Upscaling of two-phase flows in porous media

Sohely Sharmin*, Carina Bringedal◇, Iuliu Sorin Pop* and Manuela Bastidas*

*Hasselt University, ◇University of Stuttgart

UHASSELT

CMAT

Introduction

- **Motivation:** Model for **two-phase** flow in porous media.
  - The pore-scale model **(Navier-Stokes)** includes a variable **surface tension**, depending on a surfactant present in one fluid phase.
  - The fluid-fluid interfaces evolving at the pore-scale are approximated by a phase-field **(Cahn-Hilliard)**.
- **Goal:** Derivation of an **upscaled** (Darcy scale) model considering:
  - **Perforated domain** as a complex geometry.

Mathematical approach

1. Assumes **scale separation** (pore scale vs. Darcy scale) and **local periodicity**.
2. Relies on asymptotic techniques:
   - **Matched asymptotic** (sharp interface limit).
   - **Homogenization** (derivation of the Darcy-scale model).
3. **Effective parameters** involved in the Darcy-scale model determined by solving **local cell-problems**.

Summary and future work

- Extended Darcy law accounting for a varying surface tension in the presence of surfactant.
- Non-standard evolution equation for the saturation (Darcy scale).

→ Numerical method for the fully coupled problem (multi-scale scheme, fully coupled, nonlinear solver).

→ Different regimes (capillary number), Marangoni effects.

References

[1] H. Abels, H. Garcke, and G. Grün. Thermodynamically consistent, frame indifferent **diffuse interface models** for incompressible two-phase flows with different densities. *Math. Models Methods Appl. Sci.*, 22(3):1150013, 40, 2012.

[2] H. Garcke, K. F. Lam, and B. Stinner. Diffuse interface modelling of **soluble surfactants** in two-phase flow. *Commun. Math. Sci.*, 12(8):1475–1522, 2014.

[3] S. Metzger and P. Knabner. **Homogenization of Two-Phase Flow** in porous media from pore to darcy scale: A phase-field approach. *Multiscale Model. Simul.*, 19(1):320–343, 2021.

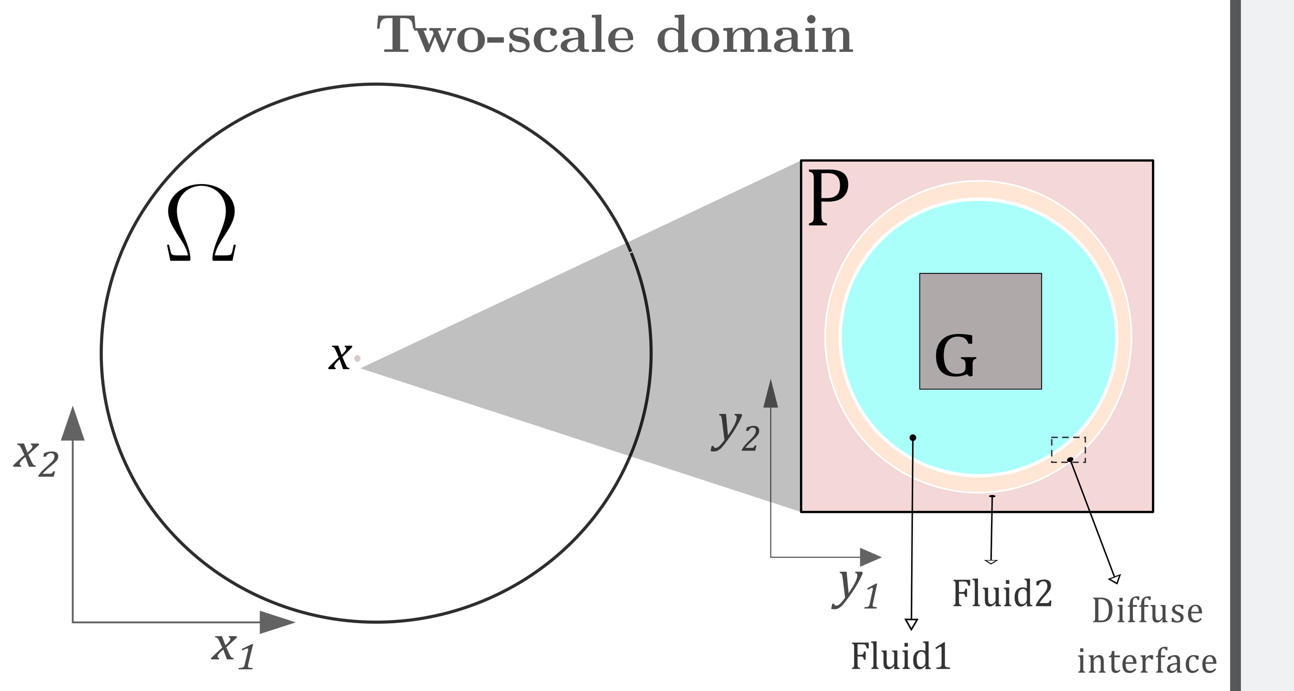
[4] S. Sharmin, C. Bringedal, and I. S. Pop. On upscaling **pore-scale models** for two-phase flow with evolving interfaces. *Advances in Water Resources*, 142:103646, 2020.

The upscaled model and cell problems

Upscaled model (Darcy scale)

$$
\begin{aligned}
\bar{\mathbf{v}} &= -\mathcal{K}\nabla p - \mathbf{M}\gamma(c), \text{ in } \Omega,\\
\nabla\cdot\bar{\mathbf{v}} &= 0, \text{ in } \Omega,\\
\partial_t S &= \frac{1}{2}\nabla\cdot\left(\mathcal{K}_\phi\nabla p + \mathbf{M}_\phi\gamma(c)\right), \text{ in } \Omega,\\
\Phi\partial_t(Sc) &= -\frac{1}{2}\nabla\cdot\left(c(\bar{\mathbf{v}}+\bar{\mathbf{v}}_\phi)\right) + \frac{1}{\mathrm{Pe}_c}\nabla\cdot\left(\mathcal{B}\nabla c + \mathbf{H}c\right) \text{ in } \Omega.
\end{aligned}
$$

Two-scale domain



Effective parameters

$$
k^{ij} = \int_P \mathbf{w}_i^j\,dy,\quad m^i = \int_P \mathbf{w}_i^0\,dy,
$$
$$
k_\phi^{ij} = \int_P \mathbf{w}_i^j\,\phi\,dy,\quad m_\phi^i = \int_P \mathbf{w}_i^0\,\phi\,dy.
$$

Pore-scale cell problem

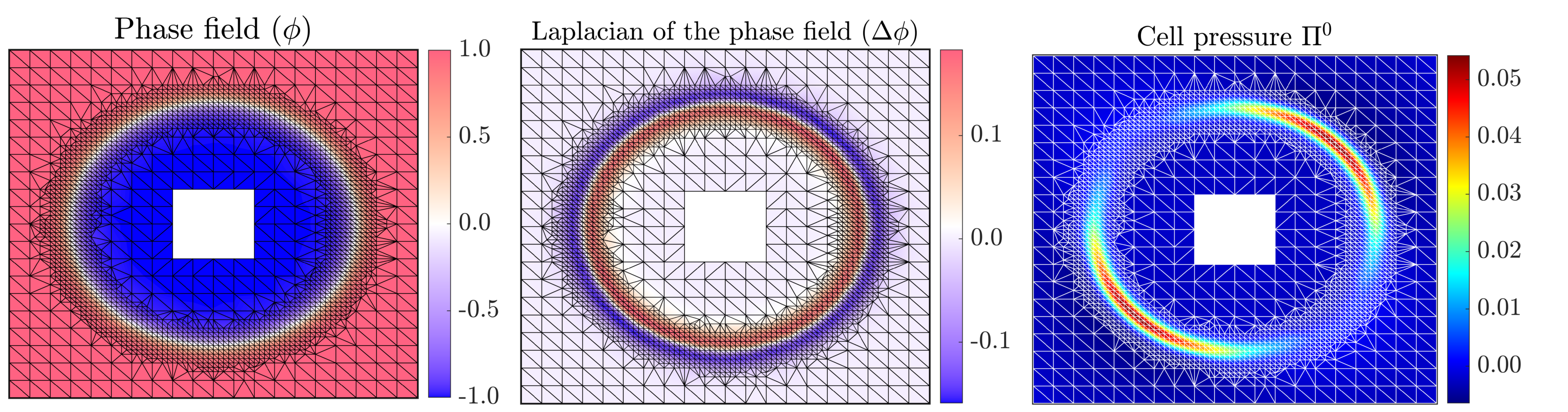
$$
\begin{aligned}
(\vec{e}_j + \nabla\Pi^j) &= -\frac{1}{\mathrm{Eu\,Re}}\nabla\cdot\left(2\mu(\phi)D(\mathbf{w}^j)\right), \text{ in } P,\\
\nabla\cdot\mathbf{w}^j &= 0, \text{ in } P,\\
\mathbf{w}^j &= \mathbf{0}, \text{ on } \partial G,
\end{aligned}
$$
$\Pi^j, \mathbf{w}^j$ are periodic in Y and $\int_P \Pi^j\,\mathbf{dy} = 0.$

Pore-scale cell problem

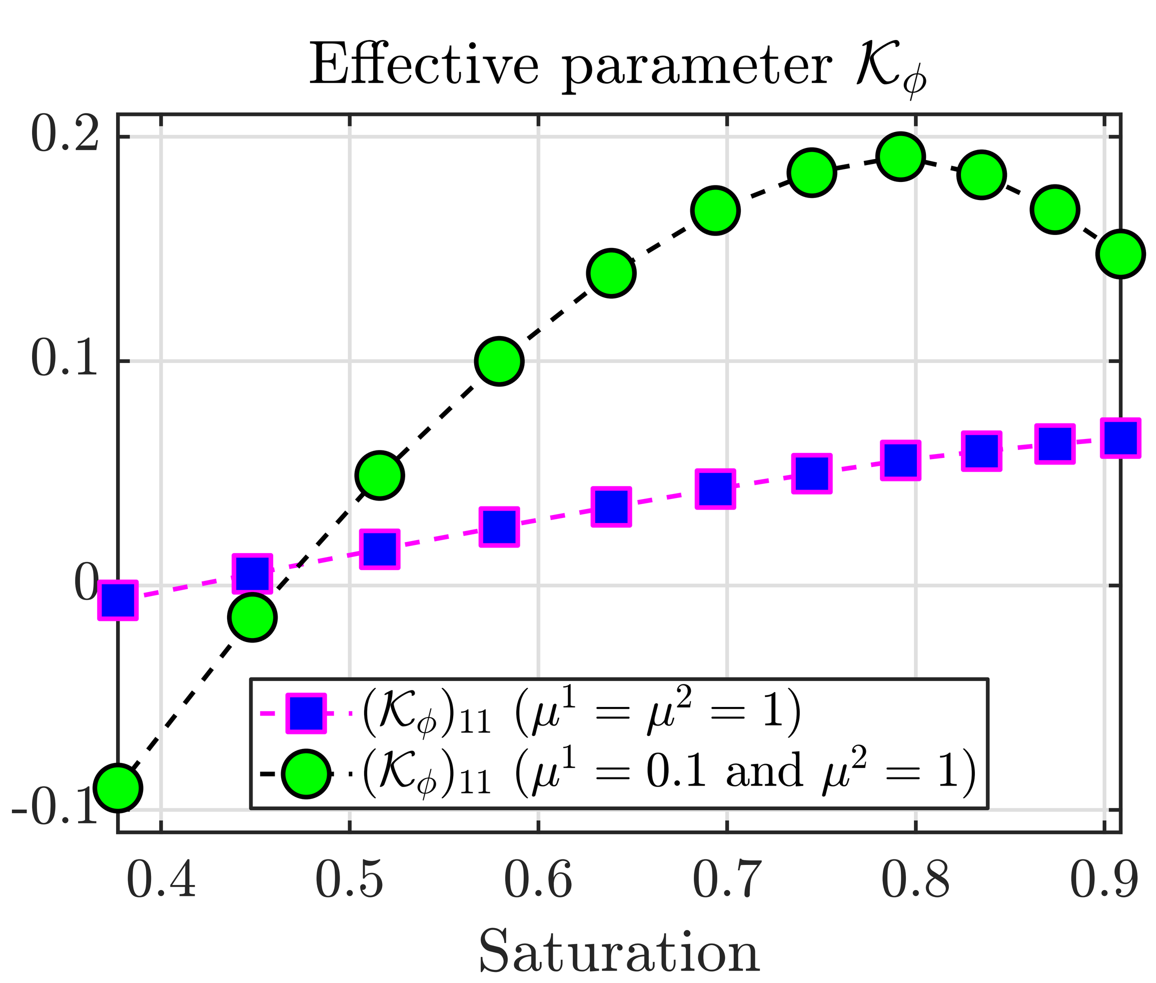
$$
\begin{aligned}
\overline{\mathrm{Eu}}\nabla\Pi^0 = &-\frac{1}{\mathrm{Re}}\nabla\cdot\left(2\mu(\phi)D(\mathbf{w}^0)\right)\\
&+\frac{1}{\mathrm{Re\,Ca}}\left(\frac{\mathcal{C}}{\lambda}P'(\phi) - \mathcal{C}\lambda\Delta\phi\right)\nabla\phi, \text{ in } P,
\end{aligned}
$$
$$
\nabla\cdot\mathbf{w}^0 = 0, \text{ in } P,
$$
$$
\mathbf{w}^0 = \mathbf{0}, \text{ on } \partial G,
$$
$\Pi^0, \mathbf{w}^0$ are periodic in Y and $\int_P \Pi^0\,\mathbf{dy} = 0.$

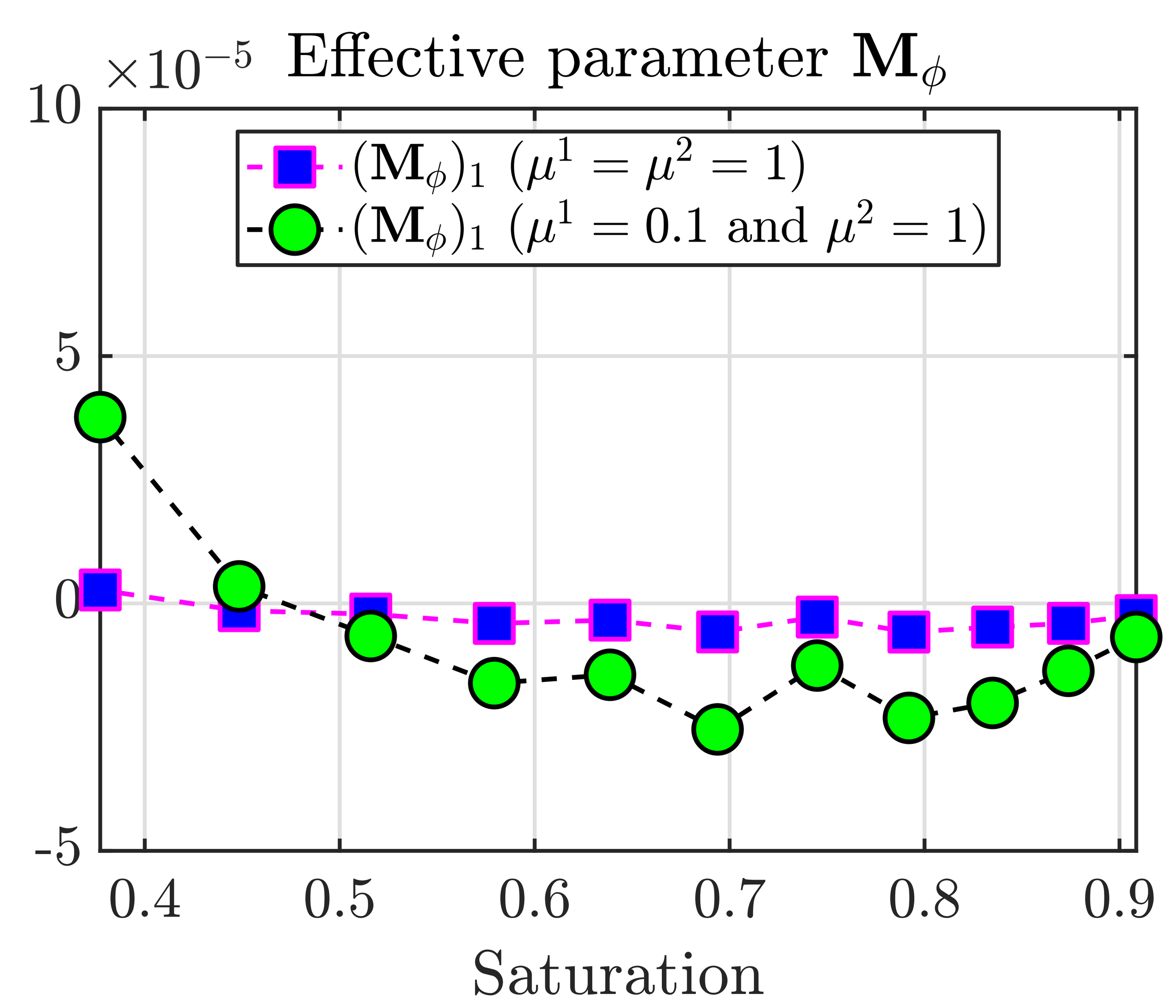
Numerical results

- **Phase field and mesh refinement**



- **The components of the effective $\mathcal{K}_\phi$ and M_ϕ depending on the saturation; results displayed for two viscosity ratios.**





Acknowledgements

fwo

ODYSSEUS

This research is part of the Project G0G1316N **"DynScale"** funded through the *Odysseus* programme of the Research Foundation Flanders FWO in Belgium.

Contact

sohely.sharmin@uhasselt.be
sorin.pop@uhasselt.be
uhasselt.be/CMAT