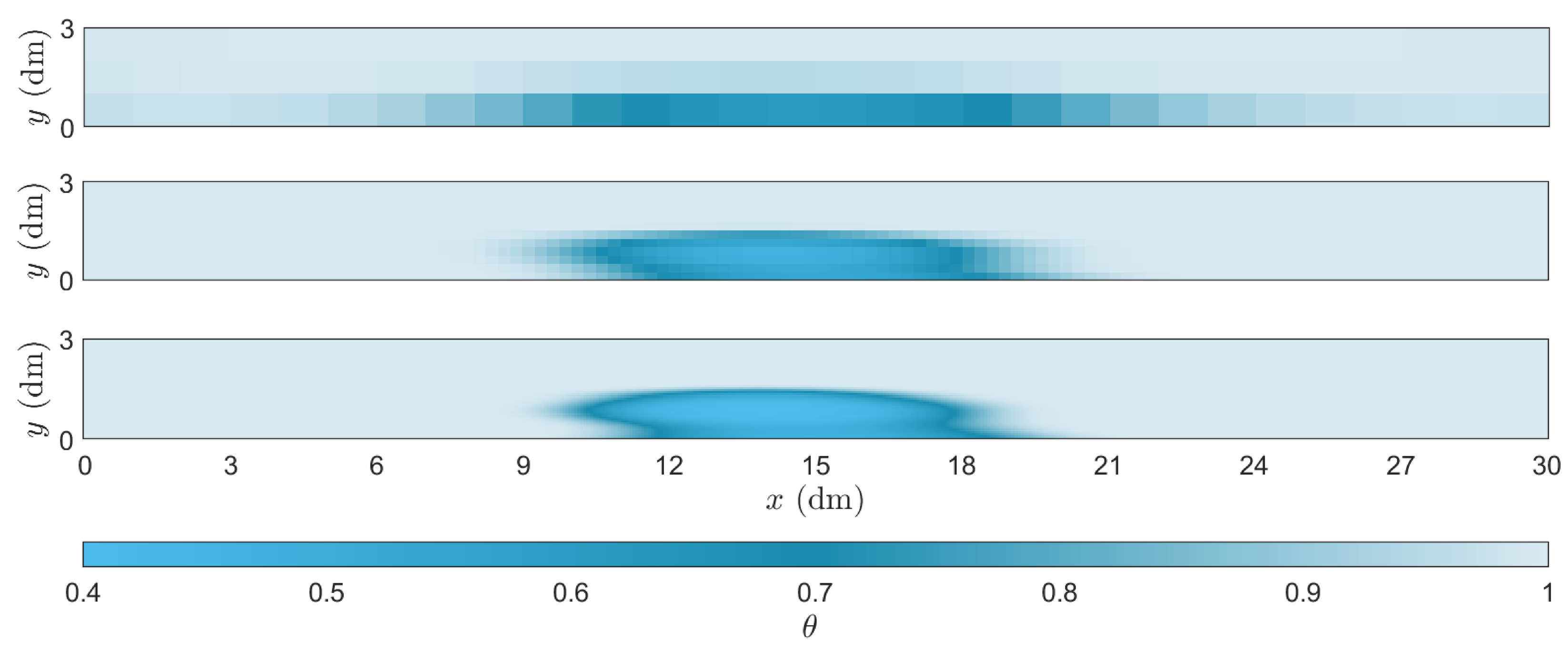
- In the test case we start from known values of θ and perturb the resulting displacements with Gaussian noise to use as **synthetic data**.
- θ is modelled as a normal field $K(\boldsymbol{x},\omega)$ which is subsequently transformed to have normalised values in $[0,1]$. $K(\boldsymbol{x},\omega)$ is approximated by means of a **Karhunen-Loève expansion**

$$K(\boldsymbol{x},\omega) = E[K(\boldsymbol{x},\cdot)] + \sum_{k=1}^{\infty} \sqrt{\lambda_k}\xi_k(\omega) b_k(\boldsymbol{x})$$

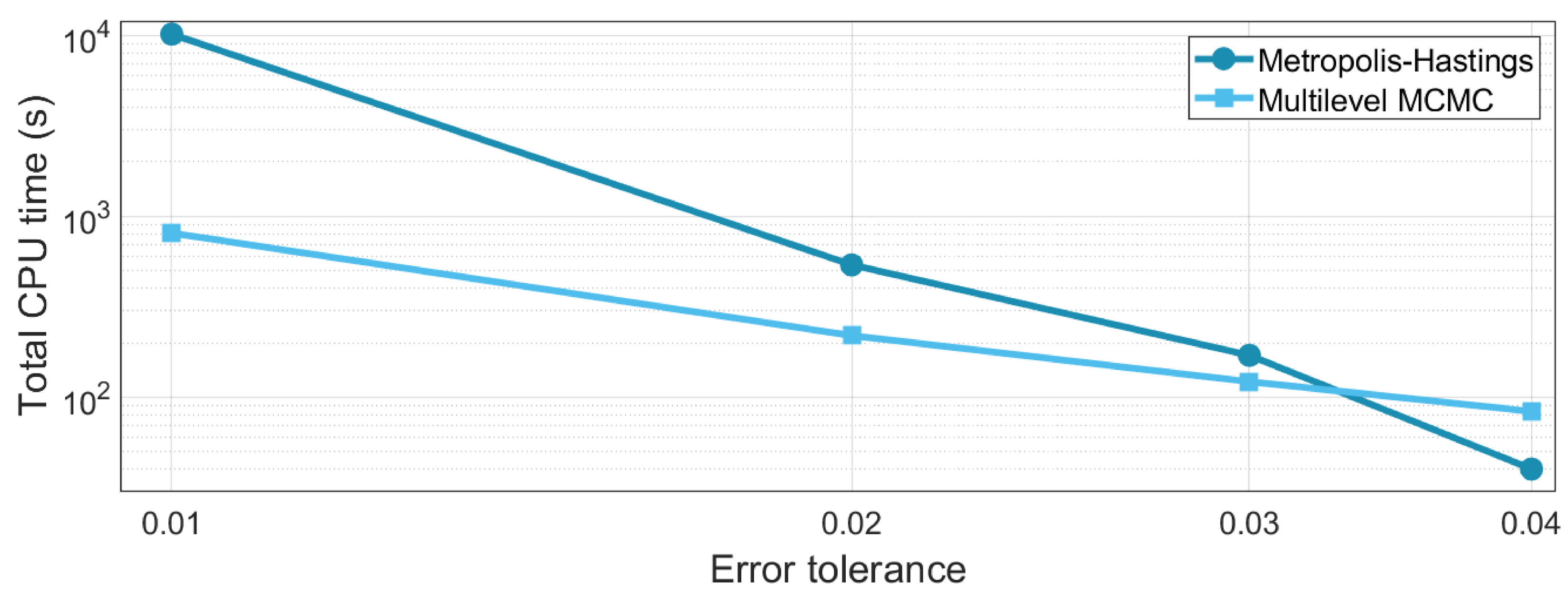
where ξ_k are independent standard normal variables.

- Finer levels are constructed by simultaneously **increasing the truncation number in the KL expansion** and **refining the FE mesh**.

Posterior mean on different levels



Efficiency



Conclusions

- With observations growing in resolution, there is a **need for efficient algorithms** to deal with the high dimension, high resolution limit.
- By using different discretisation levels, **fewer calculations** need to be done on higher resolutions.
- In cases where the coarser levels yield exceptionally large discretisation errors, an **adaptive error model** should be used to ensure proper convergence.
- Multilevel methods achieve **significant speed-up** compared to classical approaches.

References

[1] P. Reumers, C. Van Hoorickx, M. Schevenels and G. Lombaert. Density filtering regularization of finite element model updating problems. *Mechanical Systems and Signal Processing*, 128:282-294, 2019.

[2] T. J. Dodwell, C. Ketelsen, R. Scheichl and A. L. Teckentrup. Multilevel Markov Chain Monte Carlo. *SIAM Journal for Uncertainty Quantification*, 3:1075-1108, 2015.

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