# Electromagnetic Wave Simulations in Photonic Quasicrystals

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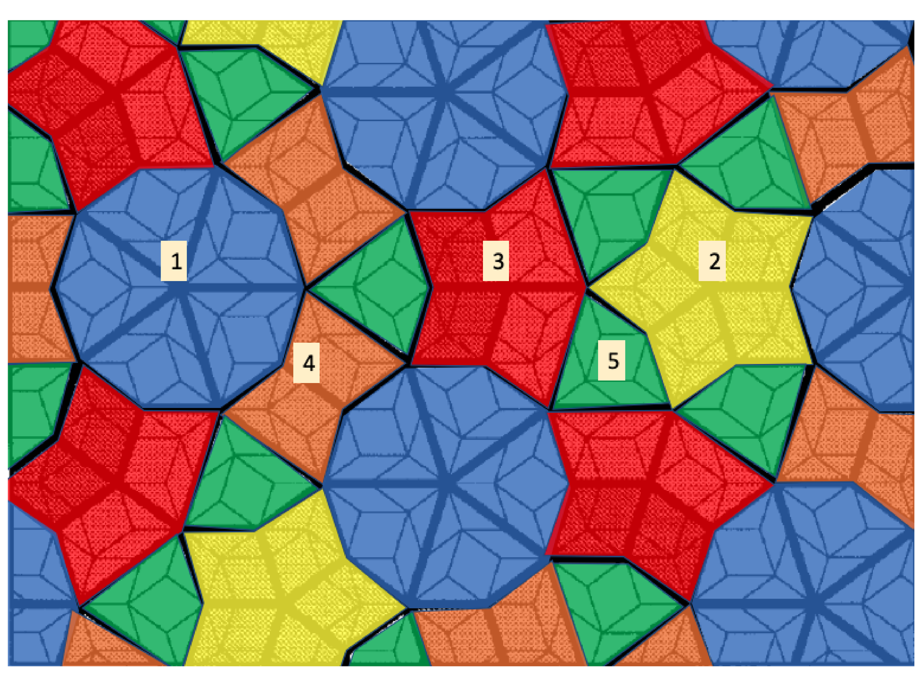
## Introduction

Photonic quasicrystals [1, 2]:

- Quasiperiodic: long-range order without translational symmetry;
- Strong ability to manipulate visible light;
- Increase light-harvesting performances.

Develop a numerical method for light propagation in quasicrystals adapted to quasiperiodicity [3, 4, 5]. Electromagnetic simulations will allow the evaluation of quasicrystals configurations.

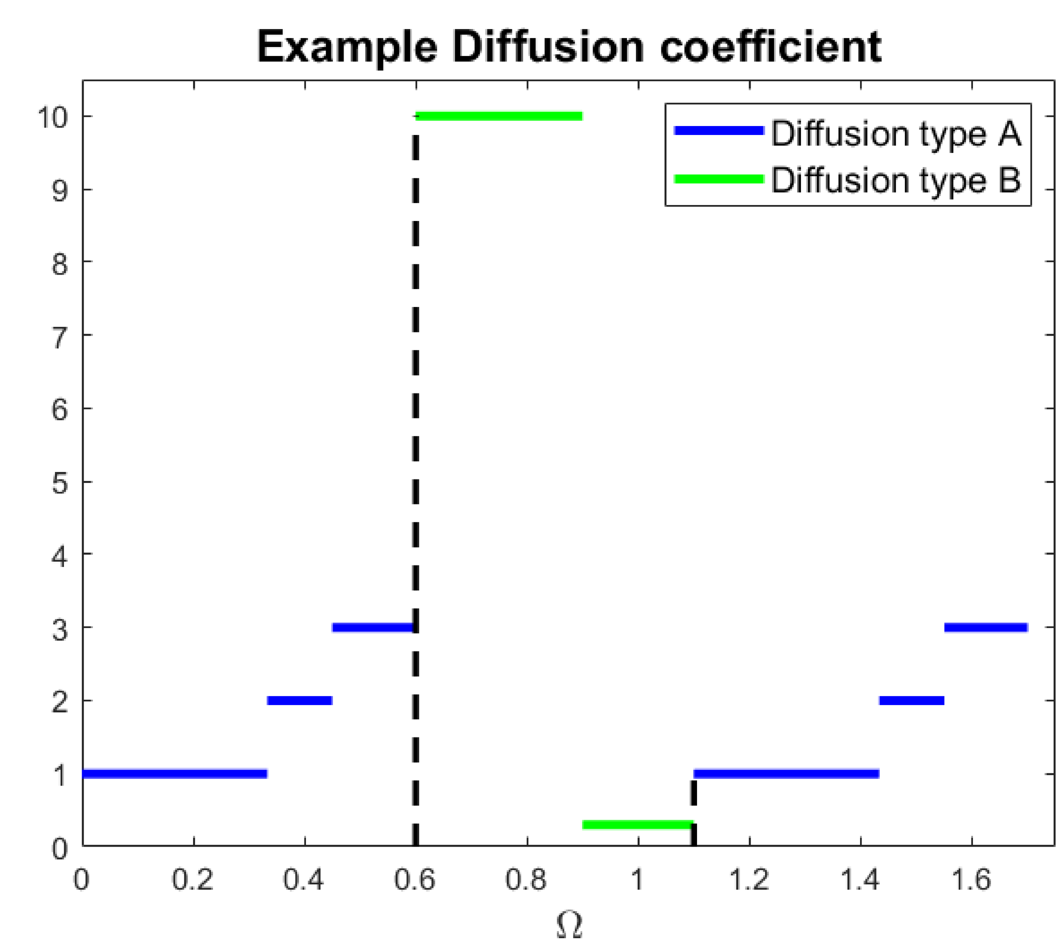



## Model Problem

$1D$ Elliptic PDE:

$$\begin{cases} -\operatorname{div}(a(x)u'(x)) = f(x), & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega, \end{cases}$$

$\Omega = \cup_{k=0,\dots,N_C} \Omega_k \subset \mathbb{R}$ bounded, $\Omega_k = (z_k, z_{k+1})$ are affine images of few reference cells $\hat{\Omega}_k$.
$f \in L^2(\Omega)$, $a(x)$ piecewise constant quasiperiodic.




## Conclusion

- Developed method adapted to quasiperiodic problem structure;
- Eigenproblem solved only on reference domains;
- Linear system has reduced dimension and cheap resolution;
- Micro-Macro convergence is at least of first order;
- For regular source term, convergence improves by at least half an order;
- Micro-Macro approximation is already quite accurate with few degrees of freedom;
- Local adaptation of approximation space is straightforward.

## The Micro-Macro Approximation

Find $u_m \in V_{N_C}^P : \int_\Omega a(x)u_m'(x)v'(x)\,dx = \int_\Omega f(x)v(x)\,dx \;\; \forall v \in V_{N_C}^P$.

$$V_{N_C}^P := \operatorname{span}\left\{\{\varphi_k\}_{k=1,\dots,N_C-1},\ \{m^{k,q}\}_{k=0,\dots,N_C-1}^{q=1,\dots,P}\right\} \subset H_0^1(\Omega).$$

**Macroscale functions:** Harmonic liftings,

$$\varphi_k \in H_0^1((z_{k-1}, z_{k+1})), \quad k = 1, \dots, N_C - 1.$$

$$\begin{cases} -\operatorname{div}(a(x)\varphi_k'(x)) = 0, & x \in \Omega_{k-1} \cup \Omega_k, \\ \varphi_k(z_{k-1}) = 0, \quad \varphi_k(z_k) = 1, \quad \varphi_k(z_{k+1}) = 0. \end{cases}$$

**Microscale functions:** $P$ smallest eigenfunctions per cell,

$$m^{k,q} \in H_0^1(\Omega_k),\ k = 0, \dots, N_C - 1,\ q = 1, \dots, P.$$

$$\begin{cases} -\operatorname{div}(a(x)m^{k,q}(x)') = \lambda_q^k m^{k,q}(x), & x \in \Omega_k, \\ m^{k,q}(x) = 0, & x \in \partial\Omega_k, \end{cases}$$

**Remark:** computation only on reference cells.

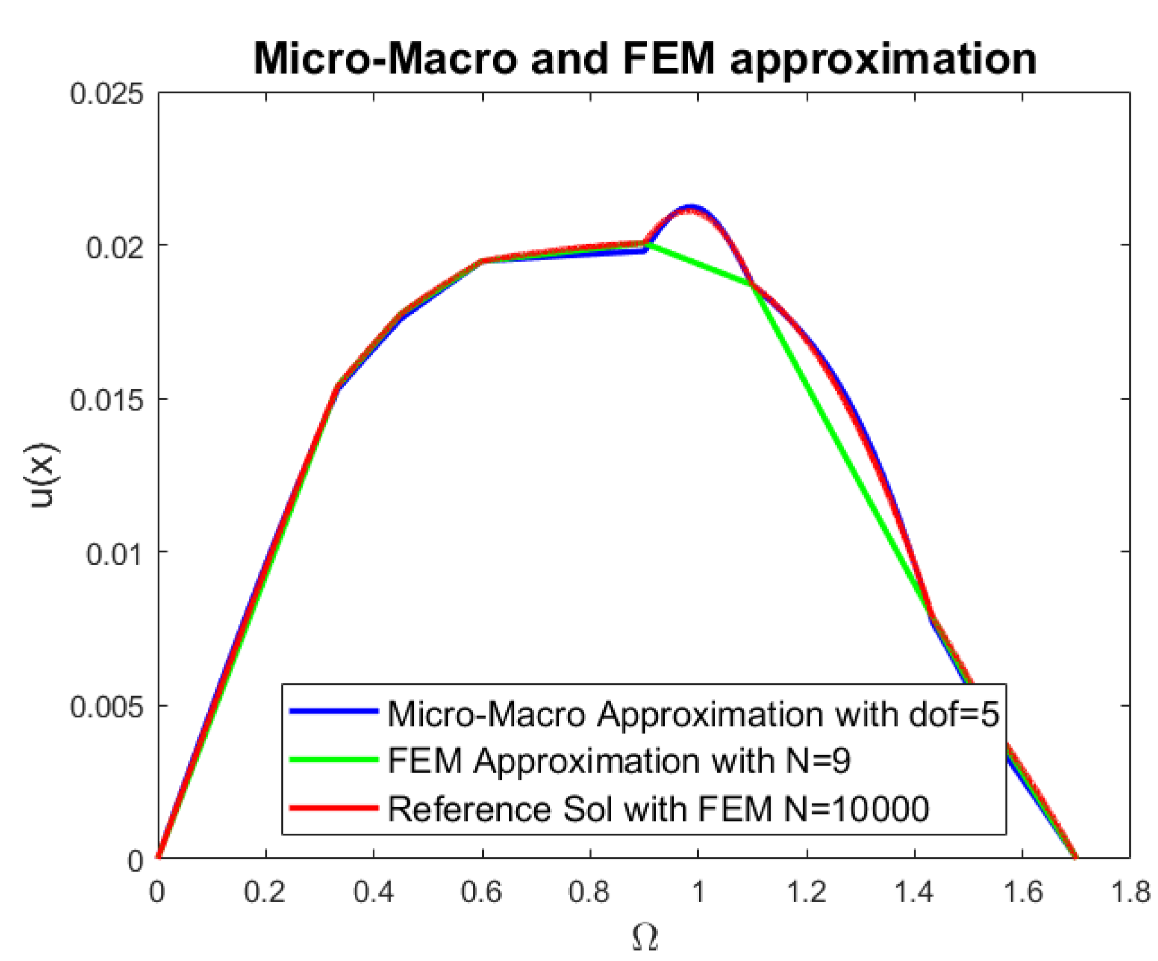



**Figure:** Input data: $\Omega$ and $a(x)$ as in example, $f(x) = \sin(\frac{x}{10})$.

## Convergence Theorem

Let $u \in H_0^1(\Omega)$ be the PDE solution, and let $u_m \in V$ be its Micro-Macro approximation. We define the projection $\Pi_P : L^2(\Omega) \to \operatorname{span}\left\{\{m^{k,q}\}_{k=0,\dots,N_C-1}^{q=1,\dots,P}\right\} \subset L^2(\Omega)$ and a constant $C > 0$ independent of the mesh.
Then it holds the following:

$$\|u - u_m\|_{L^2(\Omega)} \le \frac{C}{(P+1)^2}\|f - \Pi_P f\|_{L^2(\Omega)},$$

$$\|\sqrt{a(x)}(u - u_m)'\|_{L^2(\Omega)} \le \frac{C}{P+1}\|f - \Pi_P f\|_{L^2(\Omega)}.$$

If $f \in H^1(\Omega_k)\ \forall\, k = 0, \dots, N_C - 1$, convergence gains half an order.

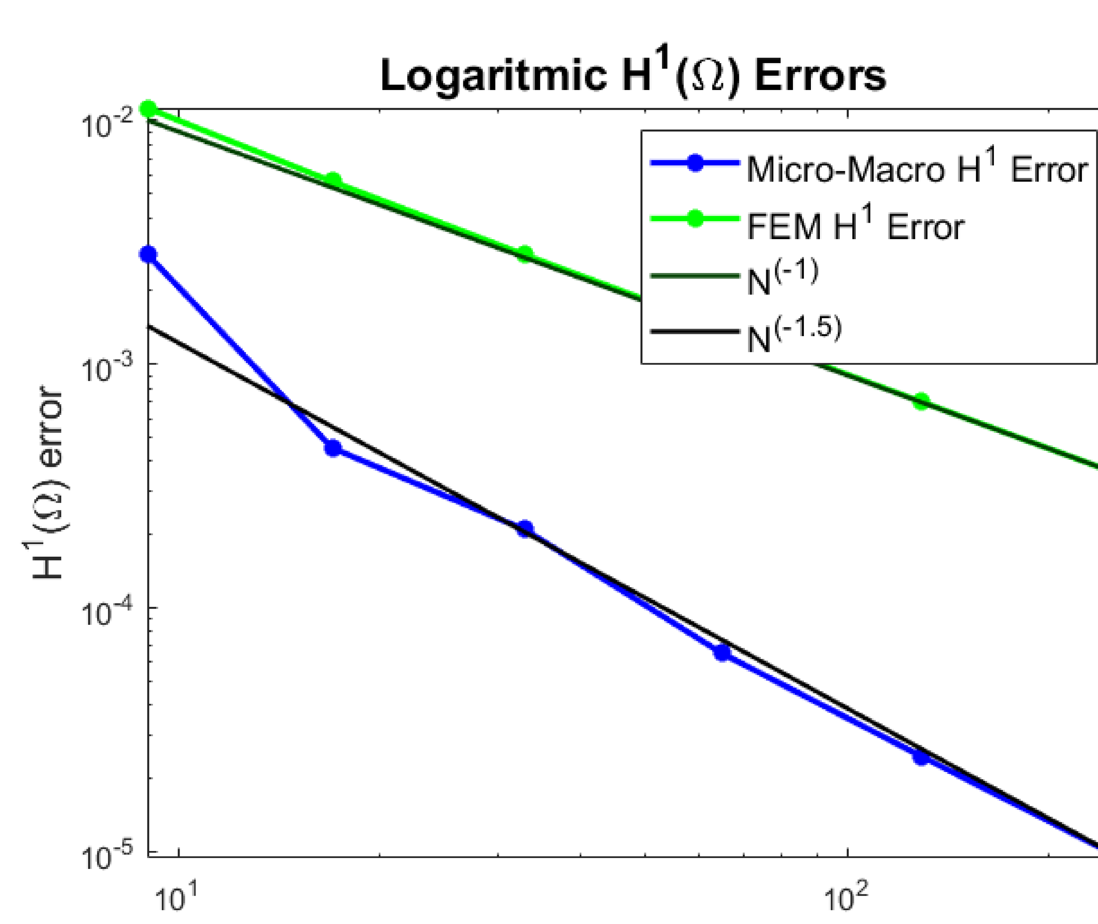



**Figure:** Input data: $\Omega$ and $a(x)$ as in example, $f(x) = \sin(\frac{x}{10})$.

## References

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## Further Information

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