Model Reduction and Simulation using Hierarchical Moment Models

Julian Koellermeier

NUMA research group, Department of Computer Science, KU Leuven

Application Background

Inaccurate classical fluid models in

(1) atmospheric reentry of spacecraft

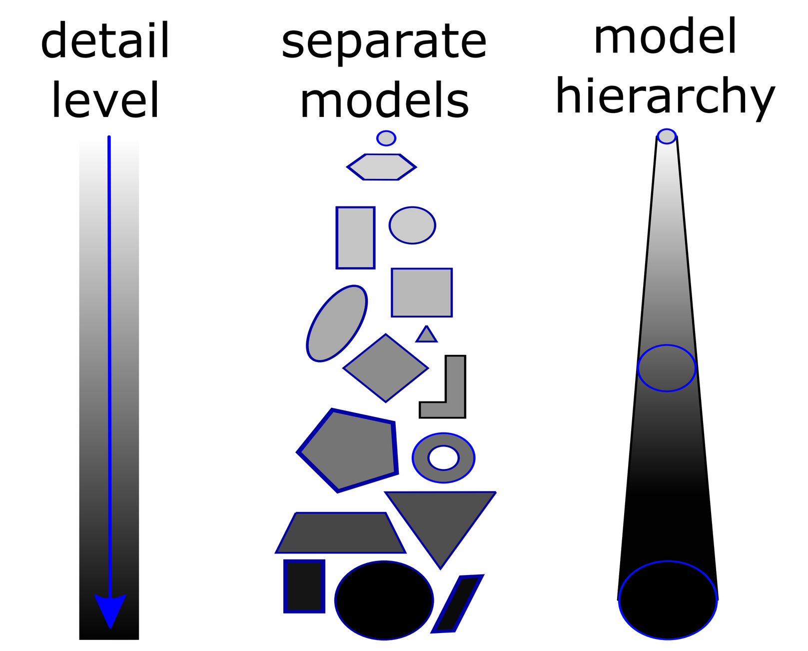
(2) fusion reactor and plasma simulations

(3) shallow free surface flows

Challenges

Modeling, analysis and numerics need to consider

- high-dimensionality
- multiple scales
- adaptivity



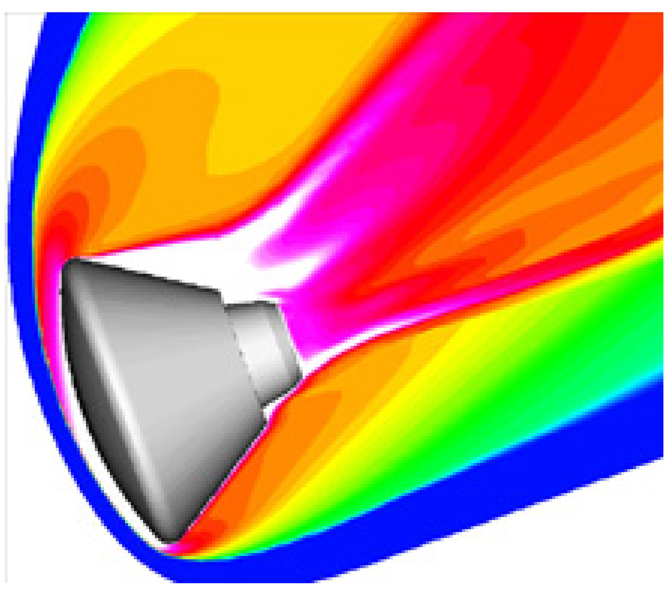
Model Hierarchies

+ preserve structure of the model
+ allow for efficient numerics
+ enable adaptivity within simulation

Goal: Self-learning models

$\Rightarrow$ moment models as hierarchical PDEs

Application 1: Atmospheric Reentry



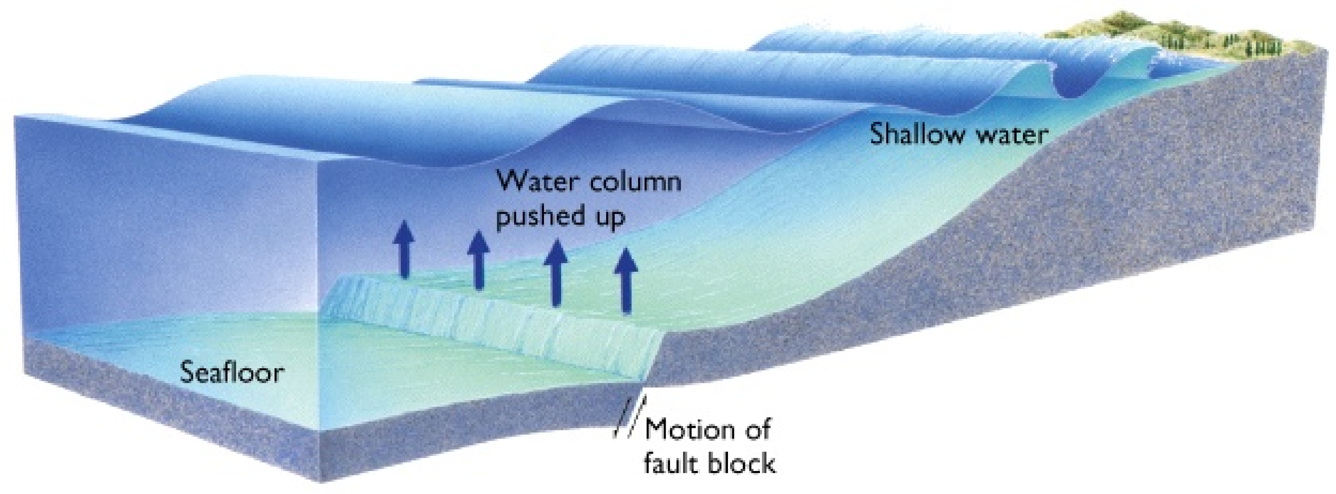
Relevant scale is the **Knudsen number**

$$\mathrm{Kn} = \frac{\text{mean free path length}}{\text{reference length}} = \frac{\ell}{L}$$

Model equation is the **Boltzmann Transport Equation**

$$\frac{\partial}{\partial t} f(t, \boldsymbol{x}, \boldsymbol{c}) + c_i \frac{\partial}{\partial x_i} f(t, \boldsymbol{x}, \boldsymbol{c}) = S(f)$$

Application 2: Shallow Flows



Relevant scale is the **shallowness**

$$S = \frac{\text{water height}}{\text{wave length}} = \frac{h}{\lambda}$$

Model equation is the **incompressible Navier-Stokes Equation**

$$\nabla \cdot \boldsymbol{u} = 0, \quad \partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \boldsymbol{\sigma} + g$$

Ansatz: Hermite Expansion [1, 2]

Expand unknown distribution function in Hermite series around equilibrium Maxwellian

$$f(t, x, c) = \sum_{i=0}^{M} f_i(t, x)\, \phi_i\left(\frac{c - v}{\sqrt{\theta}}\right)$$

Leads to **hyperbolic moment model**

$$\partial_t \boldsymbol{u}_M + \boldsymbol{A} \partial_x \boldsymbol{u}_M = \boldsymbol{S}, \quad \boldsymbol{u}_M = (\rho, v, \theta, f_3, f_4, \dots, f_M)^T$$

Ansatz: Legendre Expansion [3]

Expand unknown horizontal velocity profile in Legendre series around mean velocity

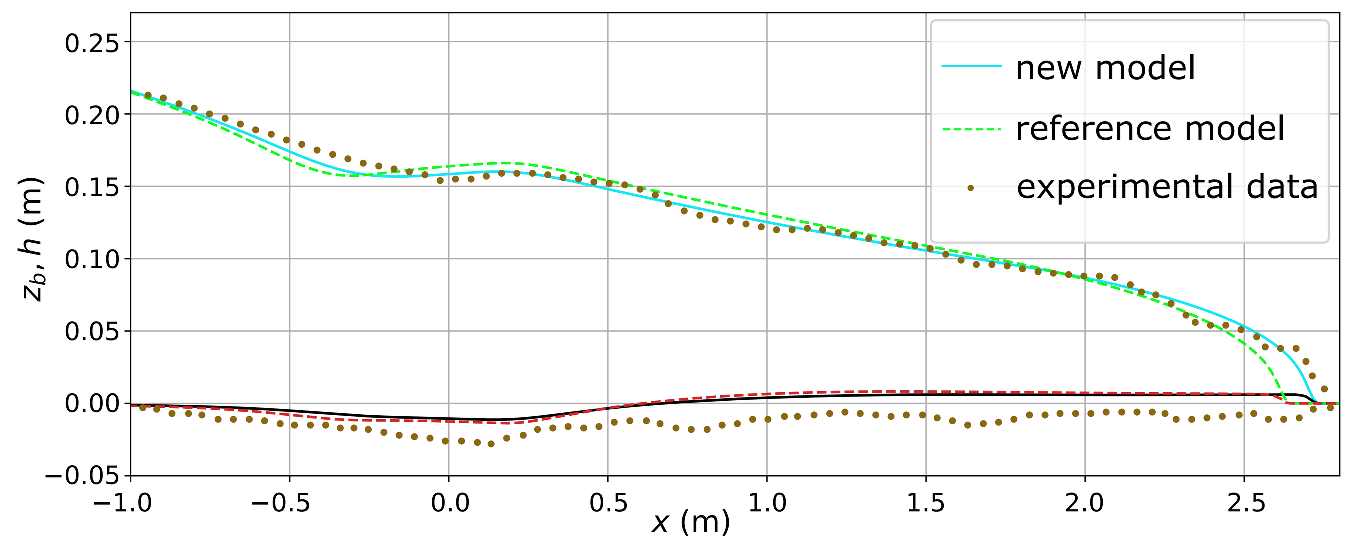
$$u(t, x, z) = u_m(t, x) + \sum_{i=1}^{M} \alpha_i(t, x) \phi_i\left(\frac{z - h_b}{h}\right)$$

Leads to **hyperbolic Shallow Water Moment Model** [4]

$$\partial_t \boldsymbol{u}_M + \boldsymbol{A}_M \partial_x \boldsymbol{u}_M = \boldsymbol{S}, \quad \boldsymbol{u}_M = (h, u_m, \alpha_1, \alpha_2, \dots, \alpha_M)^T \in \mathbb{R}^{M+2}$$

Results: Hypersonic channel flow [2, 5]

Results: Shallow sediment transport [6]



Next challenges:

(1) derivation of **hybrid moment models**

(2) development of **adaptive numerical schemes**

(3) simulation of **application tests**

Contact: julian.koellermeier@kuleuven.be

This research has been supported by the European Union's Horizon 2020 program under the Marie Sklodowska-Curie grant agreement no. 888596 and the Research Foundation - Flanders (FWO) under the FWO grant no. 0880.212.840.

References

[1] Y. Fan, et al. Model Reduction of Kinetic Equations by Operator Projection, *J. Stat. Phys.*, (2016)

[2] J. Koellermeier. Derivation and numerical solution of hyperbolic moment equations for rarefied gas flows, *RWTH Aachen University*, (2017)

[3] J. Kowalski, M. Torrilhon. Moment Approximations and Model Cascades for Shallow Flow, *Commun. Comp. Phys.*, (2019)

[4] J. Koellermeier, M. Rominger. Analysis and Numerical Simulation of Hyperbolic Shallow Water Moment Equations, *Commun. Comp. Phys.*, (2020)

[5] J. Koellermeier, G. Samaey. Projective Integration Schemes for Hyperbolic Moment Equations, *Kinet. Relat. Mod.*, (2021)

[6] J. Garres-Díaz, et al. Shallow Water Moment models for bedload transport problems, *Commun. in Comput. Phys.*, (2021)

LaTeX TikZposter