Model problem:

\[-\Delta u + b \cdot \nabla u + cu = \lambda u, \quad u = 0 \text{ on } \partial \Omega\]

where $b \in [L^\infty(\Omega)]^2$ and $c \in L^\infty(\Omega)$ is non-negative.

Traditional Approach

Call generalized eigensolver after each adaptivity step.

- Ordering of eigenvectors may change between adaptivity steps.
- For repeated eigenvalues, it may return an arbitrary linear combination.
- There is no one optimal mesh to suit all $n$ eigenfunctions.
Call generalized eigensolver **only once**:

- Pursue each eigenvalue-eigenvector pair on an individual mesh.
- Adaptive $hp$-FEM combined with Newton or Picard.
- Keeps focus on the selected eigenvalue-eigenvector pair.
- Avoids linear combinations in case of repeated eigenvalues.
Algorithm (Picard)

\[ Au^0 = \lambda^0 Bu^0 \]

\[ Au^{m+1} := \lambda^m Bu^m \]
\[ \lambda^{m+1} := \frac{(u^{m+1})^t Au^{m+1}}{(u^{m+1})^t Bu^{m+1}} \]
\[ m := m + 1 \]

Eigenfunction from coarse mesh used as initial condition on globally refined mesh. Adaptivity: Based on solution pairs, same as in standard \( hp \)-FEM.
Algorithm (Newton)

\[ 0 = f(x, \lambda) := \begin{pmatrix} Ax & - \lambda Bx \\ x^T B x & -1 \end{pmatrix} \]

Adaptivity - same as with Picard.
Example 1

Eigenfunctions #1 and #2
Example 2

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Selected Topics in Adaptive Higher-Order FEM