# Multiscale FEM for light propagation through realistic photonic crystals

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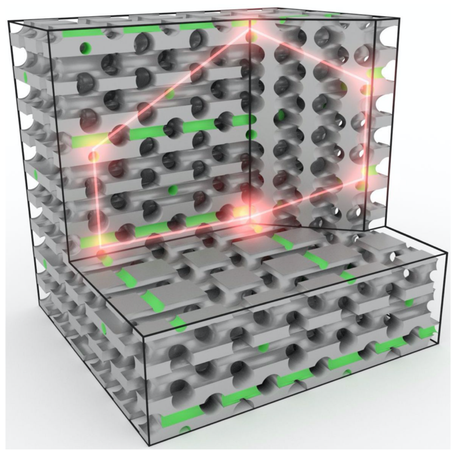
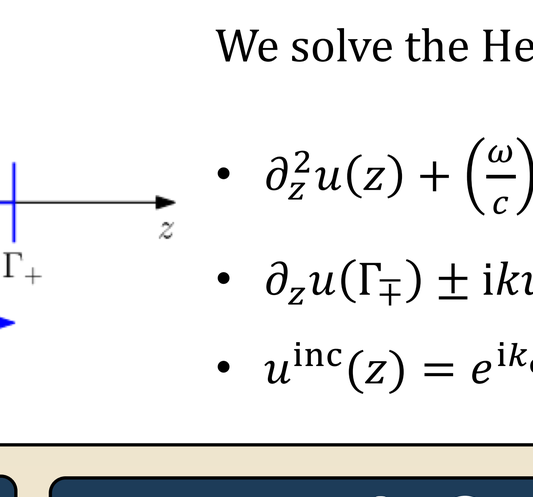
## 1. Photonic crystals

- Periodic refr. index + Complex structure => Manipulation of light

**Applications**

- Optical components
- Solar cells
- Lasers
- Quantum computing
- *Etc.*






**Multiscale character**

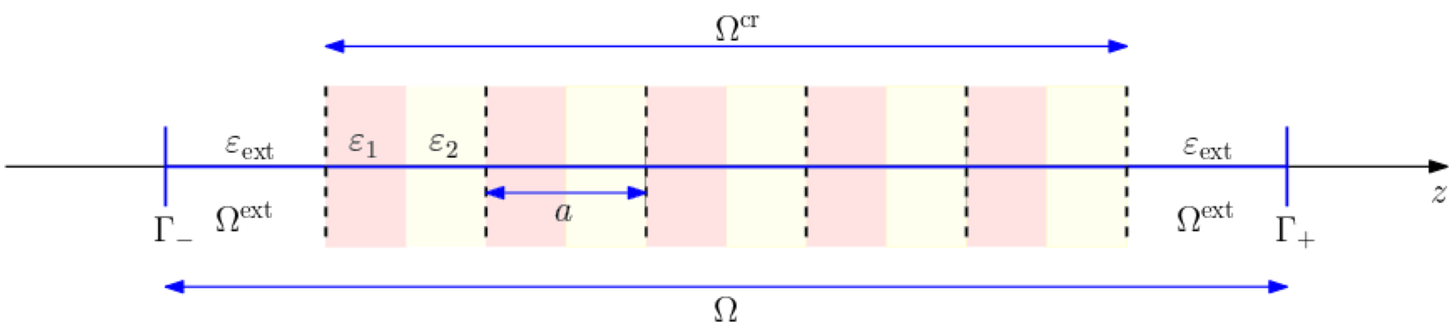
- Micro-scale: Unit cell
- Macro-scale: Whole crystal

**Computations**

- Crucial tool for physics
- In 3D extremely demanding

Goal: Obtain a size-robust numerical algorithm.

## 2. Problem formulation in 1D



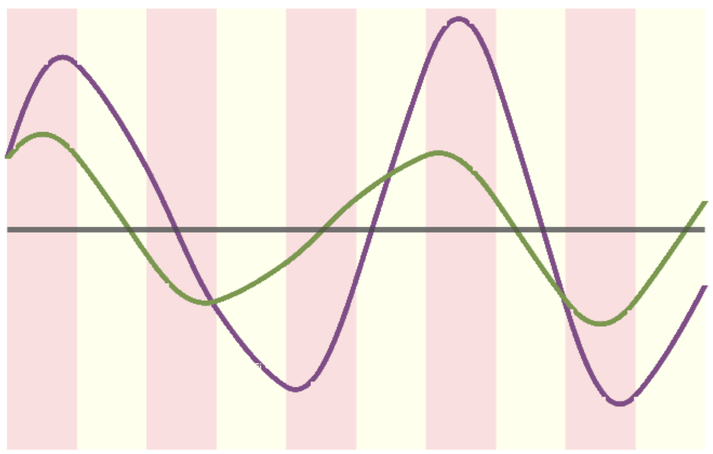

We solve the Helmholtz equation:

- $\partial_z^2 u(z) + \left(\frac{\omega}{c}\right)^2 \varepsilon(z) u(z) = 0$, for all $z \in \Omega$
- $\partial_z u(\Gamma_\mp) \pm \mathrm{i}ku(\Gamma_\mp) = \partial_z u^{\mathrm{inc}}(\Gamma_\mp) \pm \mathrm{i}ku^{\mathrm{inc}}(\Gamma_\mp)$
- $u^{\mathrm{inc}}(z) = e^{\mathrm{i}k_{\mathrm{ext}}z}$

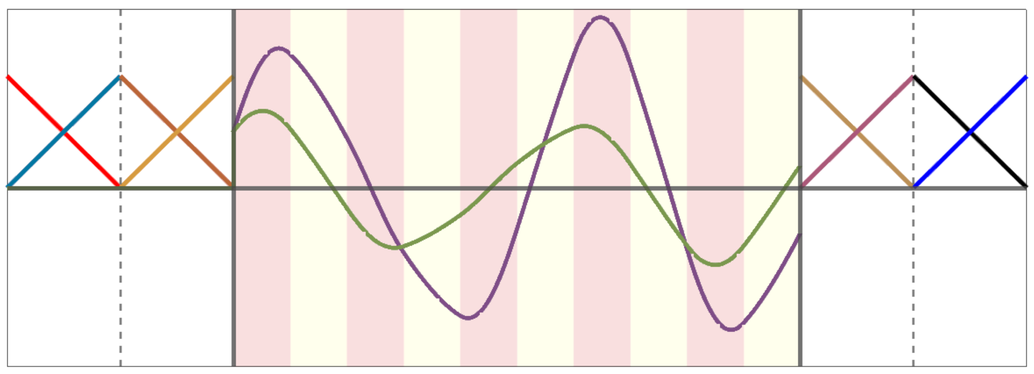
## 3. DG multiscale method

**Bloch modes** ≡ solutions to a globally periodic problem:

- $\partial_z^2 u_{\mathrm{per}}(z) + \left(\frac{\omega}{c}\right)^2 \varepsilon(z) u_{\mathrm{per}}(z) = 0$, for all $z \in [0, L]$
- $u^{\mathrm{Bloch}}(z) = u_{\mathrm{per}}(z)\, e^{\mathrm{i}kz}$, $u_{\mathrm{per}}(z) = u_{\mathrm{per}}(z + L)$



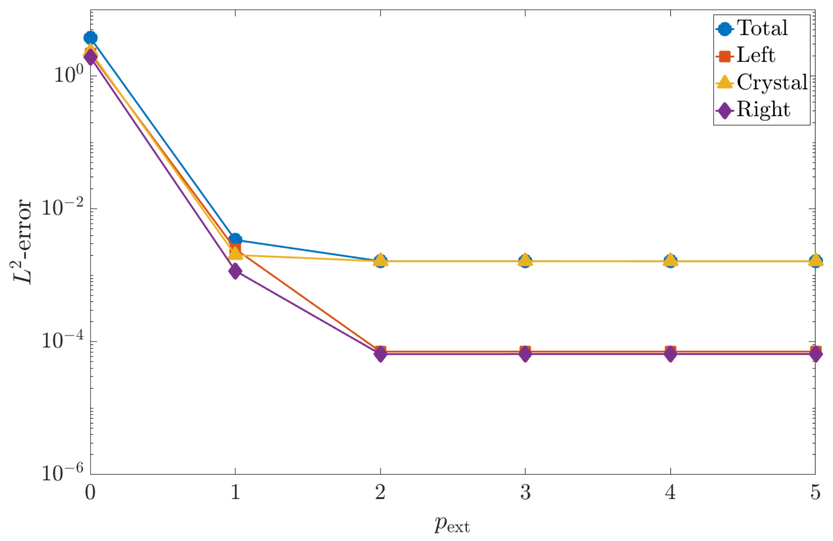
- Multiscale Discontinuous Galerkin finite element basis: Bloch modes in $\Omega^{\mathrm{cr}}$, piecewise polynomials in $\Omega^{\mathrm{ext}}$



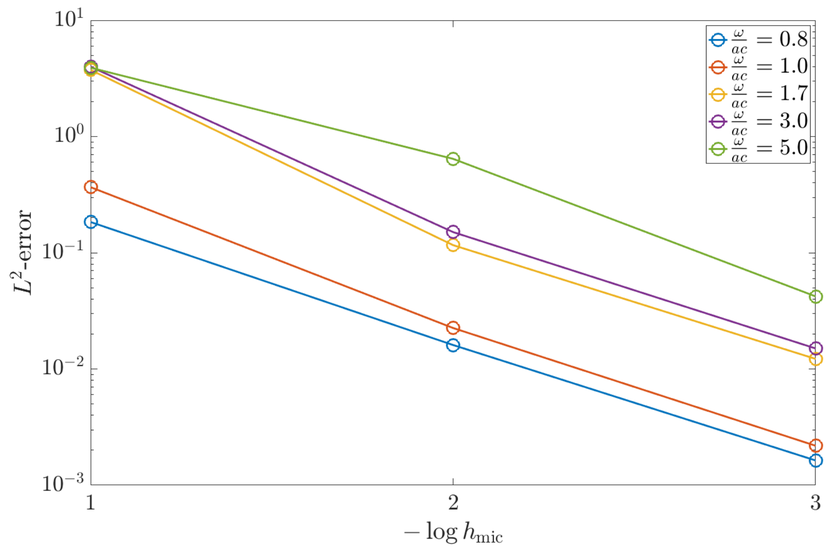
- The whole crystal treated as one finite element
- Our method is size robust

## 4. Convergence

- $\varepsilon_1 = 13\mathrm{F/m}$, $\varepsilon_2 = 3\mathrm{F/m}$, $N_{\mathrm{ext}} = 10$, $N_{\mathrm{cr}} = 10$
- $\frac{\omega}{ac} = 0.8$, $h_{\mathrm{mic}} = 10^{-3}$




- $p_{\mathrm{ext}} = 2$




## References

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