Multiphysics Problems

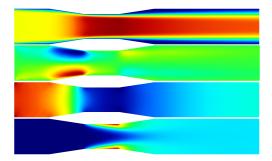


Figure: Laminar N-S equations with heat transfer

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Multiphysics Problems

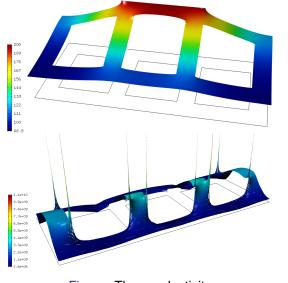


Figure: Thermoelasticity

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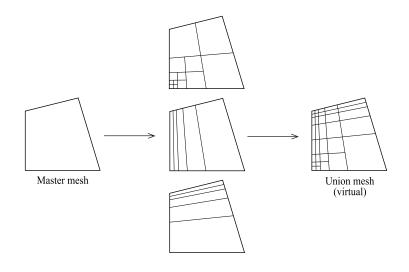
Techniques for Working with Multiple Meshes

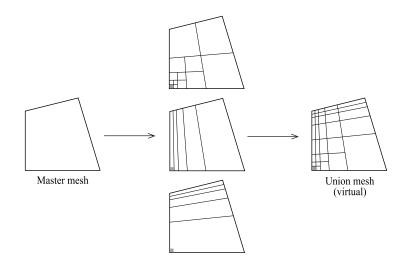
- Interpolation methods (large) data losses
- Projection methods (smaller) data losses
 - X. Jiao and M. T. Heath. Common-Refinement-Based Data Transfer Between Non-Matching Meshes in Multiphysics Simulations. Internat. J. Numer. Methods Engrg., 61(14):2402-2427, 2004.
- Monolithic multimesh FEM no data losses
 - P. Solin, J. Cerveny, L. Dubcova, D. Andrs: Monolithic Discretization of Linear Thermoelasticity Problems via Adaptive Multimesh hp-FEM, J. Comput. Appl. Math 234 (2010) 2350 - 2357.
 - L. Dubcova, P. Solin, J. Cerveny, P. Kus: Space and Time Adaptive Two-Mesh hp-FEM for Transient Microwave Heating Problems, Electromagnetics, Vol. 30, Issue 1, pp. 23 - 40, 2010.
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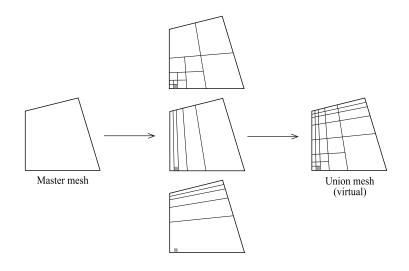
Comparison

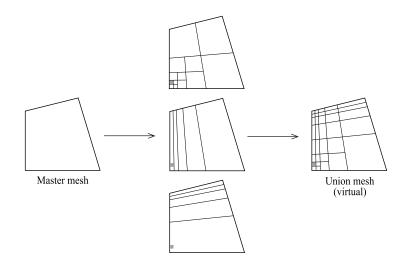
L. Dubcova, P. Solin, G. Hansen, H. Park: Comparison of Multimesh hp-FEM to Interpolation and Projection Methods for Spatial Coupling of Reactor Thermal and Neutron Diffusion Calculations, J. Comput. Phys. 230 (2011) 1182-1197.

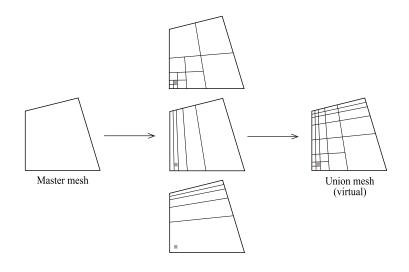
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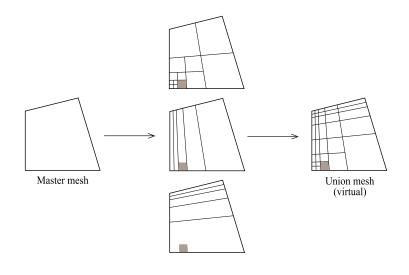




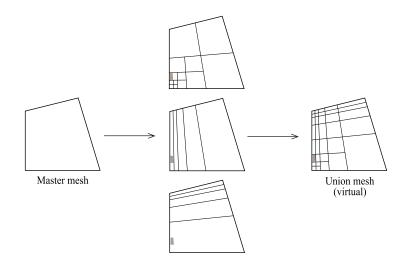


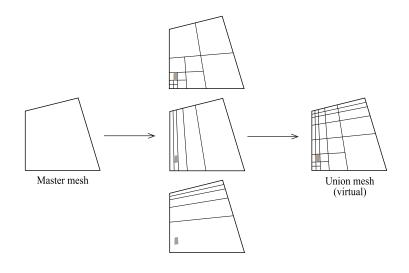


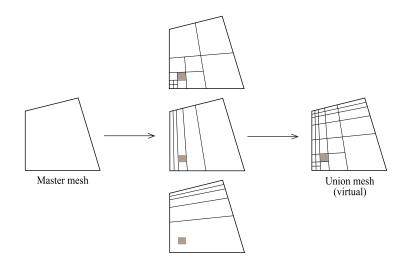




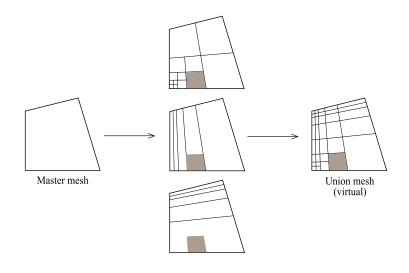
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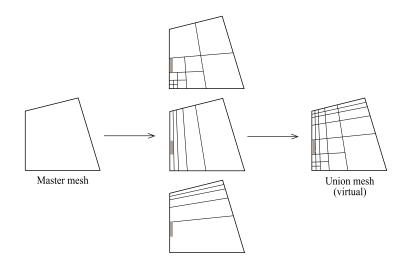


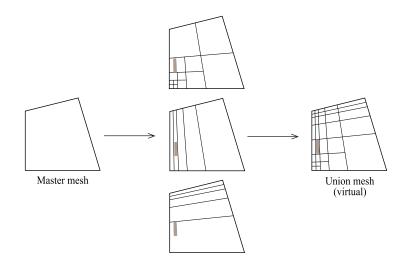


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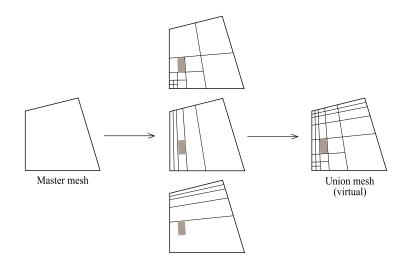


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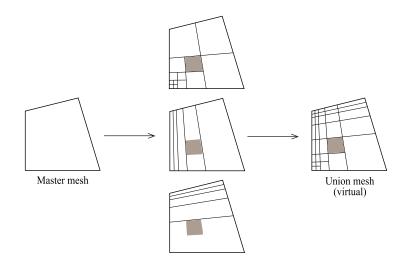




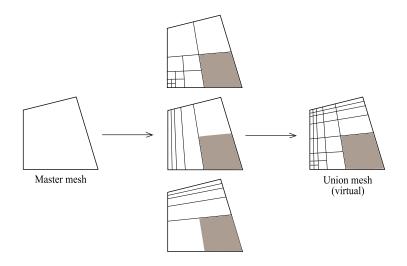
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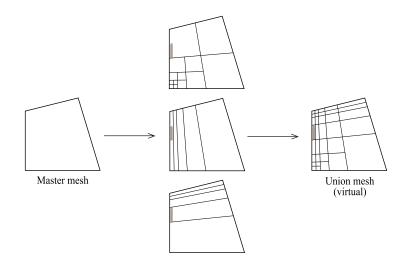
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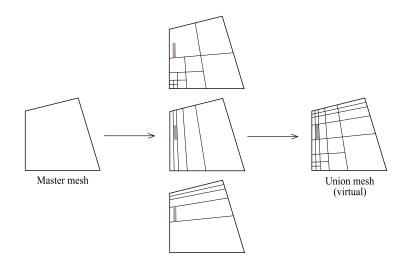


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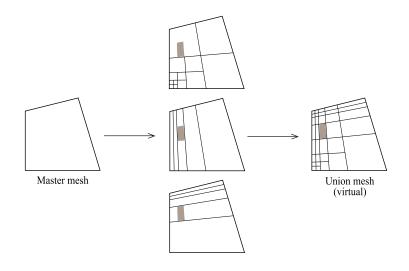


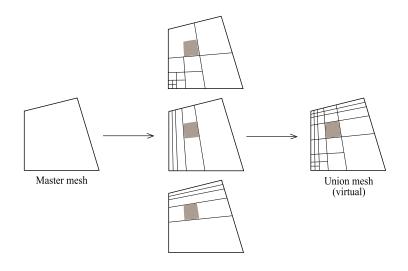
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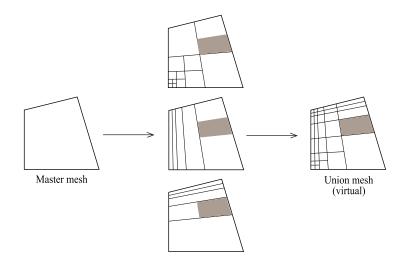


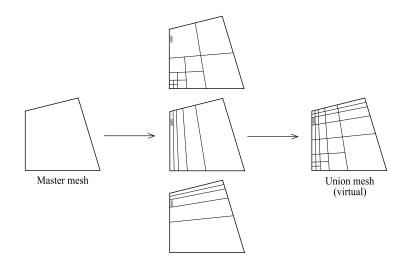


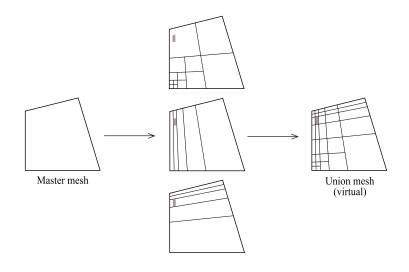
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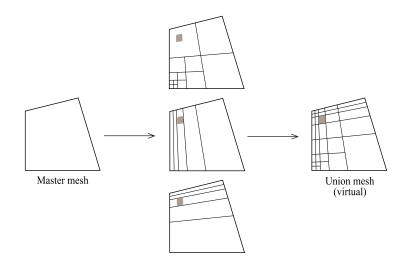


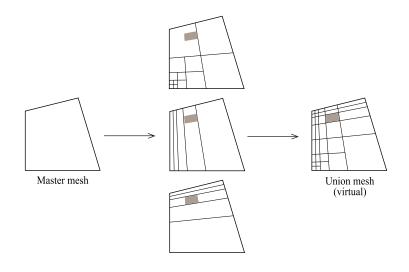




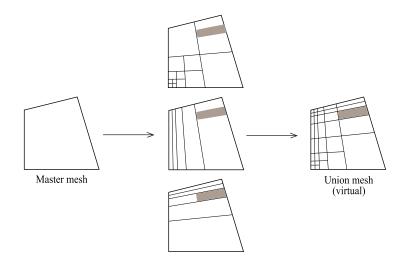




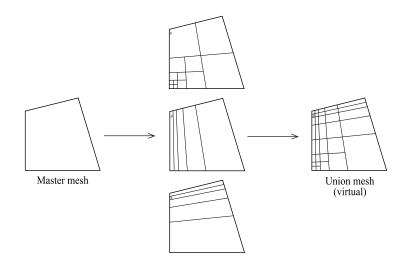


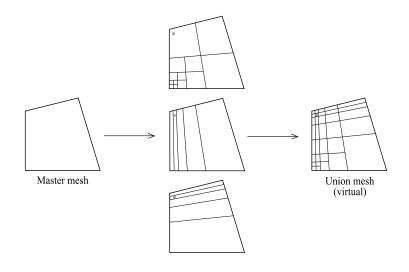


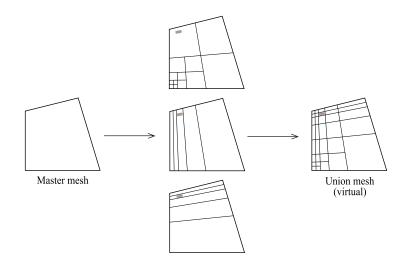
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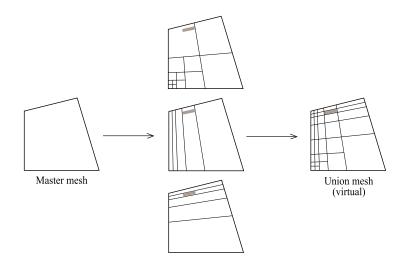
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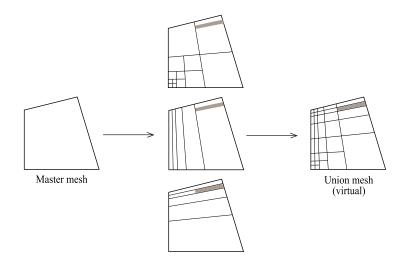


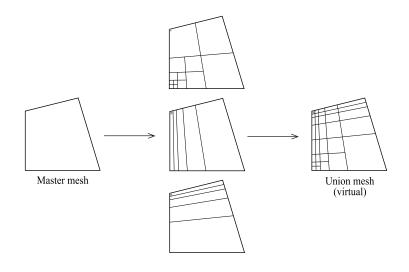


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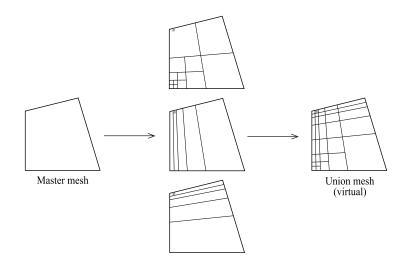


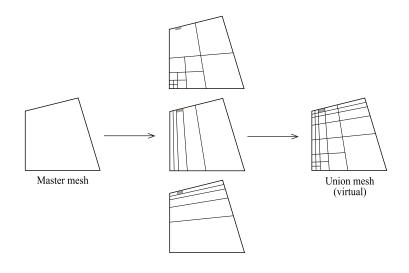
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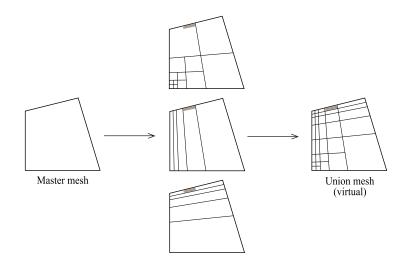


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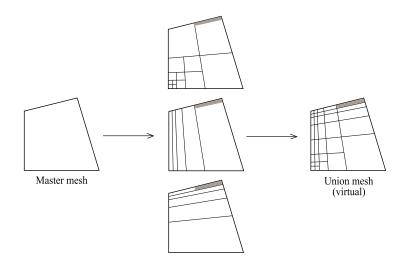




Monolithic Multimesh FEM



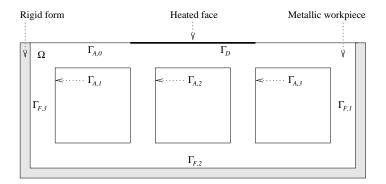
Monolithic Multimesh FEM



Automatic adaptivity:

- Almost the same as in single-mesh *hp*-FEM.
- Put all elements of all meshes into one list.
- Sort according to their error estimates.
- Refine those with the largest errors.

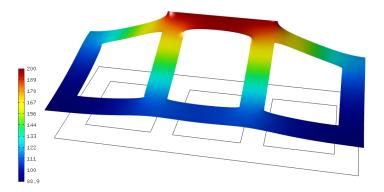
Thermoelasticity



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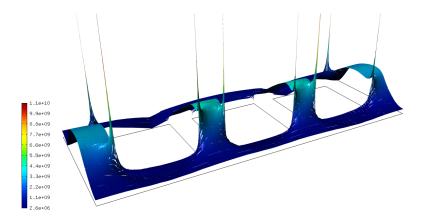
Thermoelasticity

Solution: temperature



Thermoelasticity

Solution: stress



Thermoelasticity (step 1)

2	2	2	2	2	2	2	2	2	2
2			2			2			2
z			2			2			2
z	2	2	2	2	2	2	2	2	2

2	2	2	2	2	2	2	2	2	2
2			2			2			2
2			2			2			2
2	2	2	2	2	2	2	2	2	2

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Thermoelasticity (step 2)

2		1 2 1 2	2	2	2	2	2	1 1 1 2	2
1 1 2 1 1			2			2			2 1 1 1
1 1 1 1			1 21 2			1 21 2			1 1 1 1
1 1 1 1 2			2 31 2			1 22 3			1 1 2 1 1
2			2 1 2	2 1 1 1	1 2 1 1	2	2 <u>1</u> 1 1 1 1 1 1	1 2 1 1	2
2	2	2	2	2	2	2	2	2	2
2			2			2			2
2			z			2			2
2	2	2	2	2	2	2	2	2	2

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Thermoelasticity (step 3)

2	1 2	1	2	2	2	2	2	2	1 1 1 2	2
$ \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array} $				2			2			$\begin{array}{c}1\\11\end{array}$
1 1				1 21 2			1 21 2			1 1
1 1 1	2 <u>1</u> 1 1	1	1 1 2	11	2 <u>1</u> 1	1 <u>1 2</u> 1 1 1	11	2 <u>1</u> 1 1 1 1	1 1 2	1111
2	1 1	1	1 1 1 1 1	312	1 1 1	1 1 1 1 1	312	1 1 1 1 1	1 1 1 1 1	2
2	2	2	2	2	2	2	2	2	2	2
2				2			2			2

2			2			2			2
2	2	2	2	2	2	2	2	2	2

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Thermoelasticity (step 4)

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	2	2	2	2	2	2	1 2	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
		2			2			1 111
1 1 1 1 1 1 2		1 21 2			1 21 2			1 1 1 2 1
$1 3\frac{1}{1}$. 1		H1 .	. 1	11	1 1		11113
$3 2 \qquad \begin{array}{c c} 1 & 1 \\ \hline 1 & 1 \\ \hline 2 & 1 \end{array}$	1 1 1 1 2	312	$ \begin{array}{c c} 1 \\ 1 \\ 1 \end{array} $ 1 1		312	1 1 1 2 1	1	1 1 2 312

2	2	2	2	2	2	2	2	2	2
2			2			2			2
2			2			2			2
2	2	2	2	2	2	2	2	2	2

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Thermoelasticity (step 5)

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	2	2	2	2	2	2	1 1 2 <u>1</u> 1 1	$\begin{array}{c c}1&1\\1&1\\2&1\end{array}$
$1 3\frac{1}{11}$ 1 1 2		2			2			$\frac{1}{11}$
1 112		1 21 2			1 21 2			1 2 1
$1 3 \frac{1}{1} \frac{1}{1}$ $2 13 2 \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1}$	1 2	11 1 33	1 1 2	1 2	11 1 33	1 1 2	11111	11 1 3 3 22 1
312312 2 1	$ \begin{array}{c cccccccccccccccccccccccccccccccc$	2 2	1 1 112 2 1	1 1 1 1 2	2 2	1 1 1 2 2 1 1 1 1	1 1 1 1 1 2	312312

2	2	1	2 2		2	2	2	2	1	2	2
2	2	1	1	2 2		2	2	1	1	2	2
2				2			2				2
2				2			2				2
2	2	2	2	2	2	2	2	2	2	2	2

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Thermoelasticity (step 6)

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		2	2	2	2	2	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
113		1 2			2 1		111
1 21 3		1 1			1 1		1 31 2
1 21 2	1	12112			1 21 2	, ,	1 21 2
					11	T	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1 2	33 22	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	1 2 1 2 1 1 2 1 1 2	3 3 2 2	2 1 1 2 2 1 1 2 1 1	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

2	2	1 <u>1 2</u> 1 1 1 1 1	2 1 1 1	2	2	1 2 1 1	$ \begin{array}{c cccccccccccccccccccccccccccccccc$	2	2
2		й	2			2		,	2
2			2			2			2
2	2	2	2	2	2	2	2	2	2

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Thermoelasticity (step 7)

1 1 1 2 2 1 1 4	2 3 1	1 1 1	2 3 4	1 1 2 1 2 1
2 1 1 1 1 2	213 2	1 1 2	213 4	2 1 1 1 1 2
	1 2		2 1	
1 21 3	1 1		1 1	1 31 2
1 21 2	1 21 2		1 21 2	1 21 2
	1		1	
	3223 222221		222221	
312312 2 1 1 1 1 2	2 2 2	1 1 1 1 2	2 2 2 1	1 1 1 2 312312

2	2	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c c} 2 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{array} $	2	2	$ \begin{array}{c c} 1 & \frac{1 & 2}{1 & 1} \\ 1 & 1 \end{array} $		2	2
2			2			2			2
2			2			2			2
2	2	2	2	2	2	2	2	2	2

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Thermoelasticity (step 8)

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	415 1	1 1		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
2 2 2 1 2	2	1 1	2	2 1 2 2 1 2 2
	1		1	
1 21 3	1 1		1 1	1 31 2
1 21 2	1 21 2		1 21 2	1 2 1 2
			1	
	223 2222221	213 213 1	322	2 3 1 1 2 2 1
3 23 2 2 1 1 1 2 2	2 2 2	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	2 2 2 2	1 1 1 2 3 23 2 2 1 1 1 1 2

2	2	1 1 1	-1 11	1	1	1	1	1	1	$ 1 \\ 1 \\ 1 $	1 1 1 1	2	2
_	-	$1 \begin{array}{c} 1 1 \\ 1 1 \end{array}$	1	1	2	1	1	2	1	1	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-	-
2			1	2					2	1			2
2			1	1					1	1			2
2			2						:	z			2
2	2	2	2		2	2	2	2	:	z	2	2	2

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Thermoelasticity (step 9)

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$415 \frac{2}{1} \frac{1}{2} \frac{1}{1} \frac{1}{1}$	1 2 1 1 4 5	$4 \qquad \frac{\begin{array}{cccc} 1 & 1 \\ 1 & 1 \end{array}}{2} \begin{array}{cccc} 1 & 2 \\ 1 & 2 \end{array} \begin{array}{cccc} 1 & 2 \\ 2 \end{array} \begin{array}{cccc} 2 & 2 \\ 2 \end{array} \begin{array}{ccccc} 2 & 2 \end{array}$
	1+3	113	
1 21 3	1 1	1 1	1 31 2
2 1 2 2 1 2 3 3 23 2 2 1 1 2		3 2 3 2 2 2 1 1 1 2 2 2 2	2 3 1 2 2 2 1 1 1 2 2 3 2 2 1 1 2 3 2 3 2

	2	2	1	1	1 1	$\frac{1}{1}$	1	1	1	1	1	1	$\frac{1}{1}$	1 1	1	1	2	. ,	,
	2	2	1	1	1 1 1 1	1	1 1 1	1 1 2 1	1	1	1 1 1 2	11 11	1	1 1 1 1 1 1	1	1	2	-	-
1	2					1	z					z	1					2	1
1	1					1	1					1	1					1	1
	2					1	1					1	1						
	Z					2	2					2	2					2	-
	2		,	1	2			2	- -		2		- -	2	1		-	2	,
	۷		_	1	1		2	-	-		۷		-	1	1		_	6	-

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Thermoelasticity (step 10)

	3 3 42 3 2 1 3 4 52 3 1 1	1 2 2 33 4 3 2 1 1 2 34 5 3 2	2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 1 1 2 1 1 1 2 1
	113	113	
1 21 3	1 1	1 1	1 31 2
2 1 2 2 2 1 2 3 3 23 2 2 1 1 2 2	22 213 222222 213 322222 13 2132 13	2 3 222 213 1 213 22222 1 213 22213	3 1 2 2 2 2 1 1 2 2 2 2 1 1 1 2 2 3 3 1 2 1 2 2 3 3 1
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		1 2 3
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c c} 1 & \frac{1}{1} & \frac{1}{2} \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \\ \\ \\ \\ \hline \\$
1 1	1 1	1 1	1 1

	21 -	-	1	-	1+	1	-	-	1	11	-	11	-	-	1 2		
1 1 2				1	12 11					21 11	1					21 11	1
1 1				1	1					1	1					1	1
2				1	1					1	1						,
2				2	2					2	2					2	•
2	2	1	1 2 1 1	z	13	2	1	1	2	z	13	2 1 1 1	1		2	2	2
		1	1			1	1	1	1			1	1				

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Selected Topics in Adaptive Higher-Order FEM

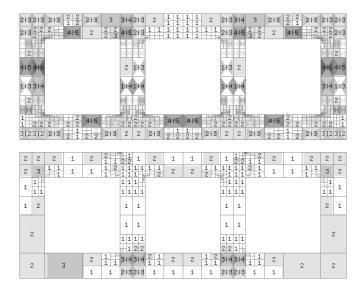
Thermoelasticity (step 11)

1 22 3		2	3	314213		1	1	2	213		3	2	22 21	2	2 31 2
21322	2 415	2	3	415213	1 1	1 1 1 1	1 1 1 1	11	213	415	3	2	415	2	² ² ₂ 213
2															2
214214				1 21 2					1 2	112					214214
1 33 4				113113					113	113					314113
2	-														2
211 2 2	2 2 1	415	2	22 2222	2	213	2 3	2	22	2 2 2	2	415	1 1 2	2	22211
312312		22	213	Z Z	213	21	1 1 2	213	2	2	213	22	1 1 2		312312
2 2	2 1	1	1 1	$\begin{array}{c} 1\\2 1\end{array}$ 1	1	1	1	1	1	1 1 2	$ 1 \\ 1 \\ 1 $	1	1	2	2 2
2 2 2 3		1	1 1 1 1 1 1			1	1			1 12 11	1 1 1 1 1 1 1 1 1 1	1	1		2 2 3 2
	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		1 1 1 1 1 1	21	1 1	_			1 1 1 1	1 12 11 11 11 11				2 1 1 1 2	
2 3			1 1 1 1 1	111	1 1	_			1 1 1 1	11	1 1 1 1 1 1				3 2 2 1
2 3 1 1 2 1 1			1 1 1 1 1 1 1 1 1 1 1		1 1	_			1 1 1 1 1 1	1 1 1 1 1					3 2 2 1 1 1
2 3 1 1 2 1 1 1 1						_			1 1 1 1 1 1 1	1 1 1 1 1					3 2 2 1 1 1 1 1
2 3 1 1 2 1 1				1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		_			1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1					3 2 2 1 1 1
2 3 1 1 2 1 1 1 1			1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1			_			1 1 1 1 1 1 1 1 1 1 2 2	1 1 1 1 1 1	2 <u>1</u> 1 1		1		3 2 2 1 1 1 1 1

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Selected Topics in Adaptive Higher-Order FEM

Thermoelasticity (step 12)



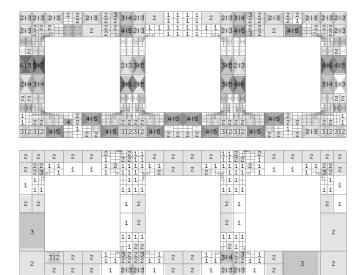
hp-FEM group, University of Nevada, Reno

Selected Topics in Adaptive Higher-Order FEM

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Thermoelasticity (step 13)



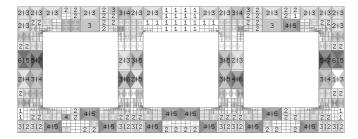
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Thermoelasticity (step 14)



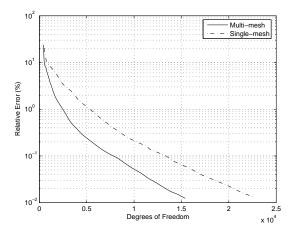
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Convergence: multi-mesh vs. single-mesh



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Adaptive hp-FEM with Dynamical Meshes

Sample time-dependent problem:

$$\frac{\partial u}{\partial t} - \Delta u = f$$

Implicit Euler:

$$\frac{u_{n+1}-u_n}{h}-\Delta u_{n+1}=f_{n+1}$$

Multimesh hp-FEM:

$$\underbrace{\frac{u_{n+1}}{h} - \Delta u_{n+1}}_{\text{solve using adaptivity}} = \underbrace{f_{n+1} - \frac{u_n}{h}}_{\text{last refined mesh}}$$

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