Challenges in simulations of electrical discharges

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$\mathcal{O}(10^{-12})$ $\mathcal{O}(10^{-6})$

Time scale (sec)

$$\frac{\partial n_e}{\partial t} - \nabla \cdot [\mu \mathbf{E} n_e + D(\nabla n_e)] = S_e(n_e, n_\pm, \mathbf{E}, \ldots)$$

$$\frac{\partial n_\pm}{\partial t} = S_i(n_e, n_\pm, \mathbf{E}, \ldots)$$

$$\nabla^2 \phi = -\frac{e(n_+ - n_- - n_e)}{\epsilon_0}, \mathbf{E} = -\nabla \phi$$

μ and D are functions of the ratio E/ρ

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + p\mathbf{I}) = 0$$

$$\frac{\partial \rho \mathcal{E}}{\partial t} + \nabla \cdot ((\rho \mathcal{E} + p)\mathbf{u}) = S_{\mathcal{E}}(n_e, \mathbf{E})$$

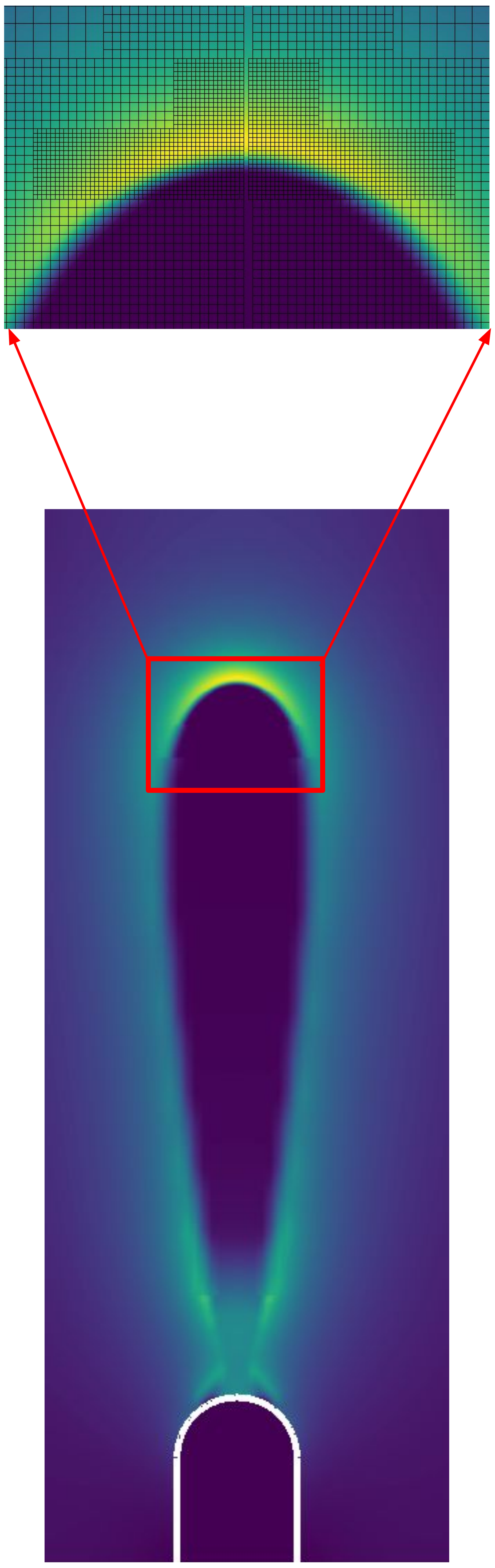
$$\mathcal{E} = \frac{p}{\gamma - 1} + \frac{1}{2}\rho \left\|\mathbf{u}\right\|_2^2, \; T = f(p, \rho)$$

highly non-linear and non-local coupling

$\mathcal{O}(10^{-6})$

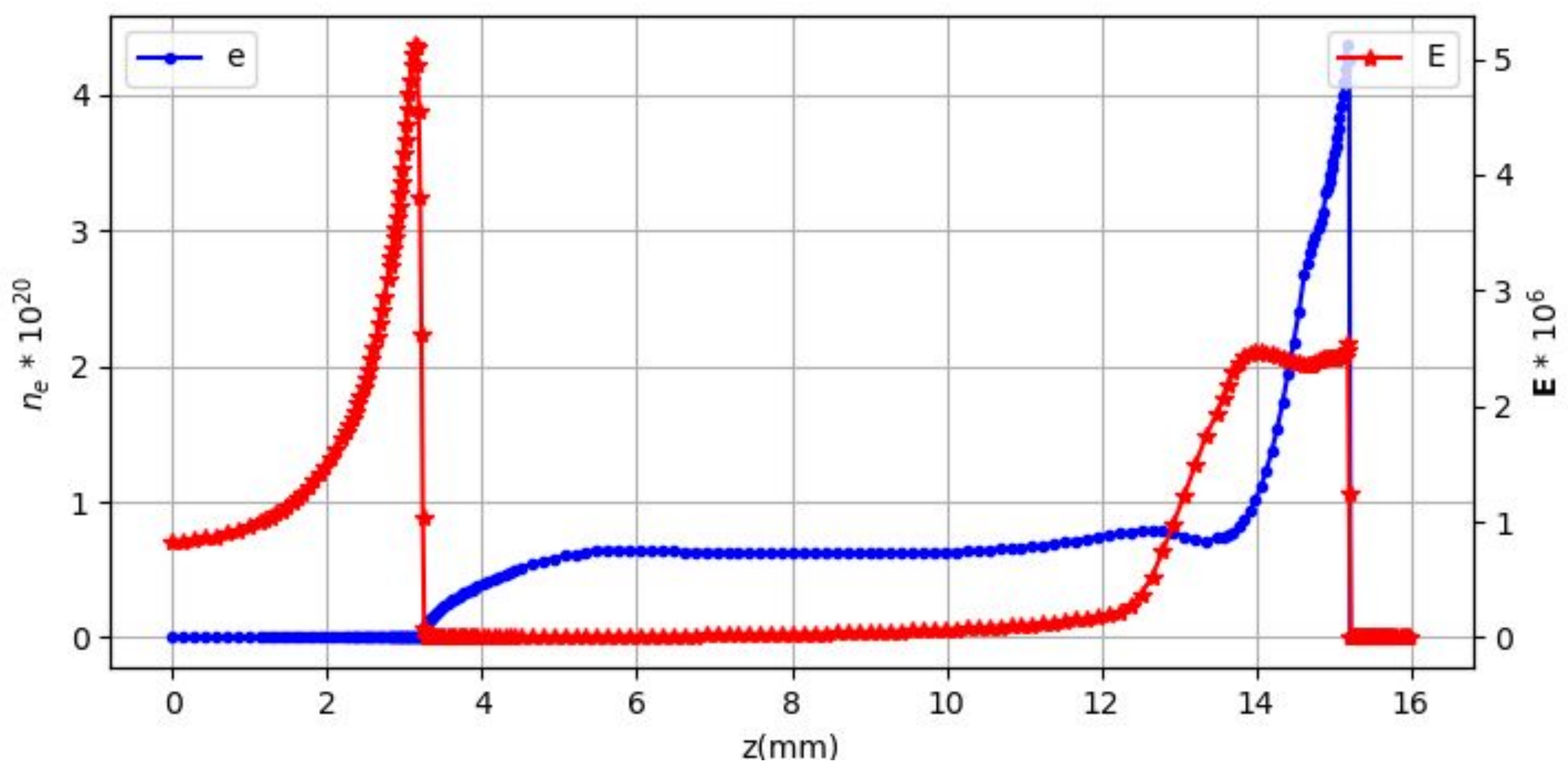
$\mathcal{O}(10^{-2})$

Spatial scale (m)



Challenges: current approaches

- Sharp gradients (“shock”-like structures): Finite volume method with flux limiters.
- Spatial, and time scales: Adaptive mesh, time stepping.
- Non-linear coupling, and non-local effects: Explicit methods
- Non-linear source: evaluated at cell centers (error prone)



Outlook

- Sharp gradients: small mesh size is computationally not practical.
  - **New, higher order discretization schemes?**
  - **Can reduced basis methods be used?**
- Non-linear coupling, and non-local effects: instabilities are amplified
  - **“Numerical fixes”, e.g., in situ correction terms?**
  - **Semi-implicit methods for larger time-stepping**