FROSch – A framework for parallel Schwarz preconditioners in Trilinos

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Introduction

Schwarz methods are an algorithmic framework for a large class of domain decomposition methods. The software FROSch (Fast and Robust Overlapping Schwarz) [2], which is part of the Trilinos package ShyLU, provides a **highly scalable implementation of the Schwarz framework**. FROSch currently focuses on Schwarz solvers that are **algebraic** in the sense that they **can be constructed from a fully assembled, parallel distributed matrix**. This is facilitated by the use of **extension-based coarse spaces**, such as generalized Dryja–Smith–Widlund (GDSW) type coarse spaces.

The GDSW preconditioner

Consider the sparse linear system $Ax = b$ arising from a finite element discretization of an elliptic boundary value problem on a domain Ω. Let Ω be decomposed into N nonoverlapping subdomains. By recursively adding layers of elements to the subdomains, we obtain overlapping subdomains $\{\Omega_i'\}_{i=1}^N$; see Figure 1.

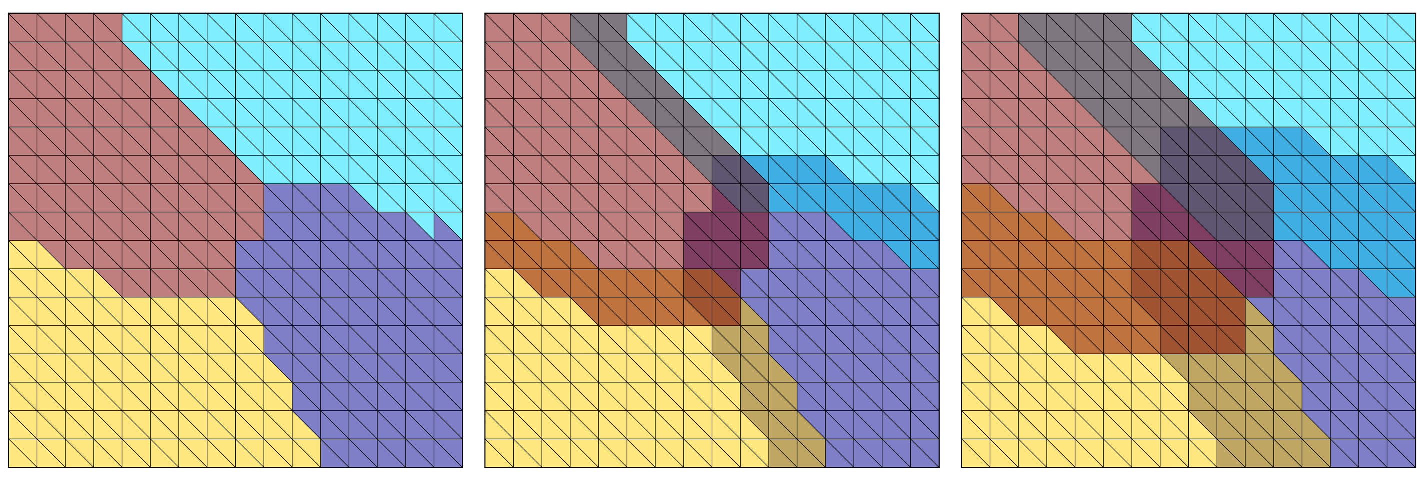

Figure 1: Recursive construction of overlapping subdomains with overlap $\delta = 0h, 1h, 2h$ (left, middle, right).

The classical GDSW preconditioner [1] is a two-level overlapping Schwarz preconditioner with exact coarse and local solvers,

$$M_{\text{GDSW}}^{-1} = \Phi A_0^{-1} \Phi^T + \sum\nolimits_{i=1}^{N} R_i^T A_i^{-1} R_i,$$

with the coarse problem matrix $A_0 = \Phi^T A \Phi$ and the matrices $A_i = R_i A R_i^T$ corresponding to the local overlapping problems. Here, R_i and R_i^T are the restriction and prolongation operators corresponding to Ω_i' and columns of the matrix Φ correspond to the coarse GDSW basis functions; cf. Figure 2.
We then obtain the condition number bound

$$\kappa(M_{\text{GDSW}}^{-1} A) \leq C \left(1 + H/\delta\right) \left(1 + \log\left(H/h\right)\right).$$

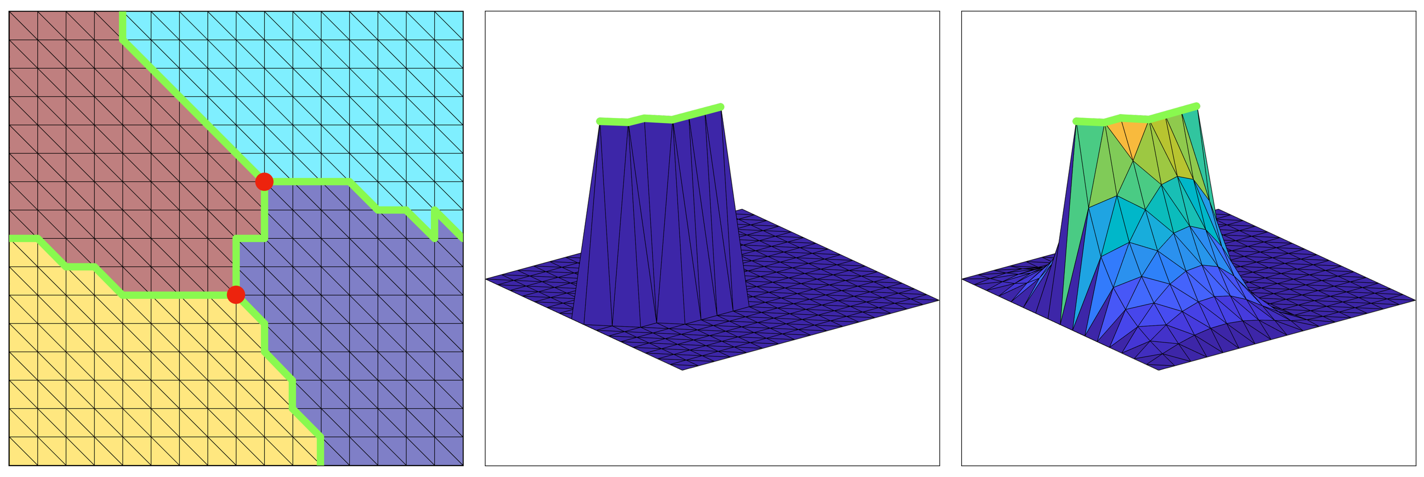

Figure 2: Construction of the GDSW coarse basis functions: interface components (left), restriction of the null space of the Neumann operator corresponding to A (middle) and energy-minimizing extension (right).

Results

Parallel scalability Parallel simulations for a 3D linear elasticity problem show the excellent parallel scalability of the three-level reduced dimension GDSW (RGDSW) preconditioner in FROSch; see Figure 3.

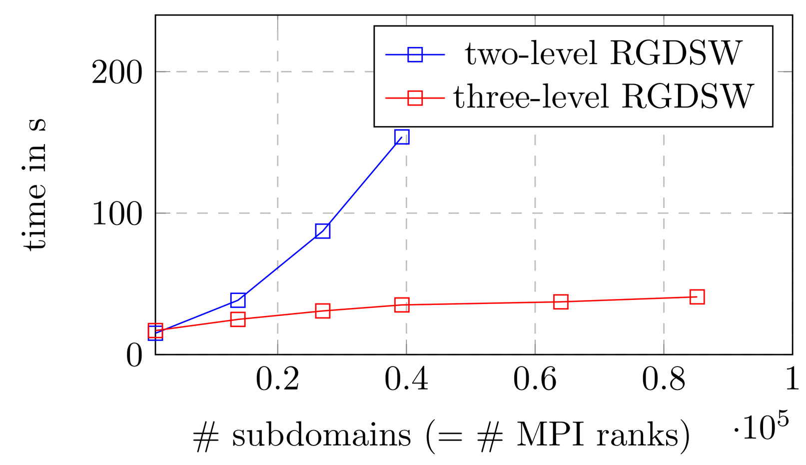

Figure 3: Weak scaling tests on the SuperMUC-NG supercomputer at LRZ, Germany. Joint work with O. Rheinbach and F. Röver (TUBAF, Germany).

Flexibility and robustness Application to challenging land ice simulations; see Table 1 for results for a velocity problem on a uniform Antarctica mesh and a coupled velocity-temperature problem on a locally refined Greenland mesh.

# MPI	Antarctica (vel.)			Greenland (vel. & temp.)		
ranks	∅ its	setup	solve	∅ its	setup	solve
512	41.9 (11)	25.10 s	12.29 s	41.3 (36)	18.78 s	4.99 s
1 024	43.3 (11)	9.18 s	5.85 s	53.0 (29)	8.68 s	4.22 s
2 048	41.4 (11)	4.15 s	2.63 s	62.2 (86)	4.47 s	4.23 s
4 096	41.2 (11)	1.66 s	1.49 s	68.9 (40)	2.52 s	2.86 s
8 192	40.2 (11)	1.26 s	1.06 s	-	-	-

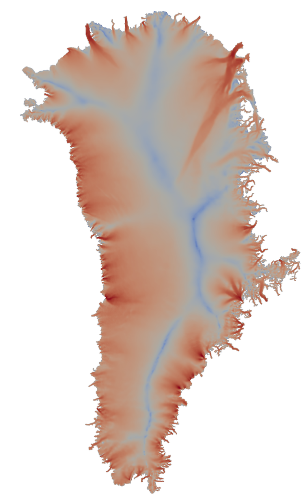

Table 1: Strong scaling tests for large-scale land ice simulations on the Cori supercomputer (NERSC, USA): GMRES iteration counts as well as setup and solve times are averaged over the numbers of Newton iterations (in parenthesis). Joint work with M. Perego and S. Rajamanickam (SNL, USA).

References

[1] C. R. Dohrmann, A. Klawonn, and O. B. Widlund. Domain decomposition for less regular subdomains: overlapping Schwarz in two dimensions. *SIAM J. Numer. Anal.*, 46(4):2153–2168, 2008.

[2] A. Heinlein, A. Klawonn, S. Rajamanickam, and O. Rheinbach. *FROSch: A Fast And Robust Overlapping Schwarz Domain Decomposition Preconditioner Based on Xpetra in Trilinos*, pages 176–184. Springer, 2020.