Averaged models for two-phase flow in a pore: The effect of hysteretic and dynamic contact angles

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Introduction

Two-phase porous-media flow models rely on:

- Constitutive laws for the averaged quantities
- Effective macroscopic parameters

Objective:

- Rational derivation of the constitutive relationships and the effective parameters
- Include dynamic and contact-angle effects in the upscaled models
- Understand the impact of the pore geometry

Approach:

- Start with pore-scale, two-phase flow models
- Explicit representation of pore geometry and evolving fluid-fluid interfaces
- Upscaling by asymptotic expansion & averaging

Dimensionless quantities

Scale ratio	ε	Velocity	$\boldsymbol{u}$
Viscosity ratio	M	Pressure	p
Reynolds number	Re	Interface position	γ
Capillary number	Ca	Mean curvature	κ
Slip length	λ	Contact angle	θ
Pore radius	R	Saturation	S

Asymptotic expansion method

1. Scale separation $\varepsilon = \frac{\text{typical radius}}{\text{length}} \ll 1$
2. Use the asymptotic expansion ansatz, e.g.

$$\boldsymbol{u}_m^\varepsilon(t,\boldsymbol{x}) = \boldsymbol{u}_m(t,\boldsymbol{x}) + \varepsilon \boldsymbol{u}_m^1(t,\boldsymbol{x}) + \mathcal{O}(\varepsilon^2)$$

3. Equate terms of the same order in ε

$\Rightarrow$ Asymptotic equations in the limit as $\varepsilon \to 0$

Main assumptions

- $\mathrm{Ca} = \mathcal{O}(\varepsilon^0)$, $\mathrm{Re} \le \mathcal{O}(\varepsilon^0)$, $\mathrm{M} \le \mathcal{O}(\varepsilon^0)$
- Transversal interface: $\partial_s \gamma_z \equiv 0$
- Constant or interface-local slip length:

  $\lambda^\varepsilon(t,z) = \lambda + \lambda_e \exp(-|z - \gamma_z^\varepsilon(t,1)|/\varepsilon)$

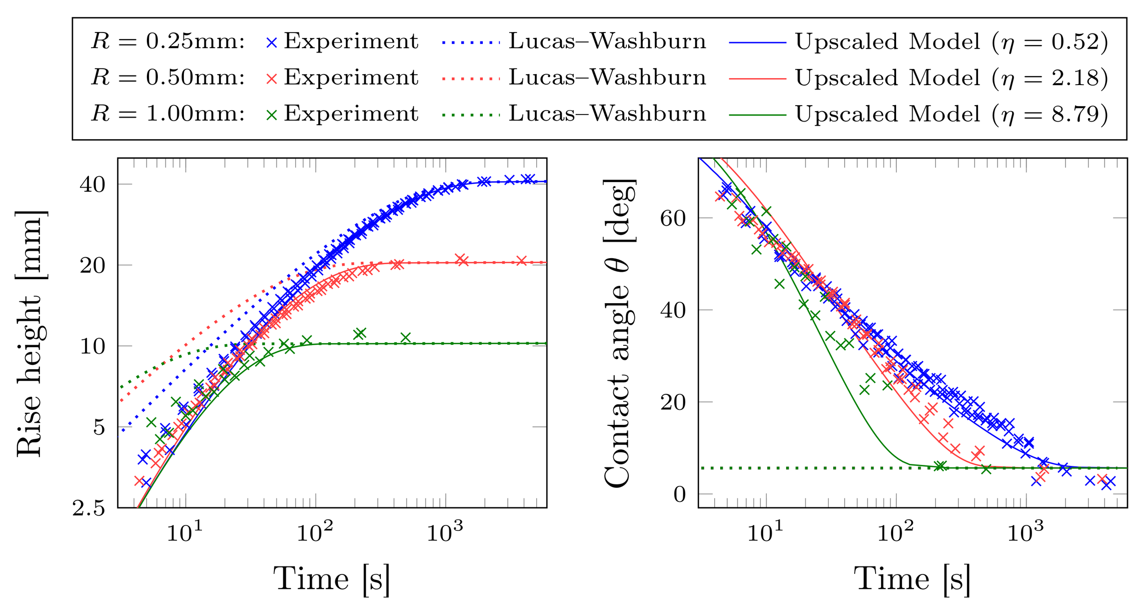
  with either $\lambda_e = 0$, or $\lambda = 0$

Validation: Capillary rise

- Rising liquid in a vertical glass tube
- Upscaled model with dynamic contact angle for the dimensionless rise height h

$$\left(\frac{8h}{1+4\lambda} + 2\eta\right)\partial_t h = 1 - h$$

- Better match with experimental data [1] than the Lucas–Washburn model. Below: Glycerol



Acknowledgements and contact

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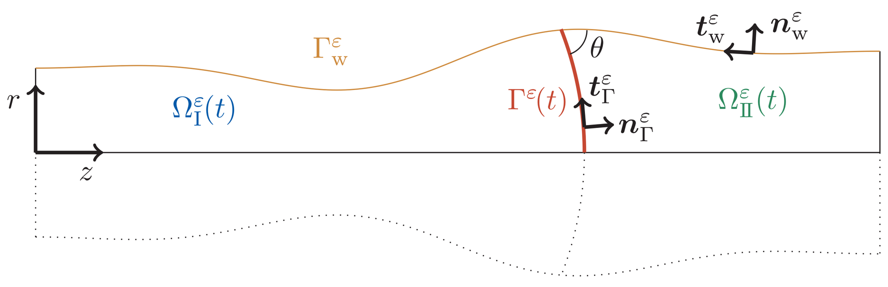
Modeling of immiscible two-phase flow in a pore

- Navier–Stokes equations for the flow of two immiscible fluids

$$\varepsilon^2 \mathrm{Re}\big(\partial_t \boldsymbol{u}_\mathrm{I}^\varepsilon + (\boldsymbol{u}_\mathrm{I}^\varepsilon \cdot \nabla^\varepsilon)\boldsymbol{u}_\mathrm{I}^\varepsilon\big) + \nabla^\varepsilon p_\mathrm{I}^\varepsilon = \varepsilon^2 \Delta^\varepsilon \boldsymbol{u}_\mathrm{I}^\varepsilon \quad \text{in } \Omega_\mathrm{I}^\varepsilon(t),$$
$$\varepsilon^2 \mathrm{Re}\big(\partial_t \boldsymbol{u}_\mathrm{II}^\varepsilon + (\boldsymbol{u}_\mathrm{II}^\varepsilon \cdot \nabla^\varepsilon)\boldsymbol{u}_\mathrm{II}^\varepsilon\big) + \nabla^\varepsilon p_\mathrm{II}^\varepsilon = \mathrm{M}\varepsilon^2 \Delta^\varepsilon \boldsymbol{u}_\mathrm{II}^\varepsilon \quad \text{in } \Omega_\mathrm{II}^\varepsilon(t),$$
$$\nabla^\varepsilon \cdot \boldsymbol{u}_m^\varepsilon = 0 \quad \text{in } \Omega_m^\varepsilon(t)$$



Sketch of the axisymmetric pore filled by two fluids

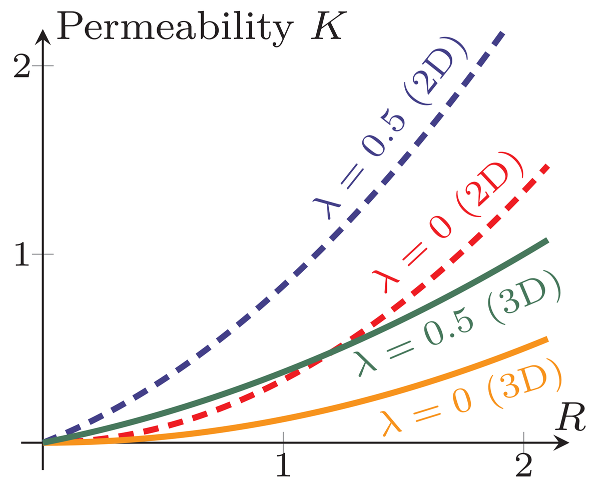
- In- and outflow conditions at $z = 0, 1$, axial symmetry at $r = 0$
- Navier-slip at the pore wall $\Gamma_\mathrm{w}^\varepsilon$: $\boldsymbol{t}_\mathrm{w}^\varepsilon \cdot (\boldsymbol{u}_m^\varepsilon + 2\varepsilon\lambda^\varepsilon \mathbf{D}^\varepsilon(\boldsymbol{u}_m^\varepsilon)\boldsymbol{n}_\mathrm{w}^\varepsilon) = 0,\quad \boldsymbol{u}_m^\varepsilon \cdot \boldsymbol{n}_\mathrm{w}^\varepsilon = 0$
- Surface tension and dynamic & hysteretic contact angle at the moving fluid-fluid interface $\Gamma^\varepsilon(t)$

$$-(p_\mathrm{I}^\varepsilon - p_\mathrm{II}^\varepsilon)\boldsymbol{n}_\Gamma^\varepsilon + 2\varepsilon^2\big(\mathbf{D}^\varepsilon(\boldsymbol{u}_\mathrm{I}^\varepsilon) - \mathrm{M}\,\mathbf{D}^\varepsilon(\boldsymbol{u}_\mathrm{II}^\varepsilon)\big)\boldsymbol{n}_\Gamma^\varepsilon = \tfrac{(d-1)}{\mathrm{Ca}}\kappa^\varepsilon \boldsymbol{n}_\Gamma^\varepsilon, \qquad \boldsymbol{u}_\mathrm{I}^\varepsilon = \boldsymbol{u}_\mathrm{II}^\varepsilon,$$
$$\partial_t(\gamma_z^\varepsilon \boldsymbol{e}_z + \varepsilon\gamma_r^\varepsilon \boldsymbol{e}_r) \cdot \boldsymbol{n}_\Gamma^\varepsilon = \boldsymbol{u}_\mathrm{I}^\varepsilon \cdot \boldsymbol{n}_\Gamma^\varepsilon, \qquad \cos(\theta(-\partial_t \boldsymbol{\gamma}^\varepsilon \cdot \boldsymbol{t}_\mathrm{w}^\varepsilon|_{z=\gamma_z^\varepsilon})) = \boldsymbol{t}_\Gamma^\varepsilon \cdot \boldsymbol{t}_\mathrm{w}^\varepsilon\big|_{z=\gamma_z^\varepsilon} \text{ at } s = 1$$

Upscaled model and averaged quantities

- Hagen–Poiseuille flow in bulk domains ⇒ **Darcy's law**

$$\bar{u} = -K\partial_z \bar{p}_m \quad \text{with} \quad K = \begin{cases} \frac{1}{3}R(3\lambda + R) & \text{(2D)} \\ \frac{1}{8}R(4\lambda + R) & \text{(3D)} \end{cases}$$



- Young–Laplace law at the interface ⇒ **Dynamic capillary pressure**

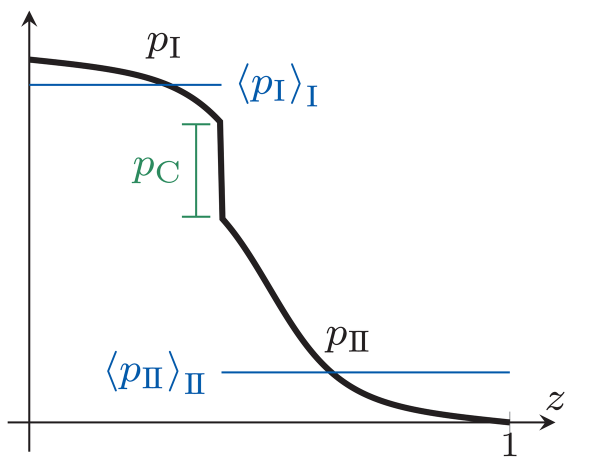
$$S = \frac{\mathcal{V}(\gamma_z)}{\mathcal{V}(1)}, \qquad \langle p_\mathrm{I}\rangle_\mathrm{I} - \langle p_\mathrm{II}\rangle_\mathrm{II} = p_\mathrm{C}(S,\partial_t S) + \tau(S)\partial_t S$$

- Local capillary pressure

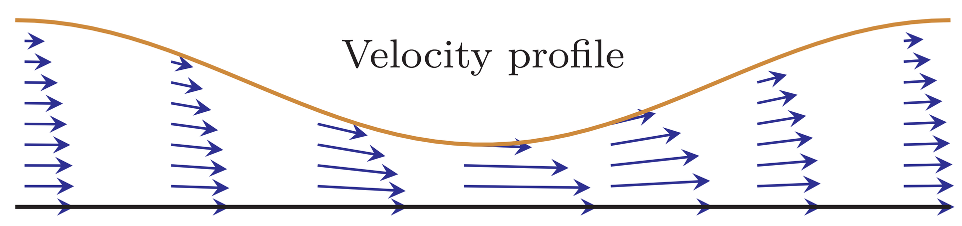
$$p_\mathrm{C}(S,\partial_t S) = \frac{(d-1)\cos\left(\theta\left(\frac{\mathcal{V}(1)}{\mathcal{A}(S)}\partial_t S\right)\right)}{\mathrm{Ca}\,\mathcal{R}(S)}$$

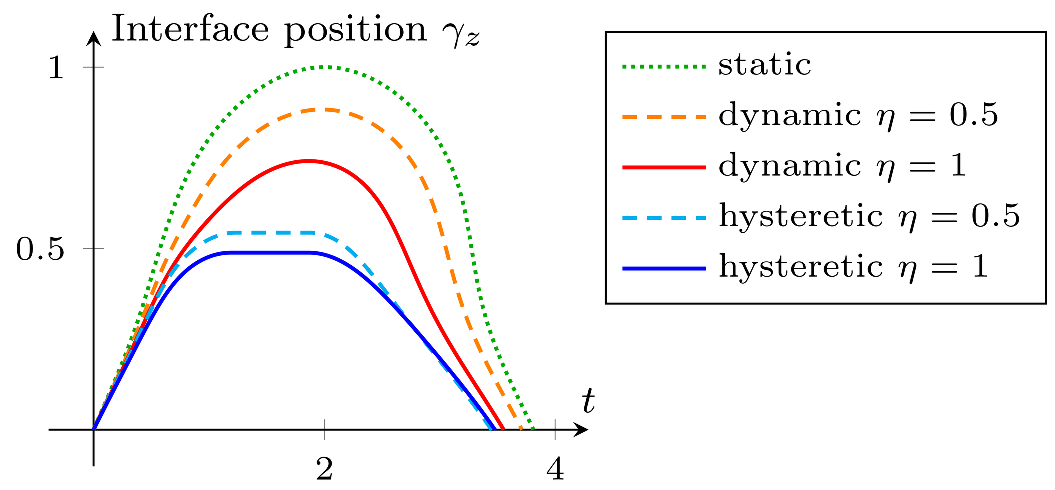
- Dynamic effect

$$\tau(S) = (\mathcal{V}(1))^2\left(\frac{1}{S}\int_0^S \frac{\sigma}{(\mathcal{A}(\sigma))^2\mathcal{K}(\sigma)}d\sigma + \frac{\mathrm{M}}{1-S}\int_S^1 \frac{1-\sigma}{(\mathcal{A}(\sigma))^2\mathcal{K}(\sigma)}d\sigma\right)$$



Numerical example: 2D "constricted pore"



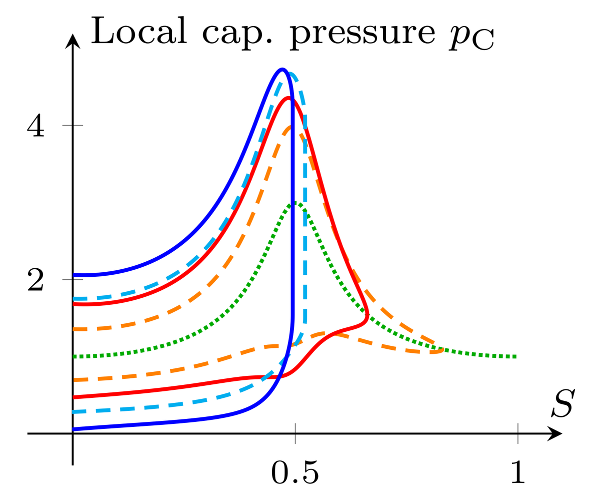


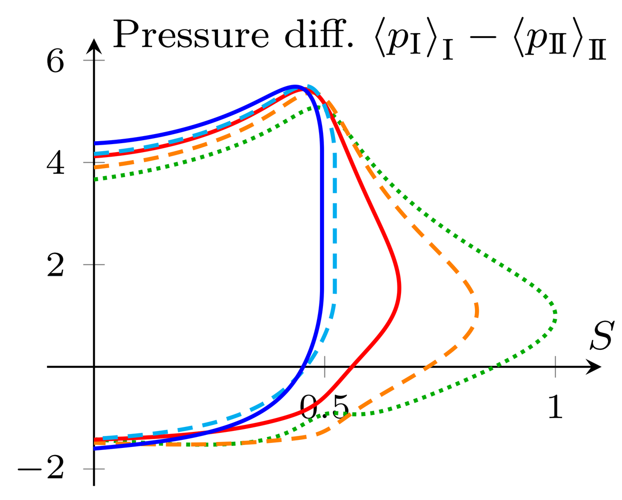
- Pressure cycle $p_\mathrm{in}(t) = 9 - 4t$
- Dynamic and hysteretic contact angle models

$$\cos\theta(u) = \cos\left(\frac{\pi}{3} - \frac{\pi \mathrm{sgn}(u)}{12}\right) - \eta \mathrm{Ca} u$$

→ Reduced interface velocity

→ Enhanced local capillary pressure and phase-pressure difference





Conclusion and future work

- Upscaled model: Darcy's law + dynamic and hysteretic capillary pressure
- Asymptotic solution for two-phase flow model with dynamic/hysteretic contact angle derived
- Extension for gravity / external forces straightforward
- Model validated using capillary rise experiments

◦ Implementation of the upscaled model for the pore throats in a dynamic pore-network model

◦ Upscaling of the dynamic pore-network model to obtain effective core-scale properties / models

References

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[2] S. B. Lunowa, C. Bringedal, and I. S. Pop, *On an averaged model for immiscible two-phase flow with surface tension and dynamic contact angle in a thin strip*, Stud. Appl. Math., 147 (2021). doi.org/10.1111/sapm.12376

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