SPARSE MULTI-DIMENSIONAL EXPONENTIAL ANALYSIS

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CMA COMPUTATIONAL MATHEMATICS

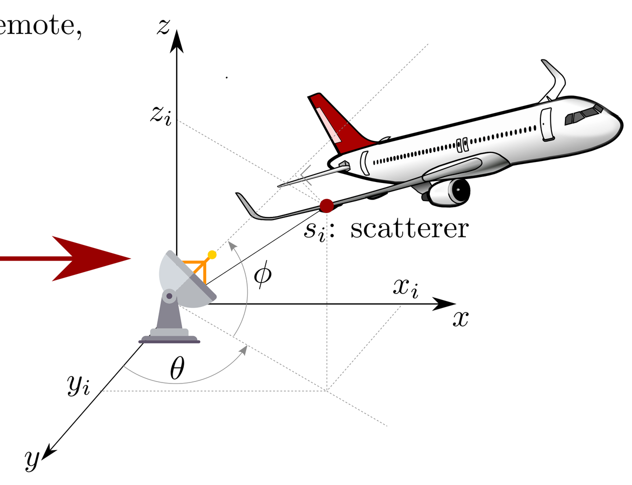
SCAN ME

CHALLENGE

1 APPLICATION DOMAINS

Multi-dimensional exponential analysis may sound remote, but touches our lives in many ways:

- Radio-frequency identification (RFID)
- Direction of arrival estimation (DOA)
- Inverse synthetic-aperture radar (ISAR)
- Global Navigation Satellite System (GNSS)
- Wireless communication
- Magnetic resonance spectroscopy (MRS)
- ...

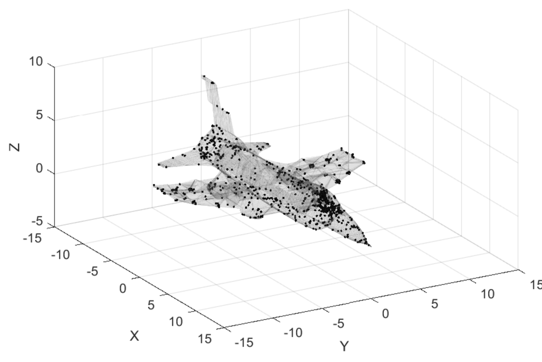


We focus on the ISAR application.

Here a stationary emitter locates a moving target in space. The location is determined from the backscattered signal.

This directly leads to a 3-D exponential analysis problem. The sampling dimensions are the frequency, azimuth and elevation angle.

2 ILLUSTRATION



As illustration we look at a large scale example consisting of 1000 scatterers depicting a fighter jet.

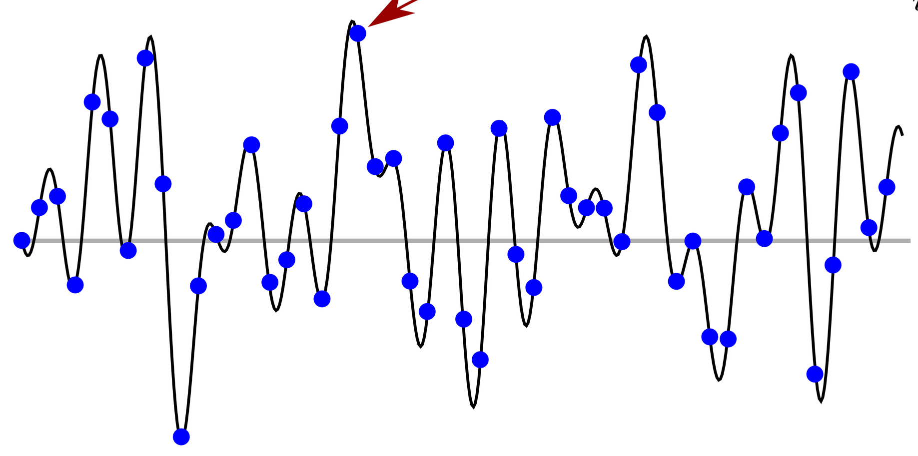
Some 90000 samples are collected with a signal-to-noise ratio of 20 dB.

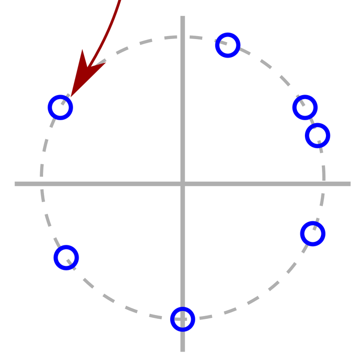
For the results, see the figures at the bottom.

MODEL

1 ONE-DIMENSIONAL

$$f(k\Delta) = \sum_{i=1}^{n} \alpha_i \exp(\phi_i \Delta k)$$



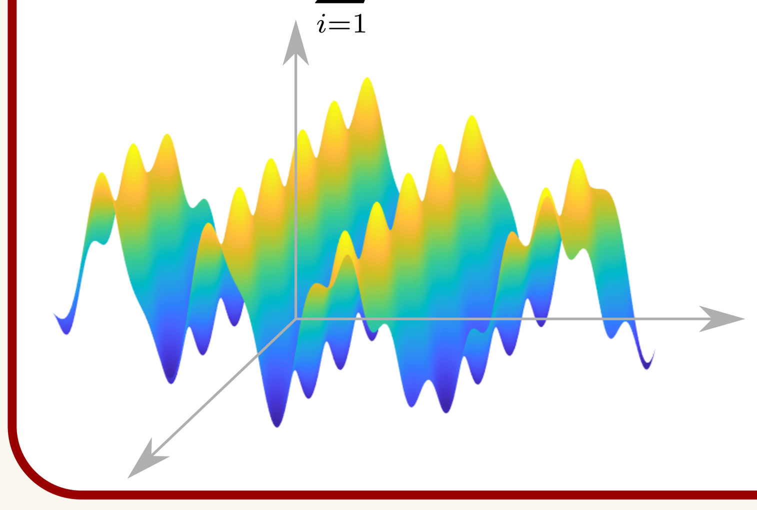


Equidistant samples are collected with spacing Δ, which satisfies the Shannon-Nyquist condition.

This directly leads to a system of non-linear equations, which can be solved by any parametric method such as matrix pencil, MUSIC, ESPRIT, ...

2 MULTI-DIMENSIONAL

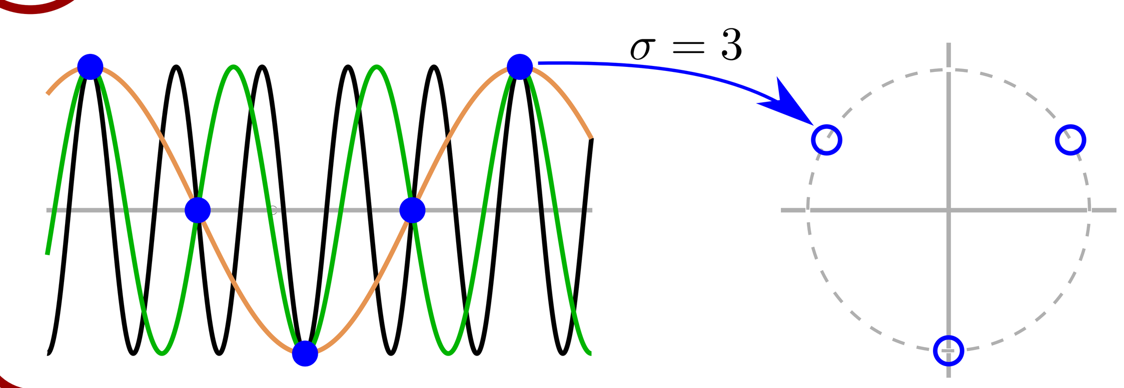
$$f(k\Delta) = \sum_{i=1}^{n} \alpha_i \exp(\langle \phi_i, \Delta \rangle k), \qquad \Delta = (\Delta_1, \ldots, \Delta_d), \qquad \phi_i = (\phi_{i1}, \ldots, \phi_{id})$$



The multi-dimensional model is a direct generalization of the one-dimensional model. The argument $\phi_i \Delta$ is replaced by the inner product of the exponent vector ϕ_i and sampling direction Δ. The coefficients α_i are still scalars.

METHOD

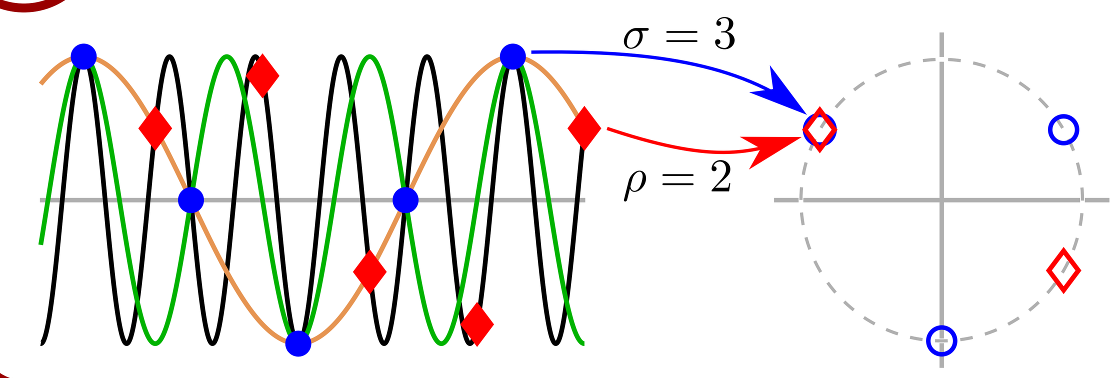
1 ALIASING



Sampling $\sigma = 3$ times coarser, at $\sigma \Delta k$, than the Shannon-Nyquist rate introduces aliasing.

There is no way to distinguish between the 3 possible frequencies

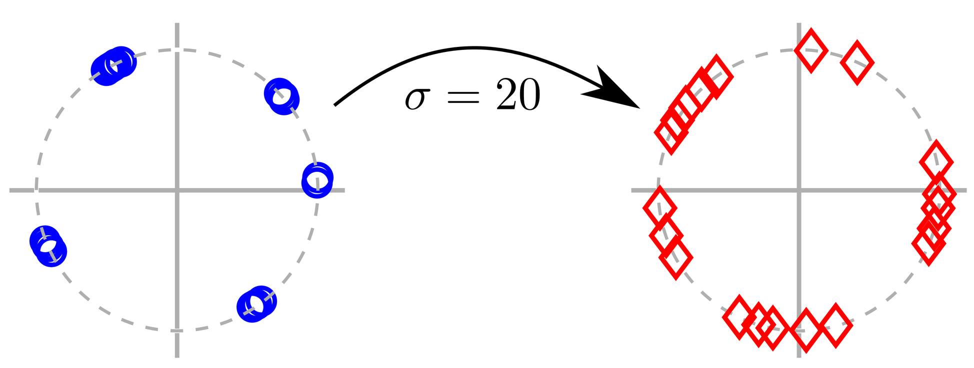
2 ANTI-ALIASING



Using an extra set of samples, shifted over $\rho\Delta$, we obtain another set of $\rho = 2$ possible frequencies.

When σ and ρ are chosen coprime, their intersection yields the unique solution.

3 RESOLUTION

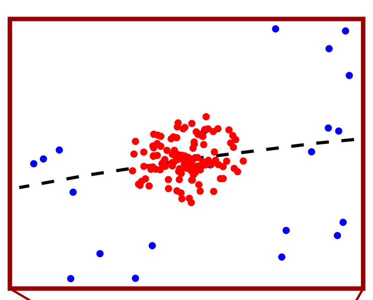


Subsampling the problem may have the benefit of pulling clustered frequencies apart.

We can easier distinguish between closeby frequencies, hence, the resolution has increased.

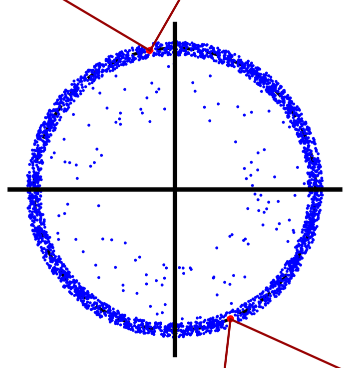
4 VALIDATION

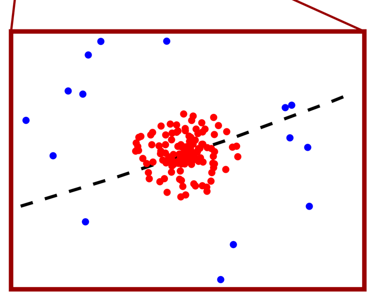
Through the connection with Padé approximation theory, and in particular Froissart doublets, a divide-and-conquer approach can be used to estimate the number of exponential terms and to stabilise the frequency computation.



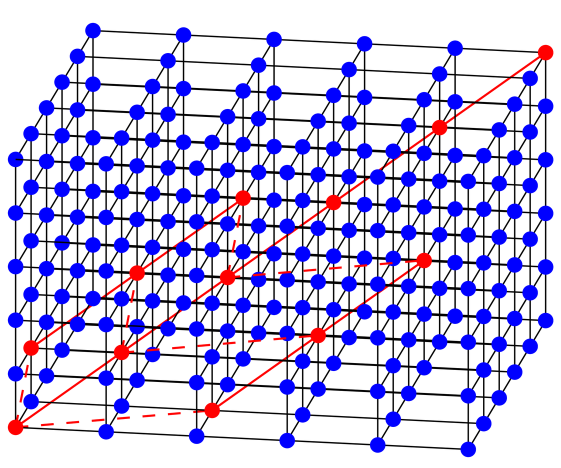
The correct frequencies form clusters, while the randomly scattered ones represent the noise.

Using a density based clustering algorithm we can easily separate the signal from the noise.





5 SAMPLING



Traditionally a whole d-dimensional grid of samples is used for the multi-dimensional analysis.

Here only $(d+1)n$ measurements are required.

6 CURSE OF DIMENSIONALITY

The theoretical minimal number of samples is $(d+1)n$, since there are exactly $(d+1)n$ unknown parameters. This bound has not been reached by alternative methods. In particular for higher dimensions the gain is very proncounced.

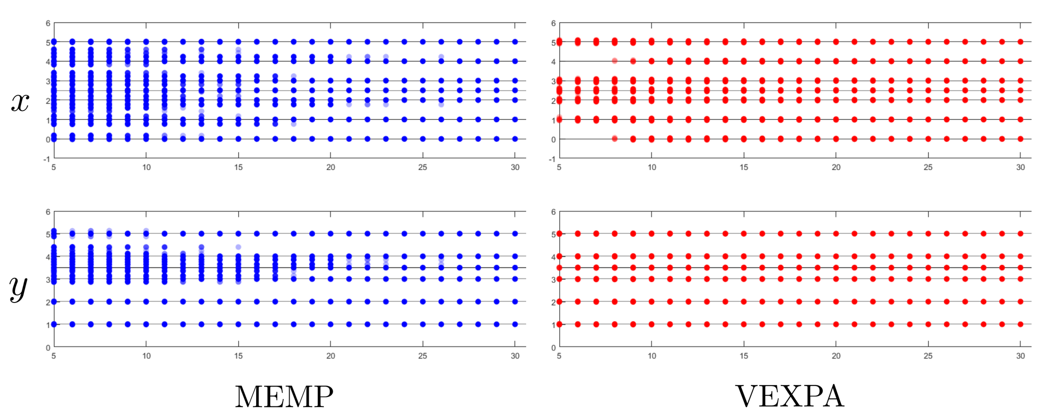
		#Samples		
d	n	$\mathcal{O}\left((d+1)n^2 \log^{2d-2} n\right)$	$\mathcal{O}\left(2^d n\right)$	$\mathcal{O}\left((d+1)n\right)$
3	100	1.80×10^7	8.00×10^2	4.00×10^2
3	500	1.49×10^9	4.00×10^3	2.00×10^3
3	1000	9.11×10^9	8.00×10^3	4.00×10^3
7	100	7.28×10^{12}	1.28×10^4	8.00×10^2
7	500	6.64×10^{15}	6.40×10^4	4.00×10^3
7	1000	9.44×10^{16}	1.28×10^5	8.00×10^3

RESULTS

1 VALIDATION ILLUSTRATION

To illustrate the validation aspect we look at a small-scale ISAR problem consisting of 12 scatterers. We compare the new method (VEXPA) with a competitor (MEMP).

250 runs for every SNR level (x-axis)



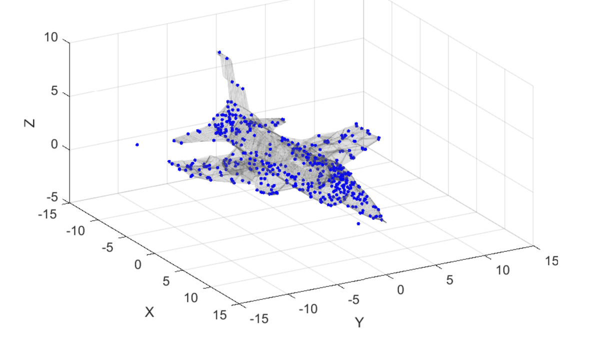
We observe that MEMP returns erroneous results for lower SNR levels, while VEXPA only returns correct locations, albeit not all because of the strict validation step.

Also note that we do not need to pass the number of scatterers to VEXPA, which detects it automatically through the clustering approach.

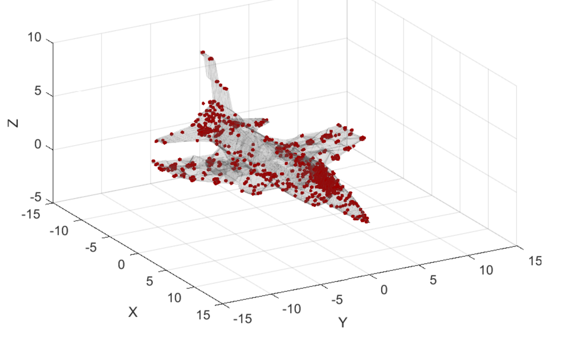
2 FULL-SCALE ILLUSTRATION

For the full-scale illustration we look at the 3-D fighter jet reconstruction of 1000 scatterers. In this case we compare with ND-ESPRIT. VEXPA combines the results of 4 different subsampling parameters to increase the resolution (1, 2, 3 and 4).

ND-ESPRIT only recovers 467 scatterers, while VEXPA finds a combined total of 934 of the original scatterers within 20 cm accuracy.



ND-ESPRIT



VEXPA