Multiphysics Problems

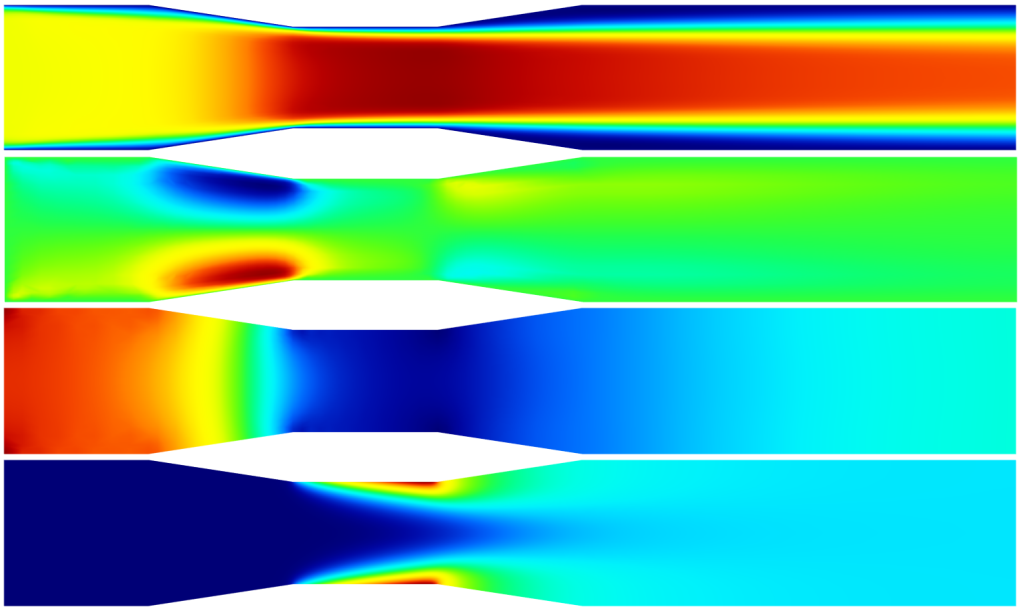


Figure: Laminar N-S equations with heat transfer

Multiphysics Problems

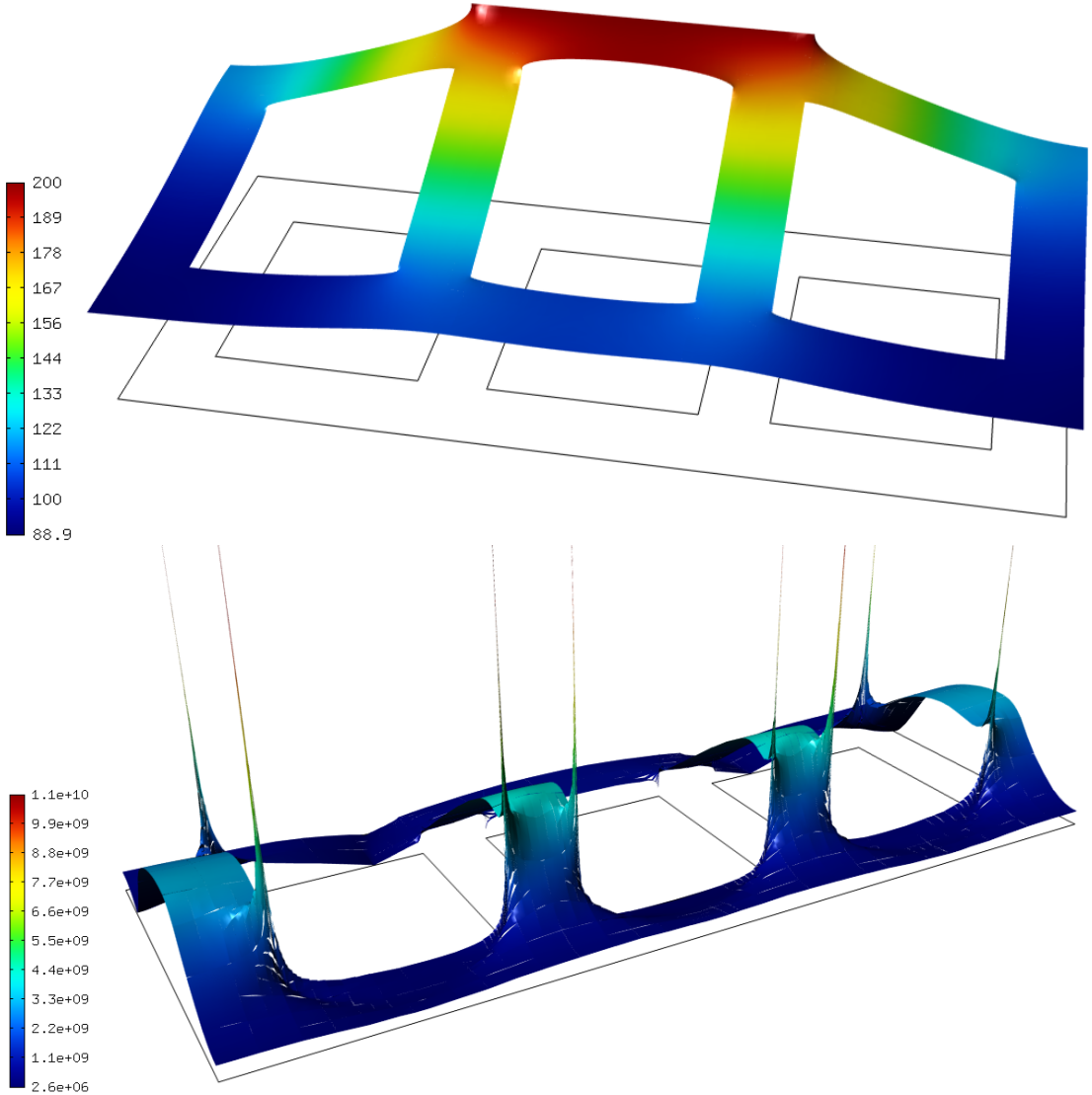


Figure: Thermoelasticity

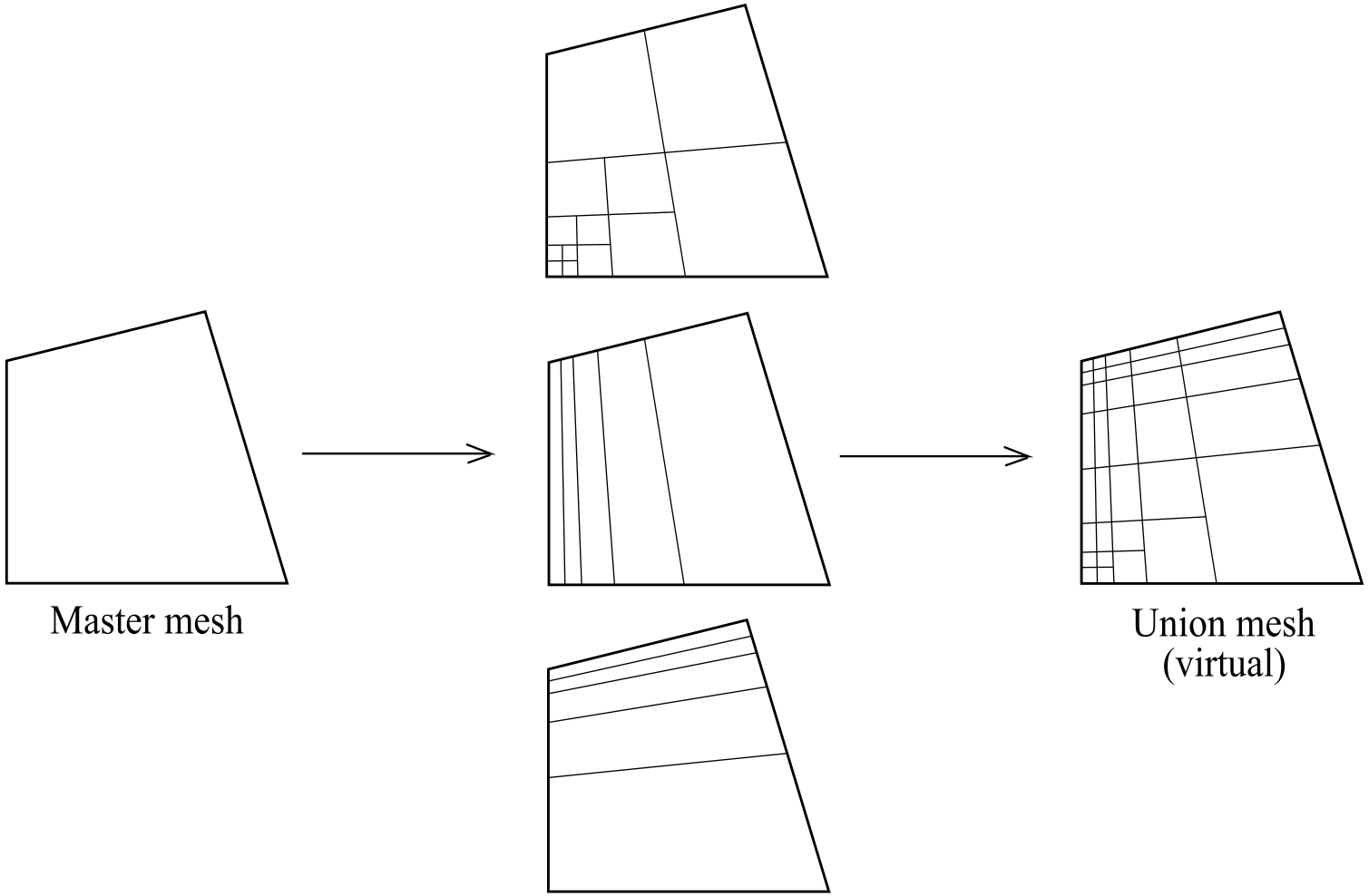
Techniques for Working with Multiple Meshes

- Interpolation methods – (large) data losses
- Projection methods – (smaller) data losses
  - X. Jiao and M. T. Heath. Common-Refinement-Based Data Transfer Between Non-Matching Meshes in Multiphysics Simulations. Internat. J. Numer. Methods Engrg., 61(14):2402-2427, 2004.
- Monolithic multimesh FEM – **no** data losses
  - P. Solin, J. Cerveny, L. Dubcova, D. Andrs: Monolithic Discretization of Linear Thermoelasticity Problems via Adaptive Multimesh hp-FEM, J. Comput. Appl. Math 234 (2010) 2350 - 2357.
  - L. Dubcova, P. Solin, J. Cerveny, P. Kus: Space and Time Adaptive Two-Mesh hp-FEM for Transient Microwave Heating Problems, Electromagnetics, Vol. 30, Issue 1, pp. 23 - 40, 2010.
  - P. Solin, L. Dubcova, J. Kruis: Adaptive hp-FEM with Dynamical Meshes for Transient Heat and Moisture Transfer Problems, J. Comput. Appl. Math. 233 (2010) 3103-3112.

Comparison

L. Dubcova, P. Solin, G. Hansen, H. Park: Comparison of Multimesh hp-FEM to Interpolation and Projection Methods for Spatial Coupling of Reactor Thermal and Neutron Diffusion Calculations, J. Comput. Phys. 230 (2011) 1182-1197.

Monolithic Multimesh FEM



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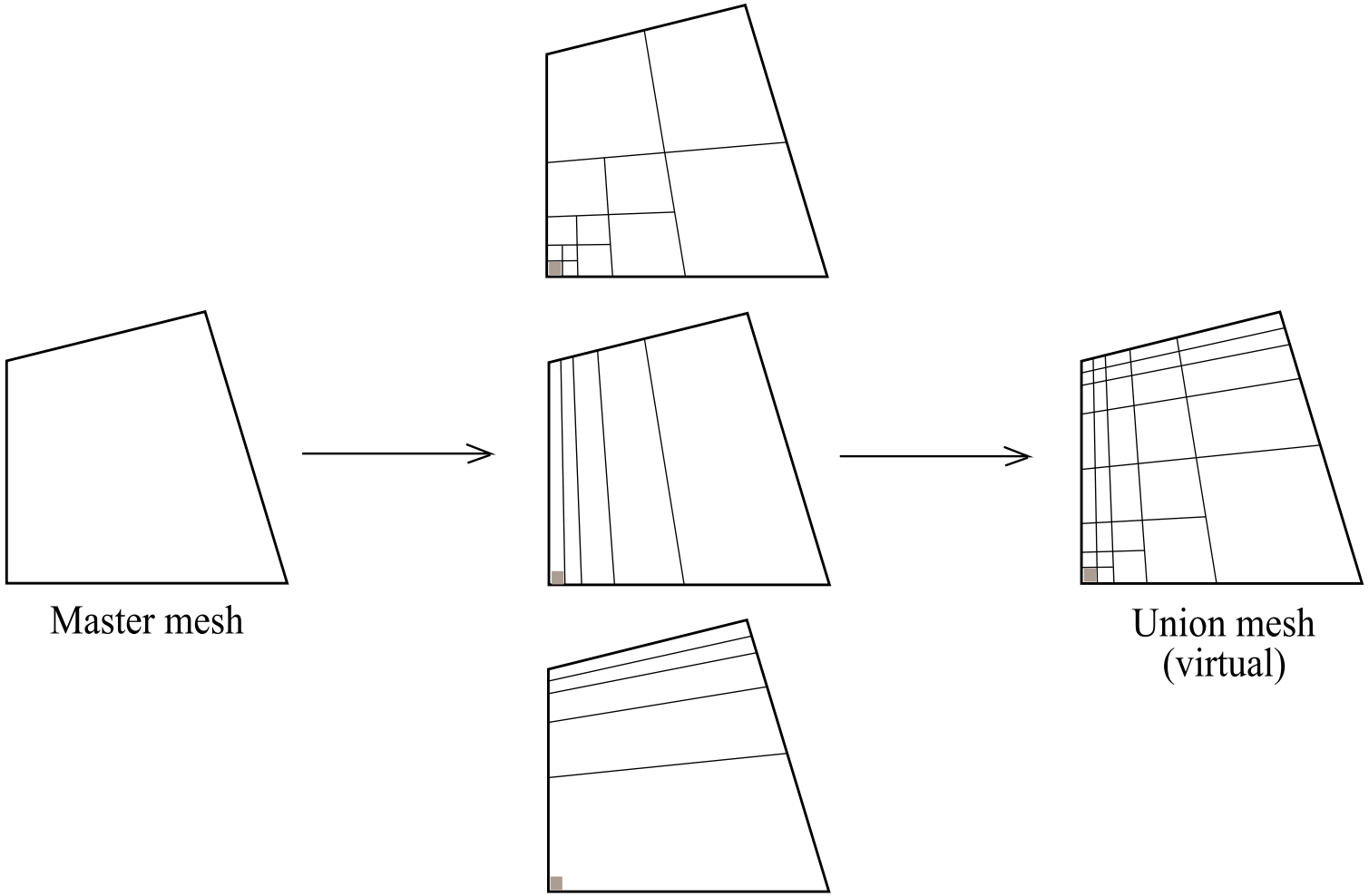

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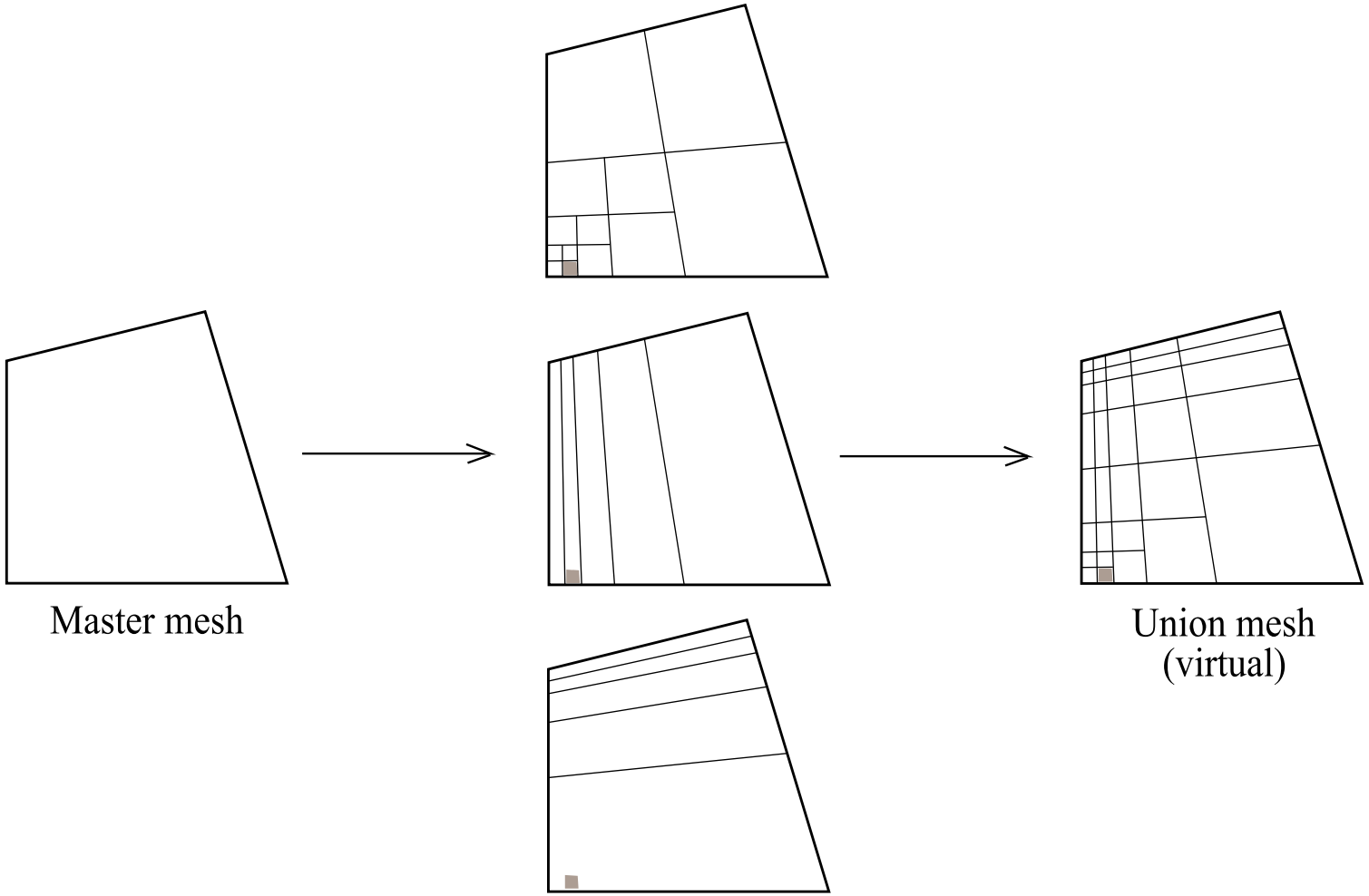

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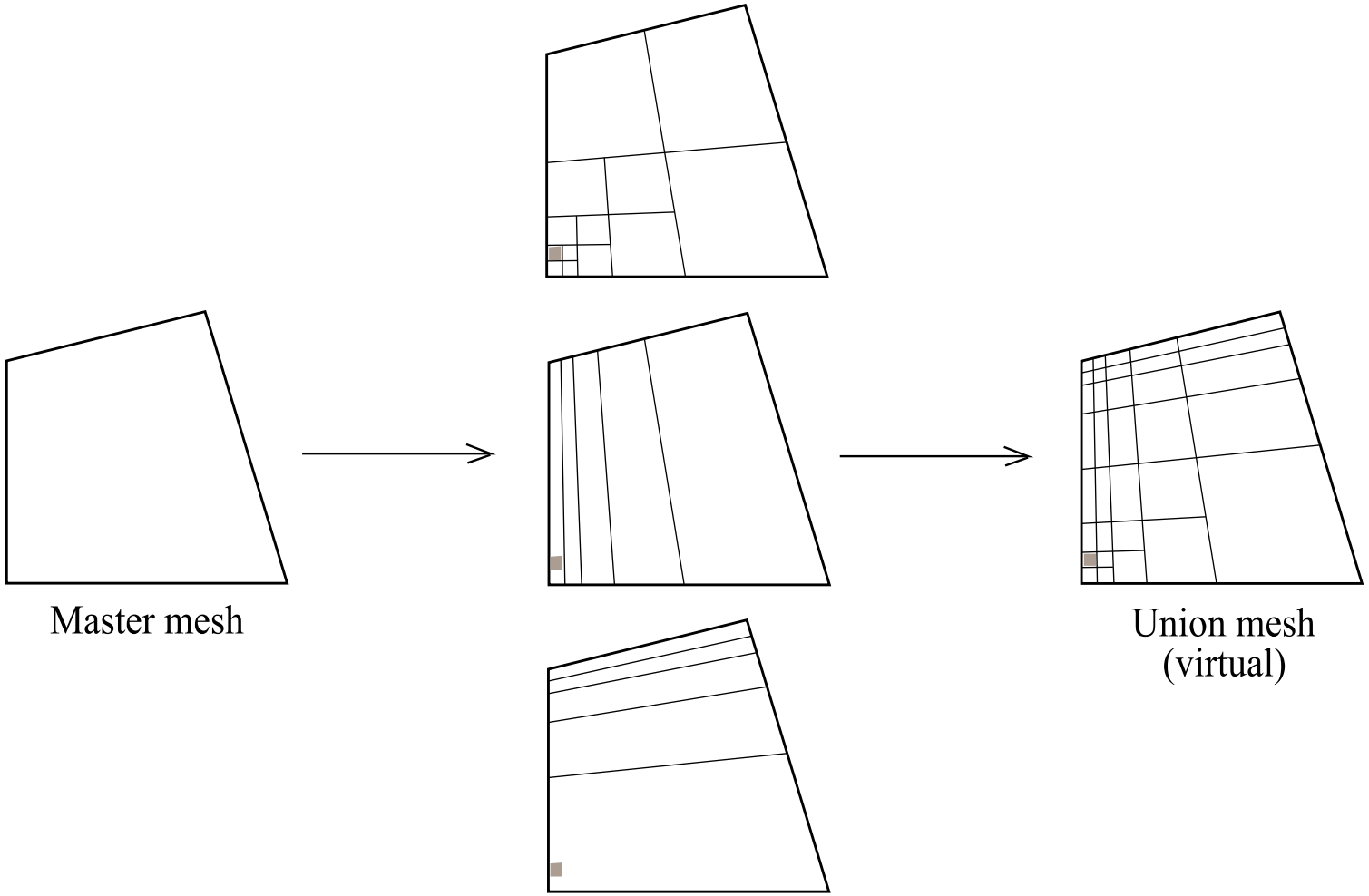

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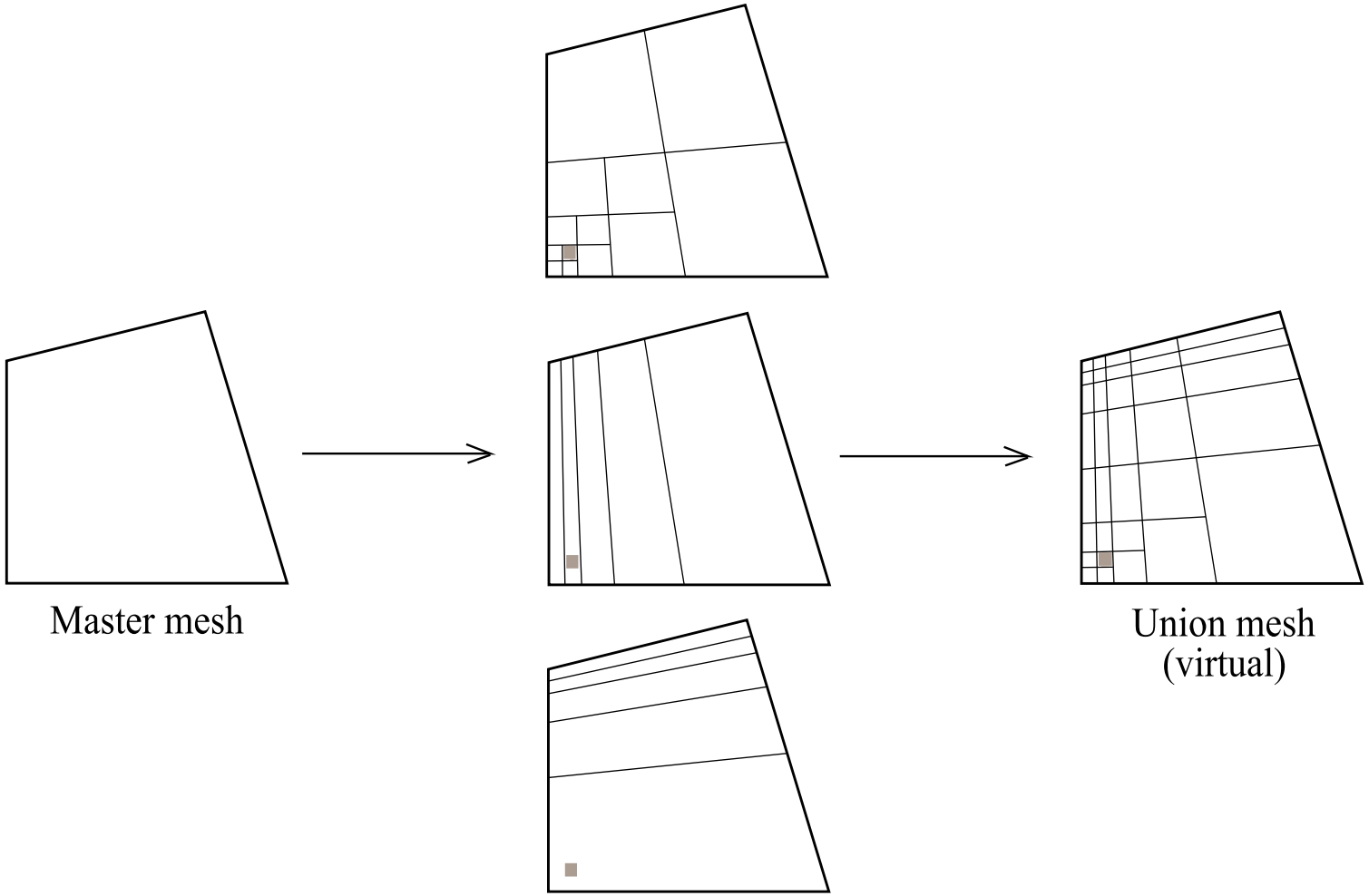

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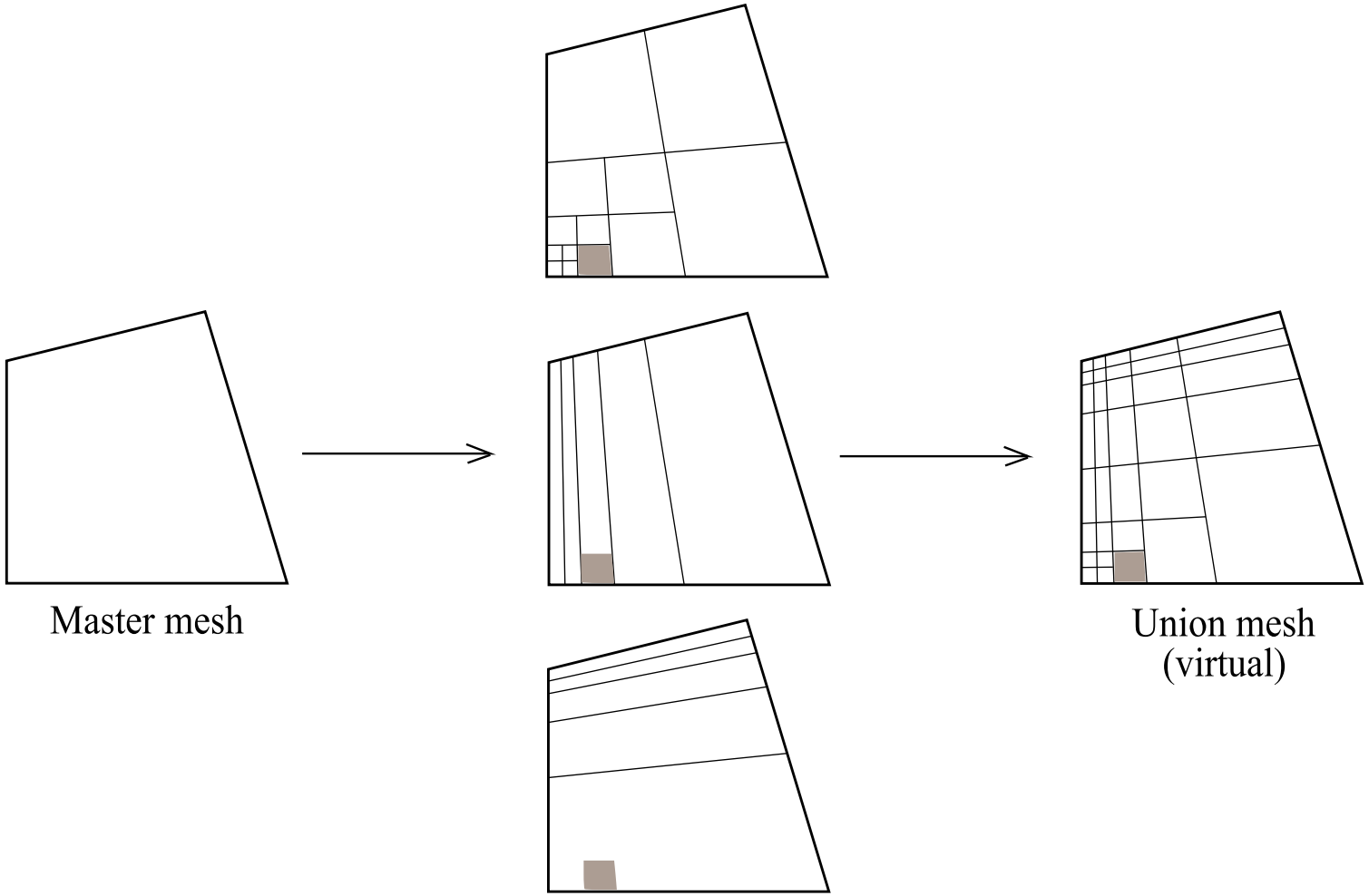

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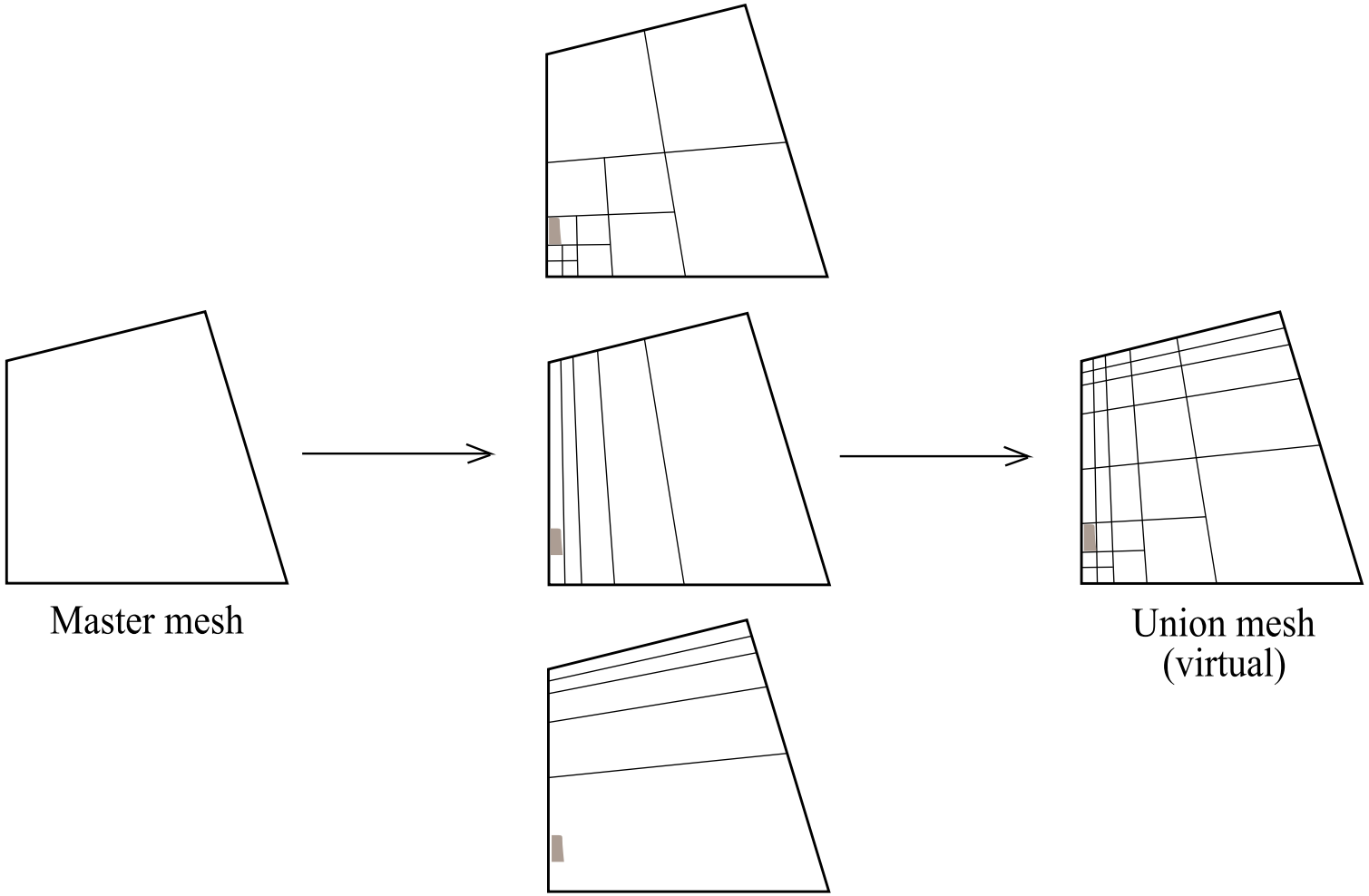

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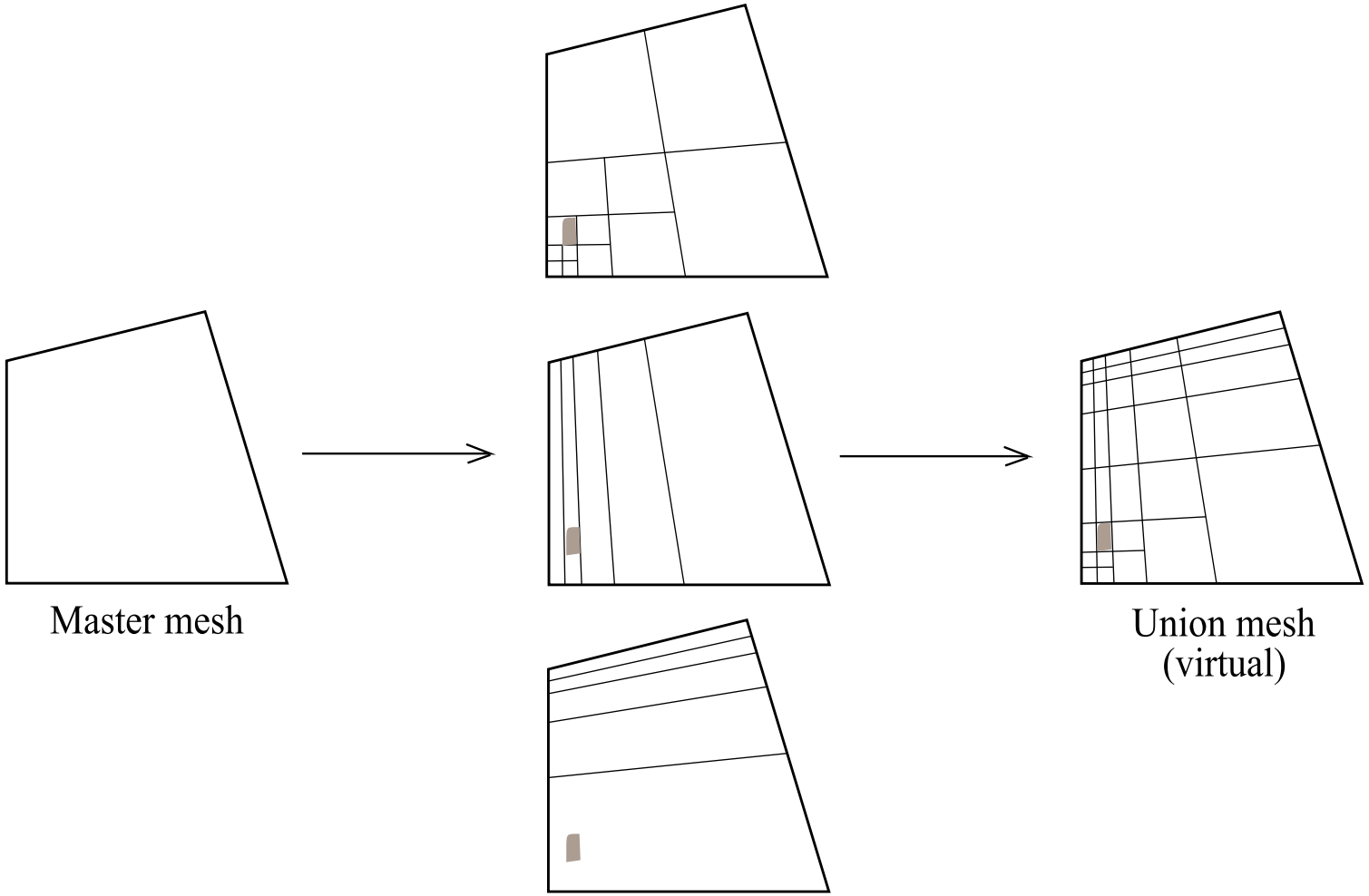

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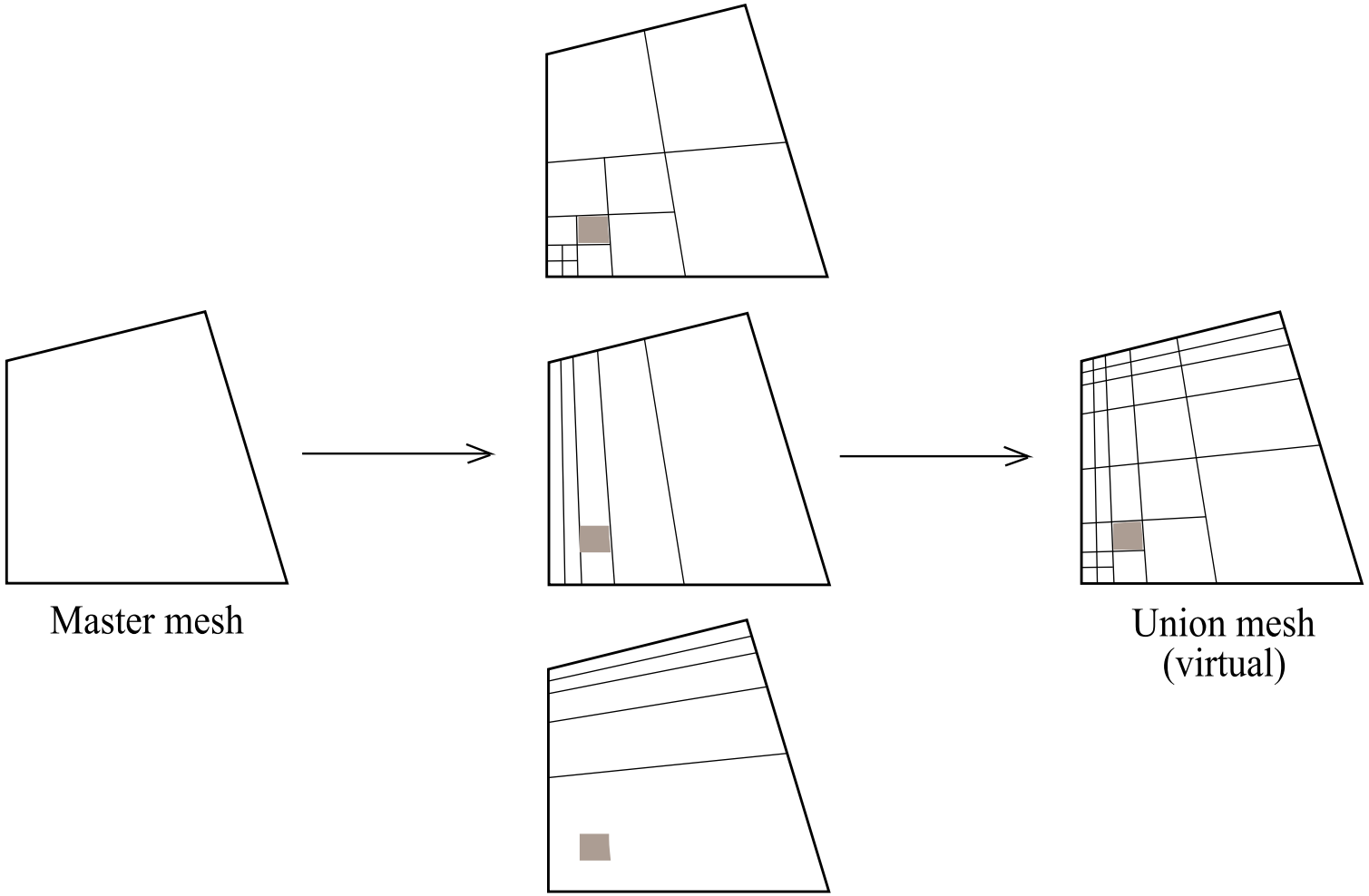

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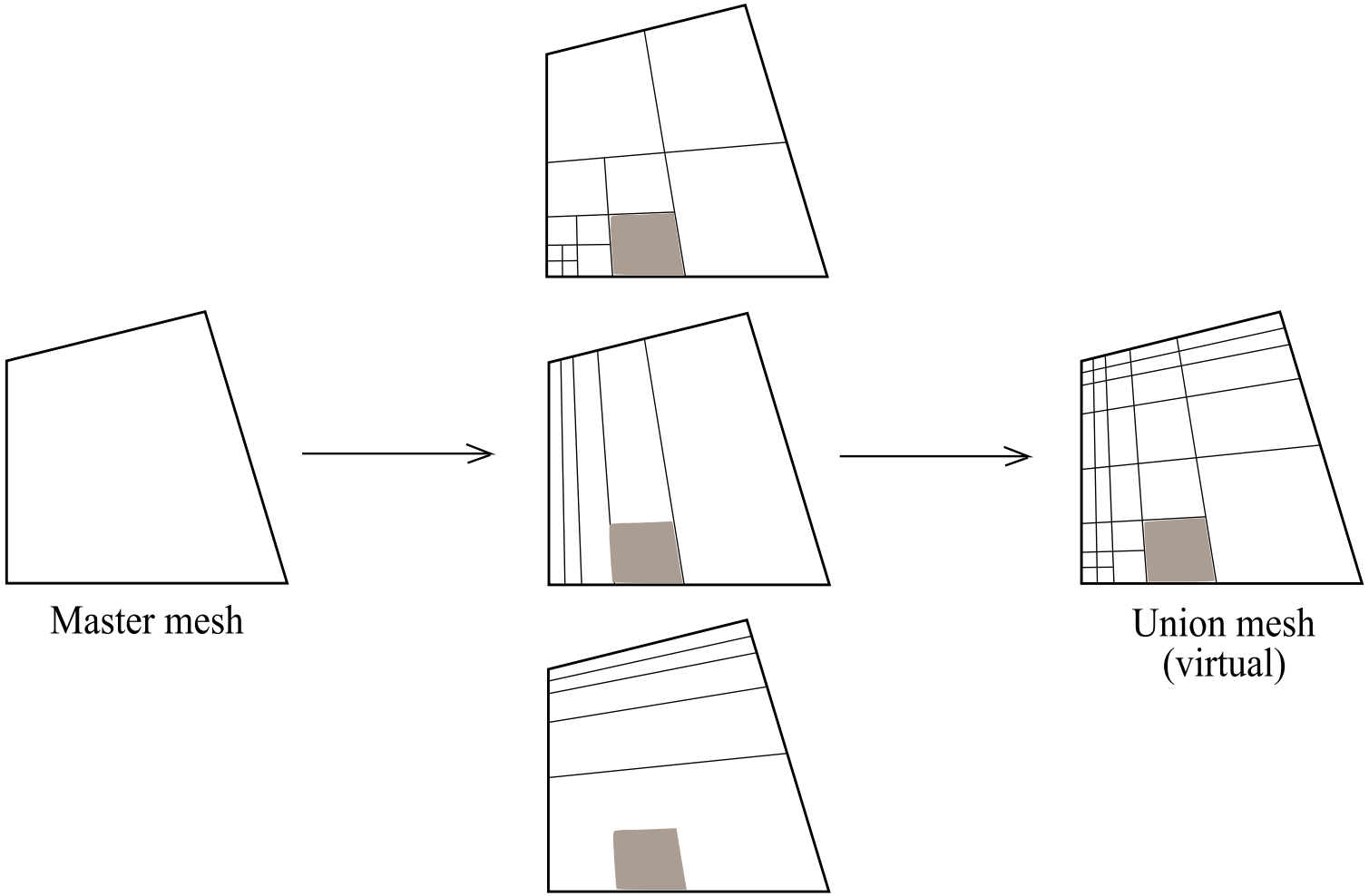

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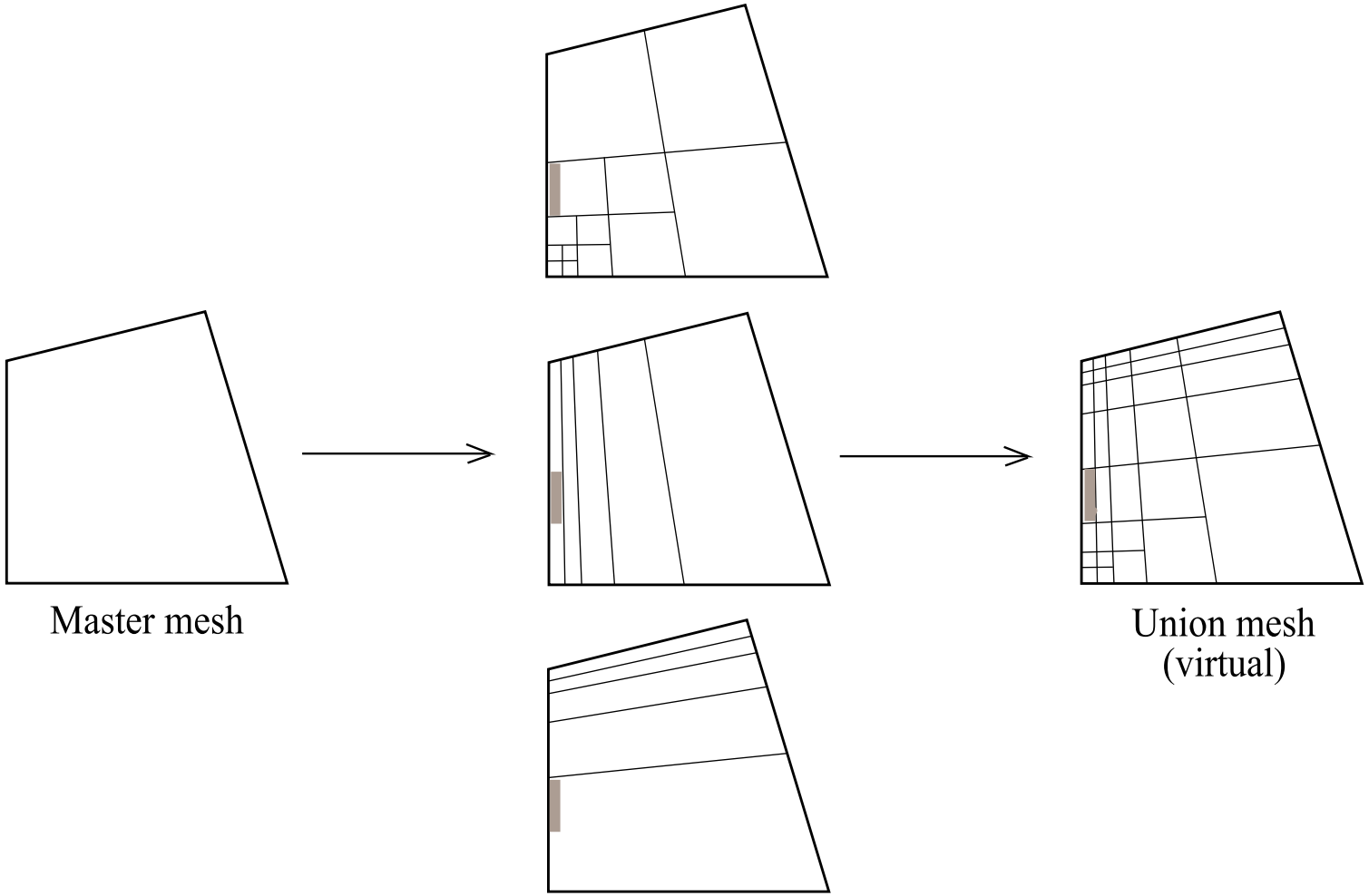

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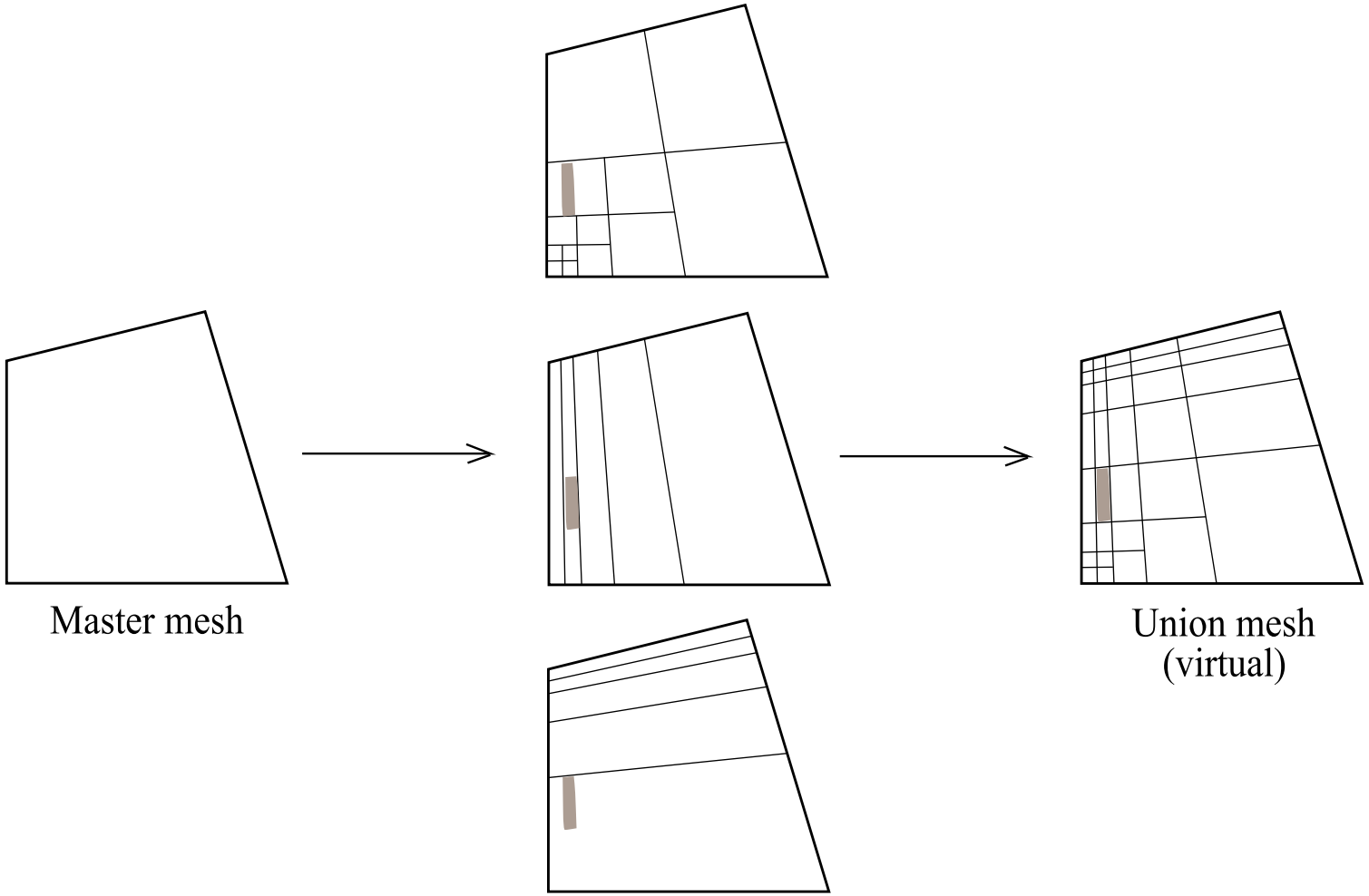

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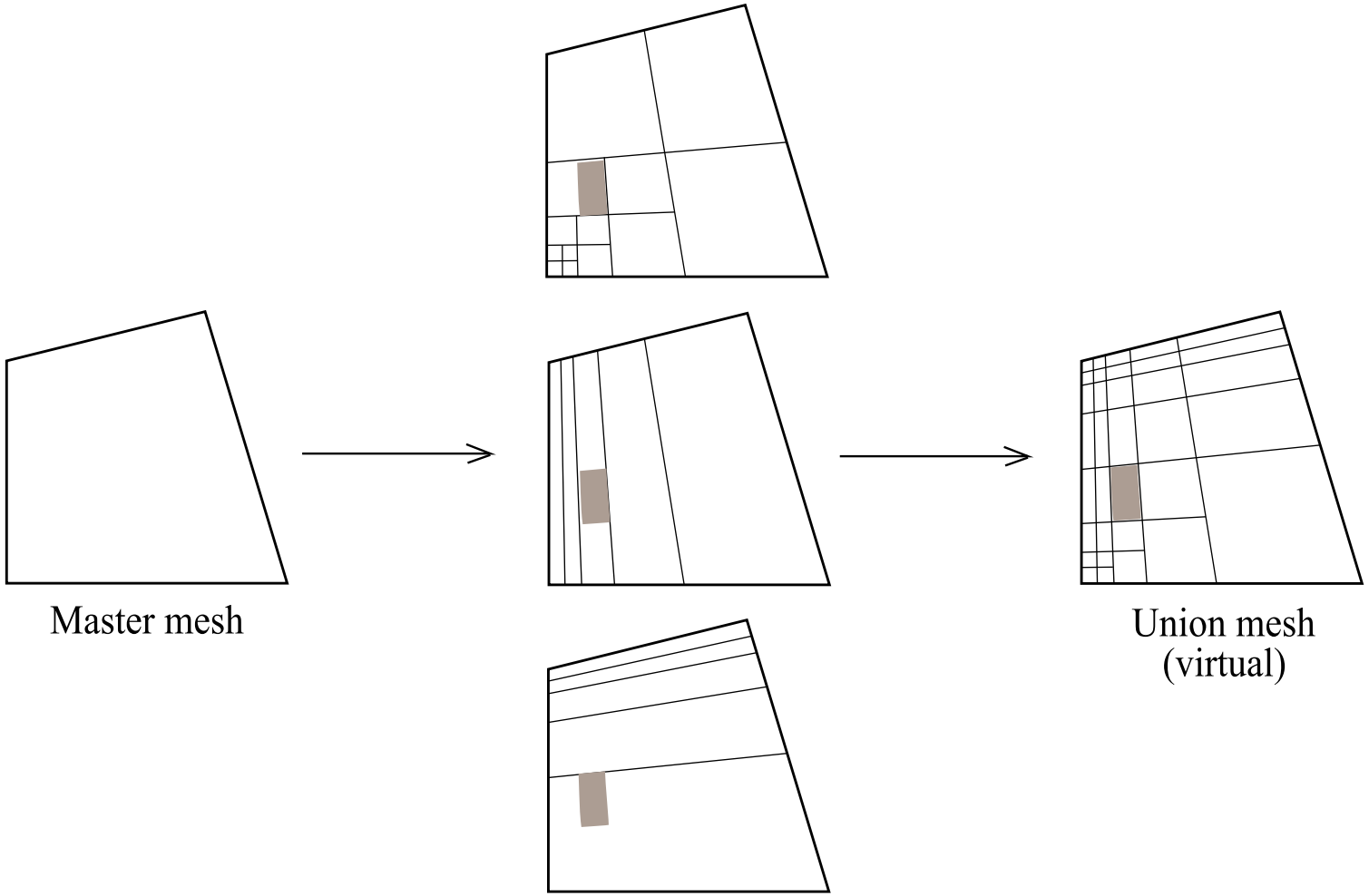

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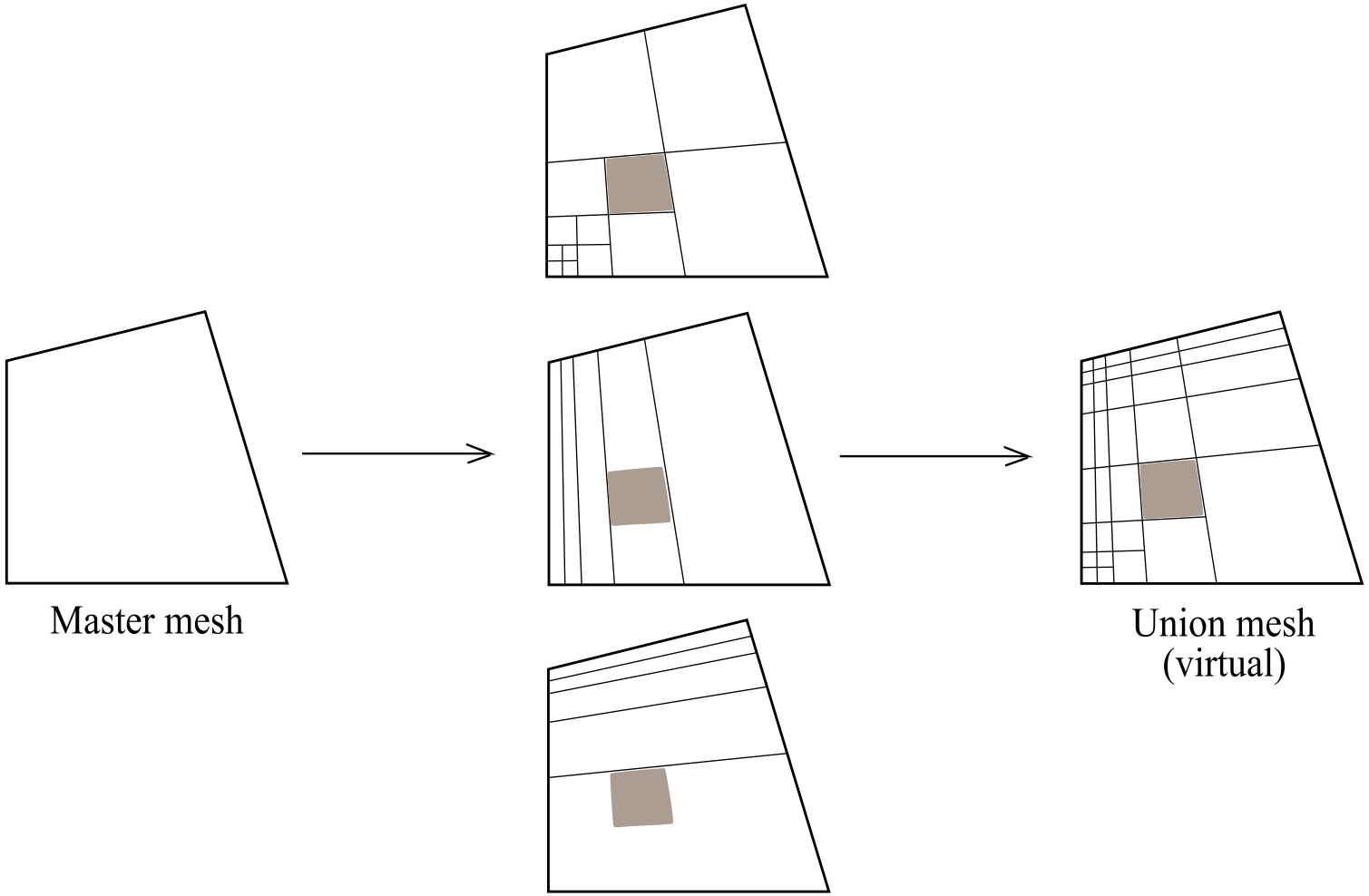

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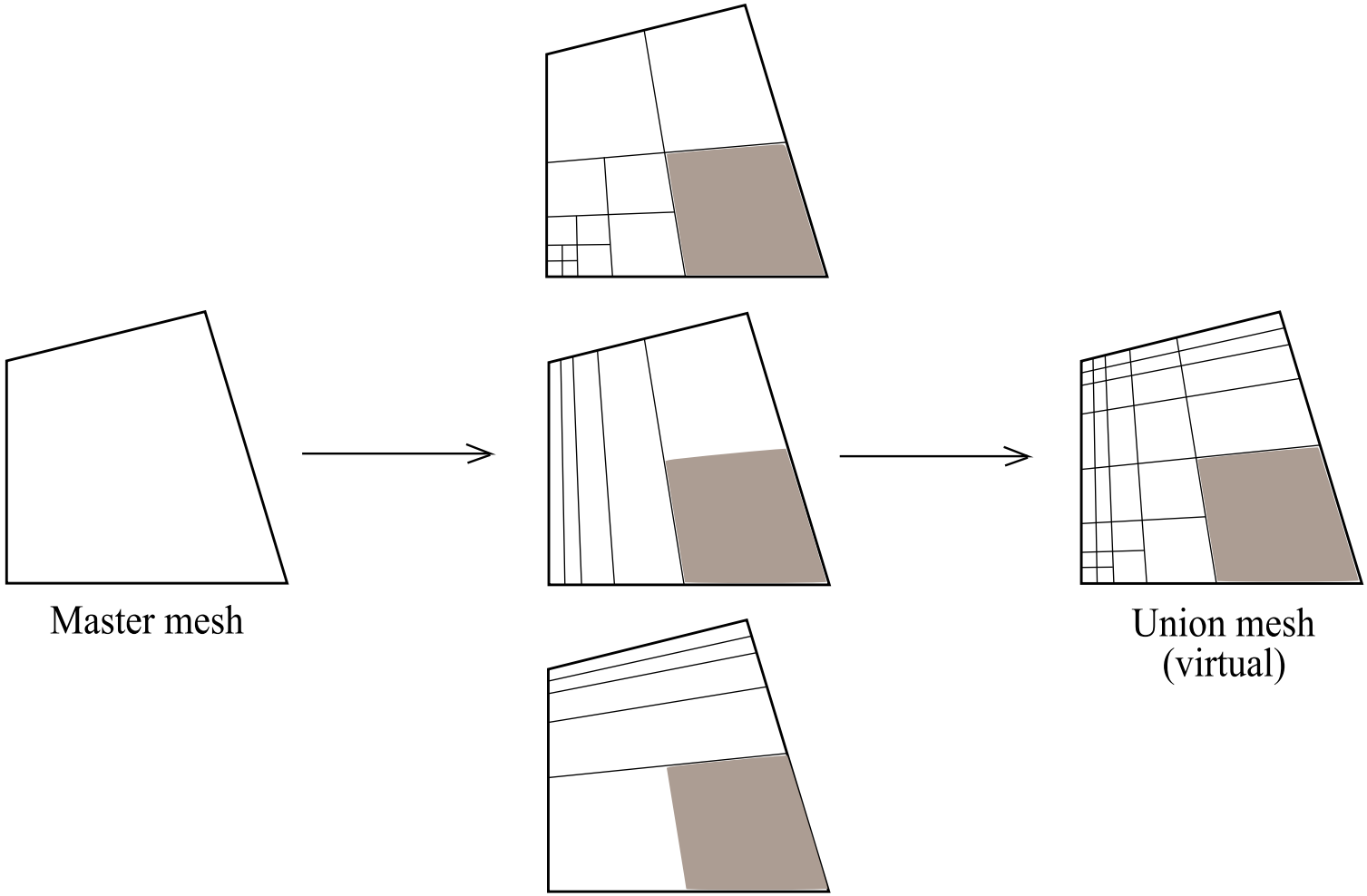

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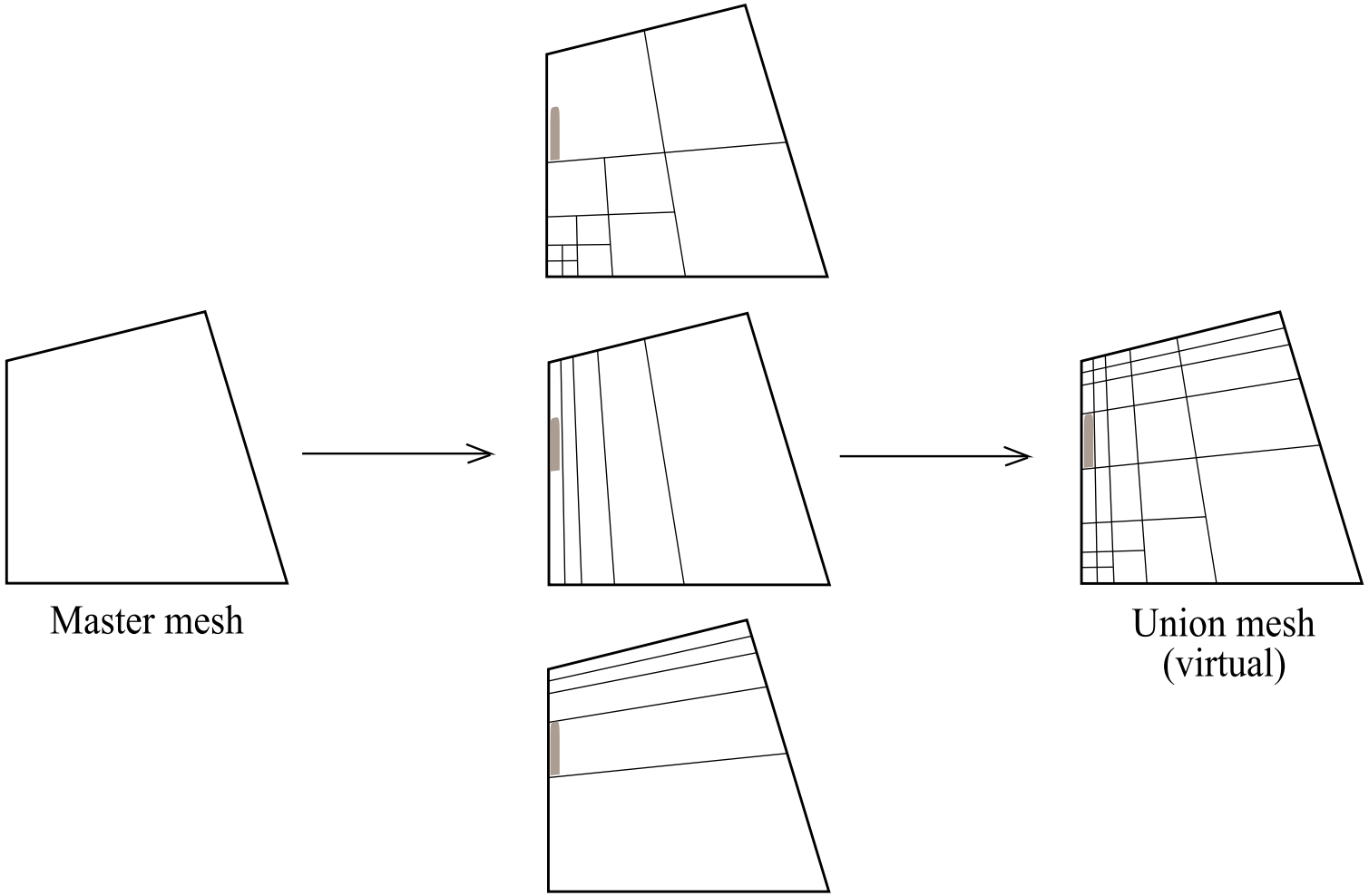

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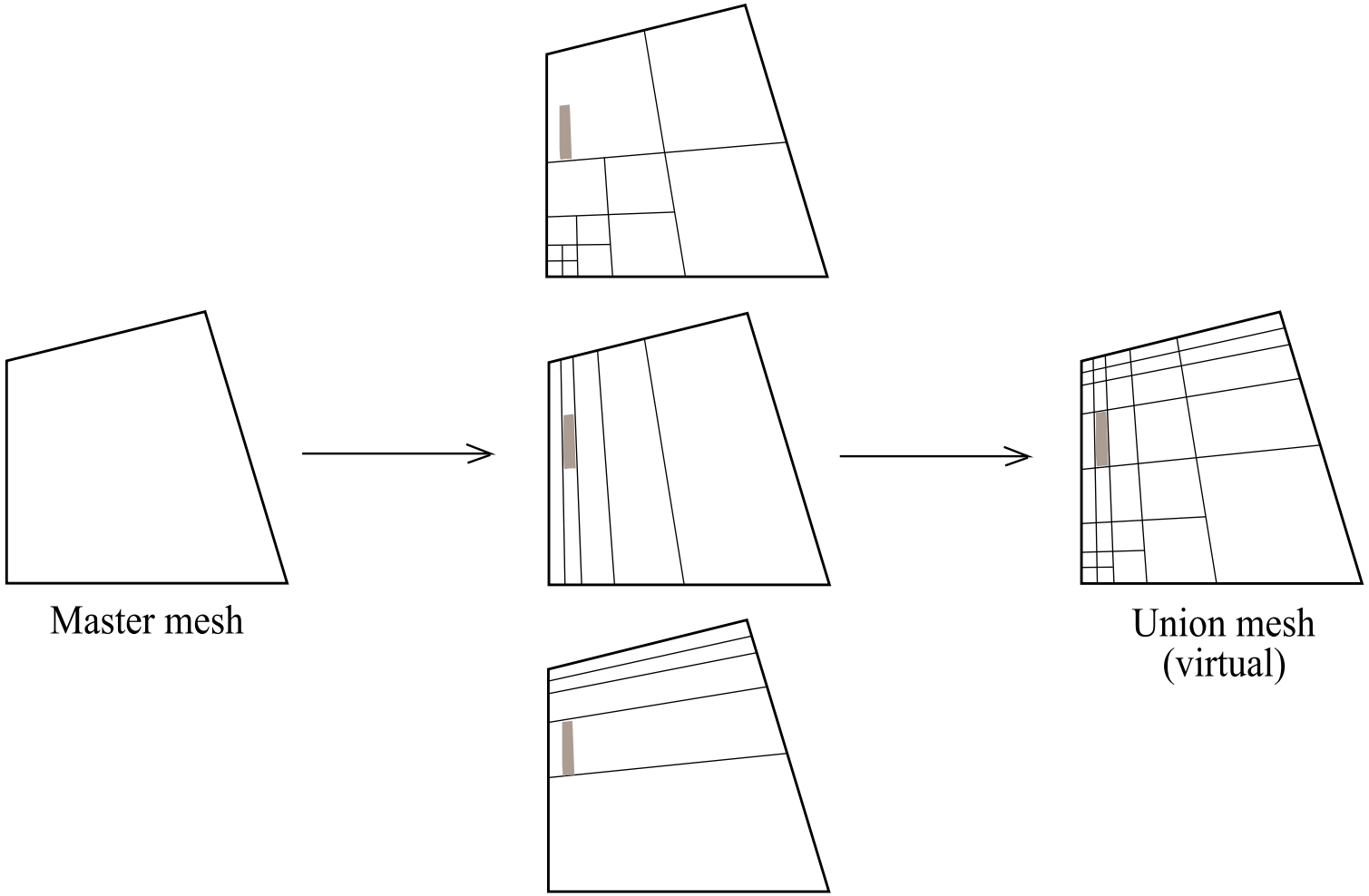

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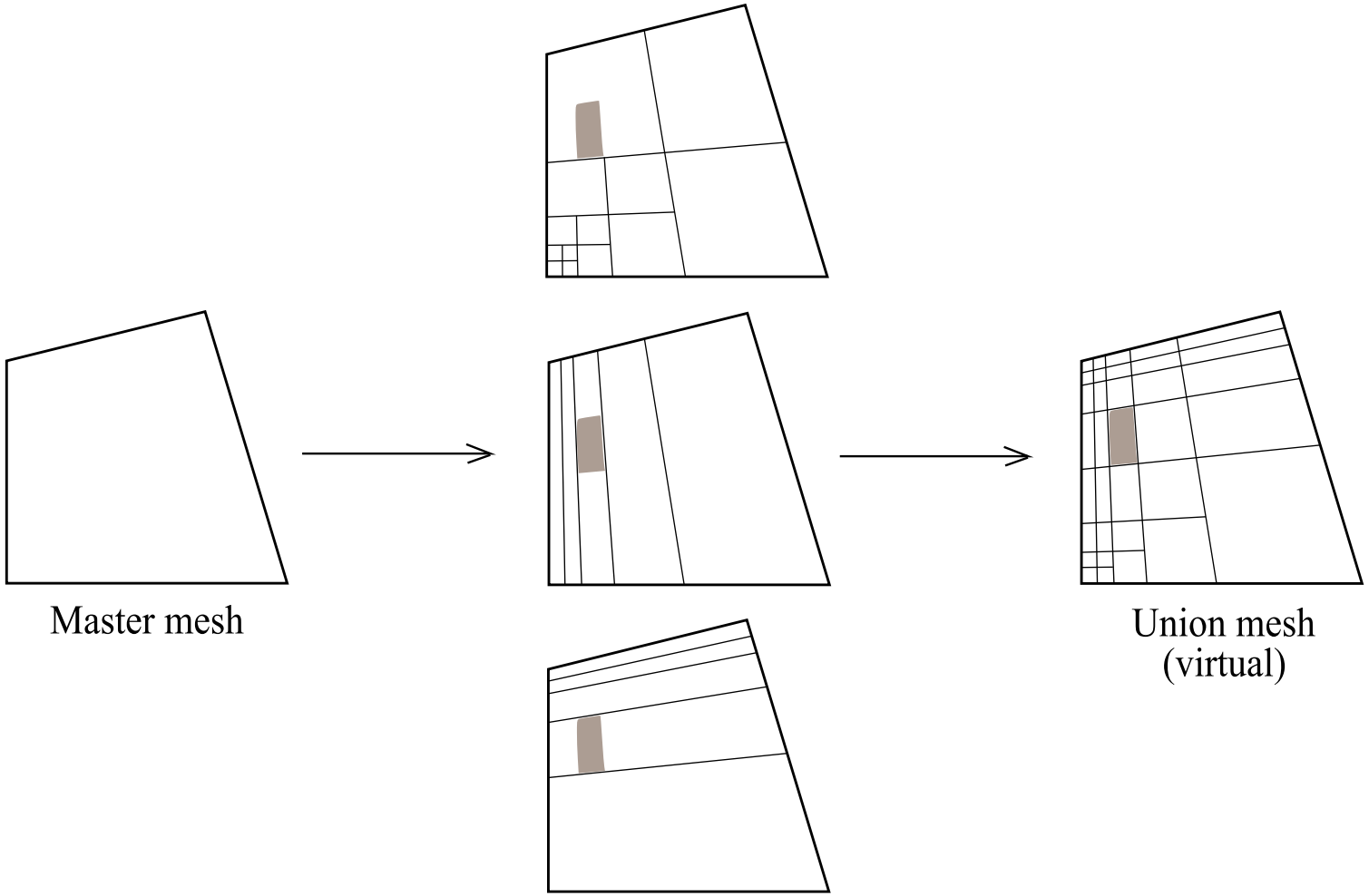

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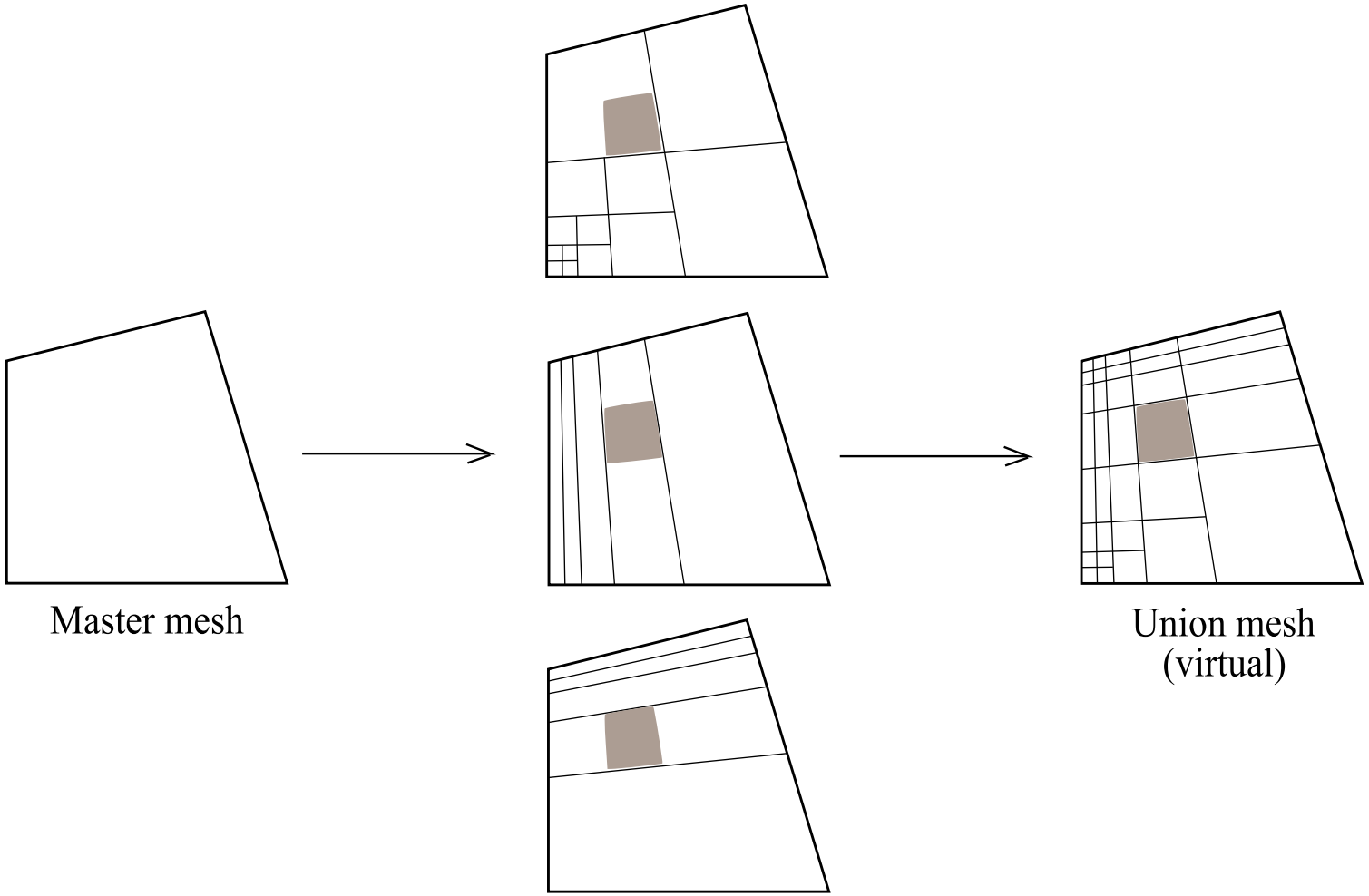

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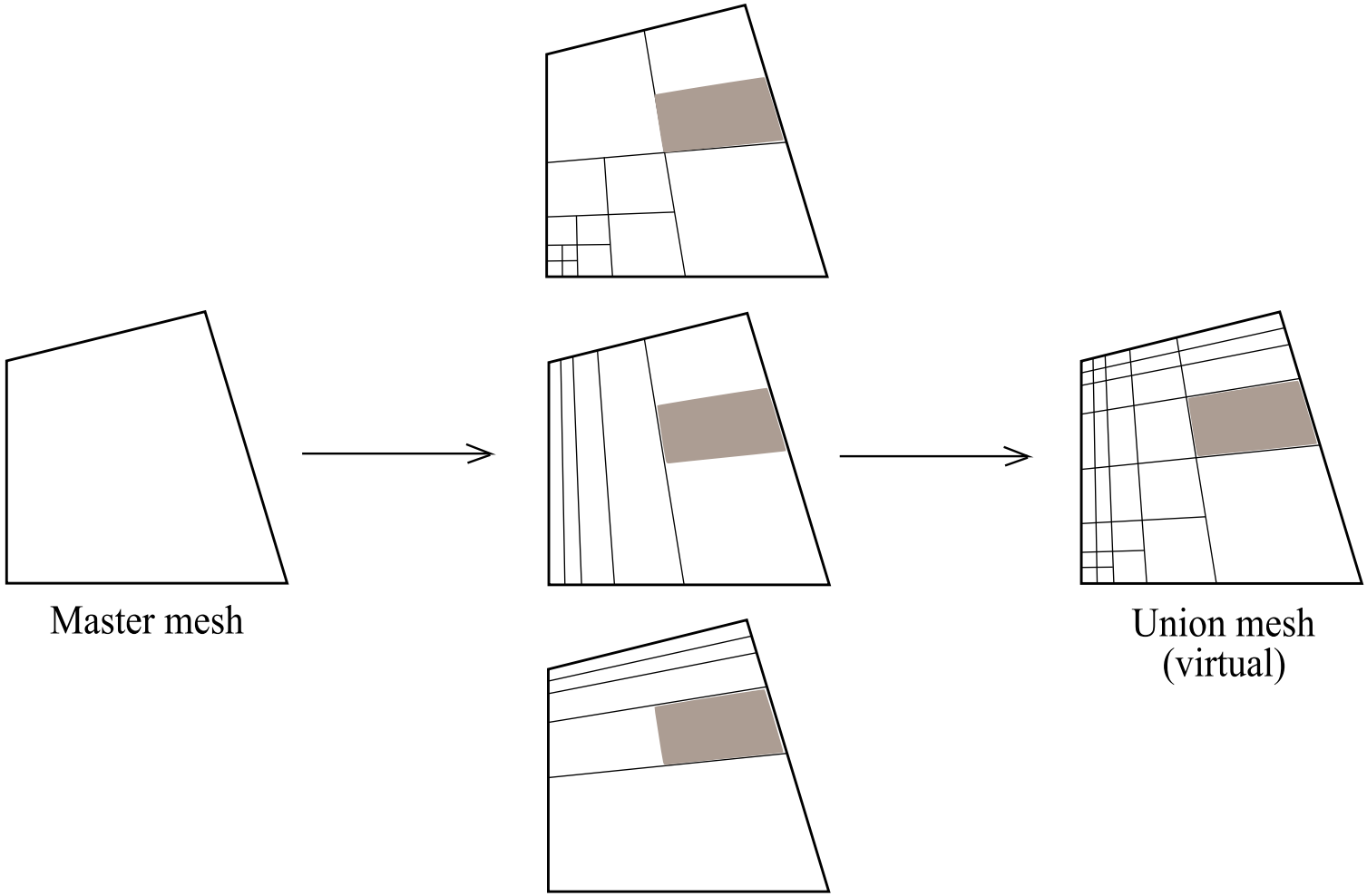

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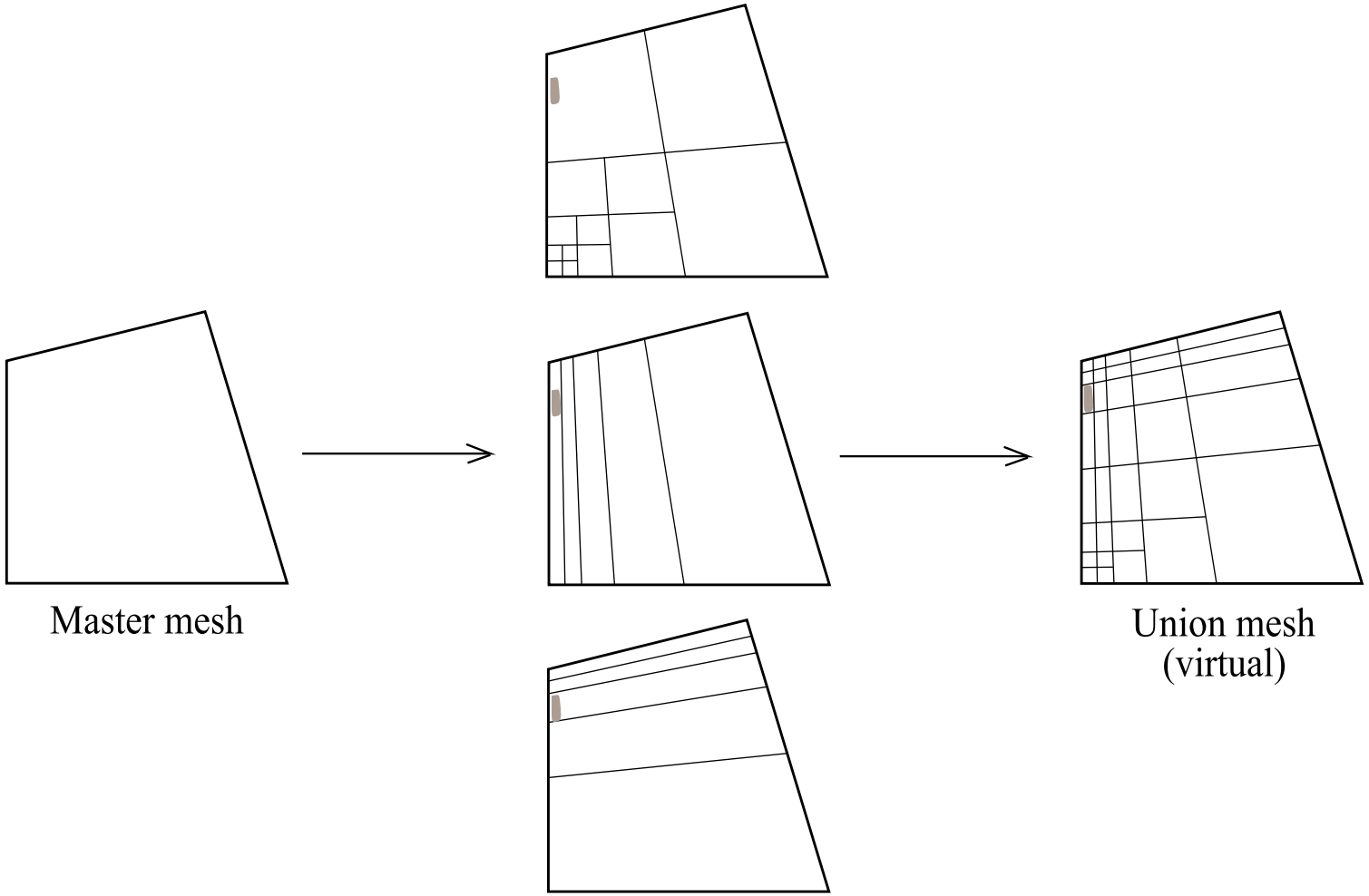

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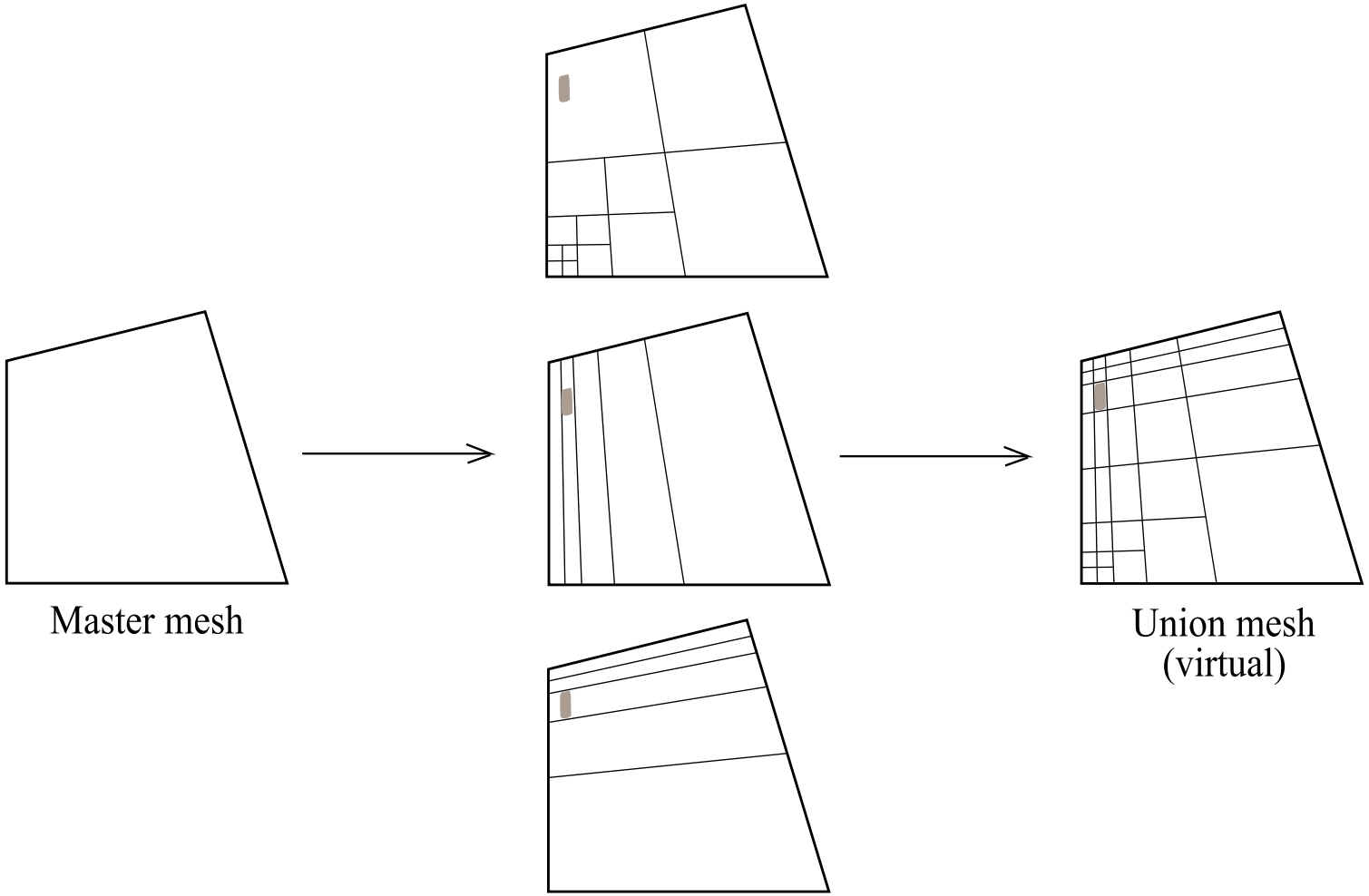

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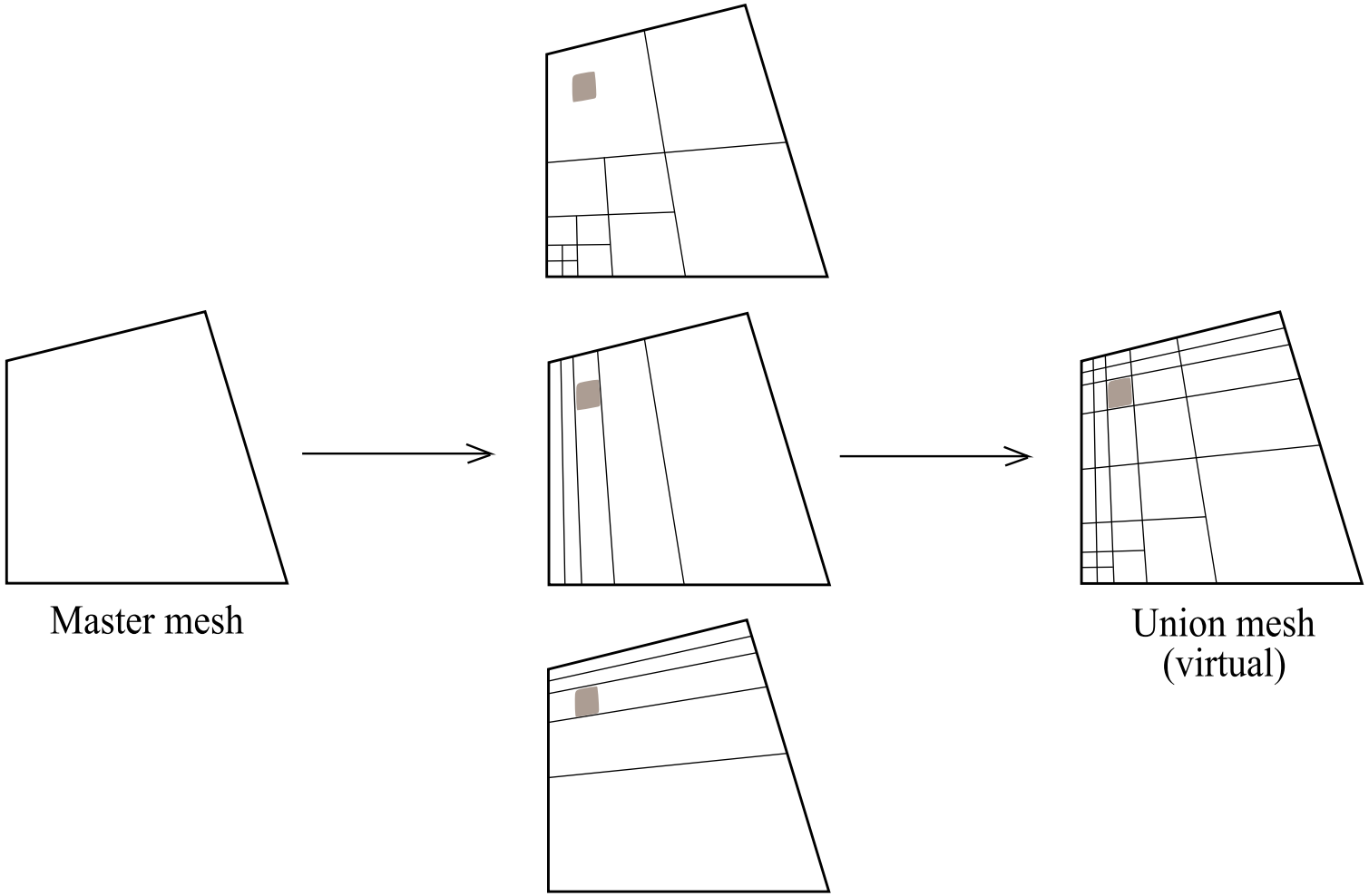

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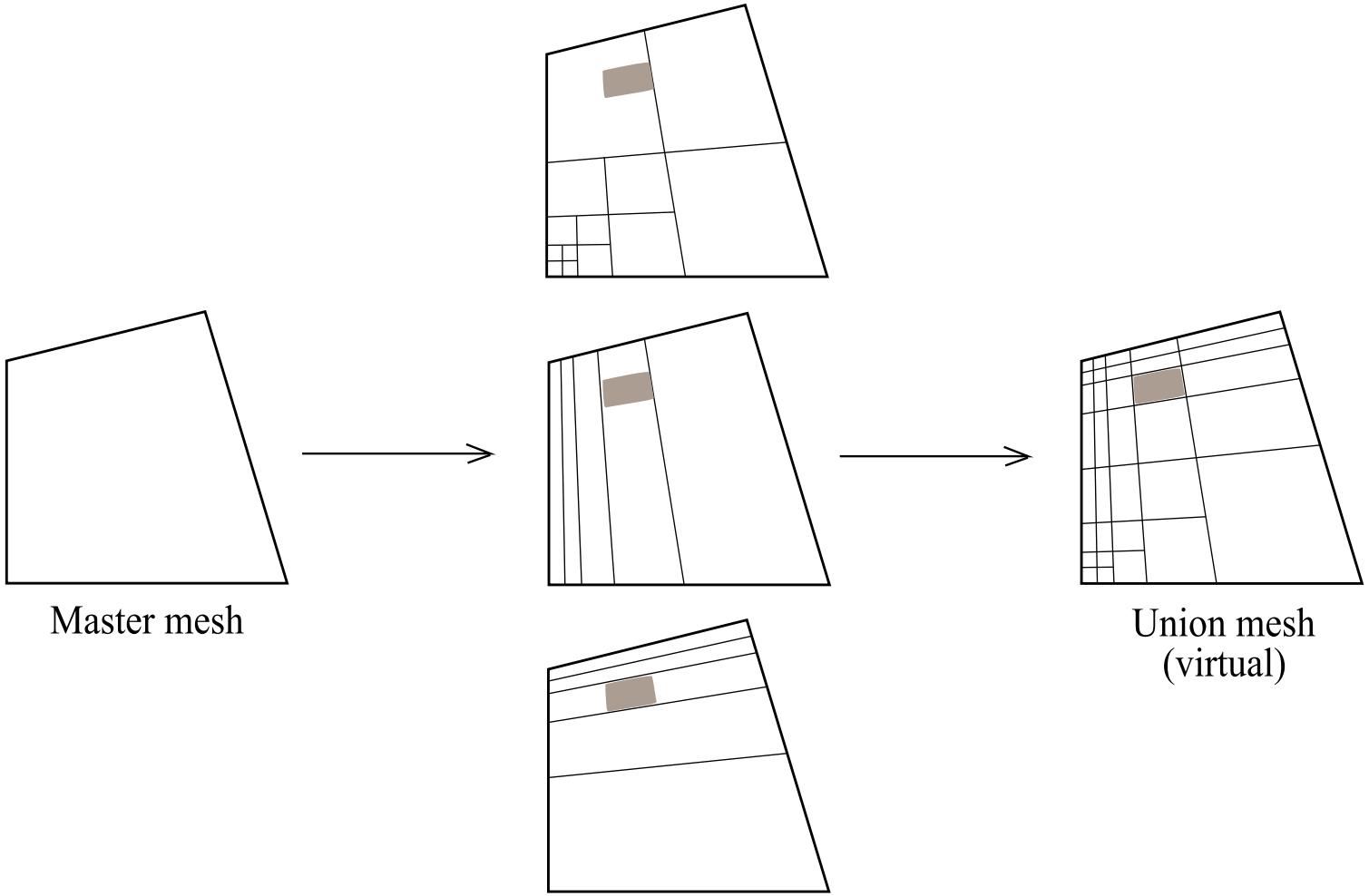

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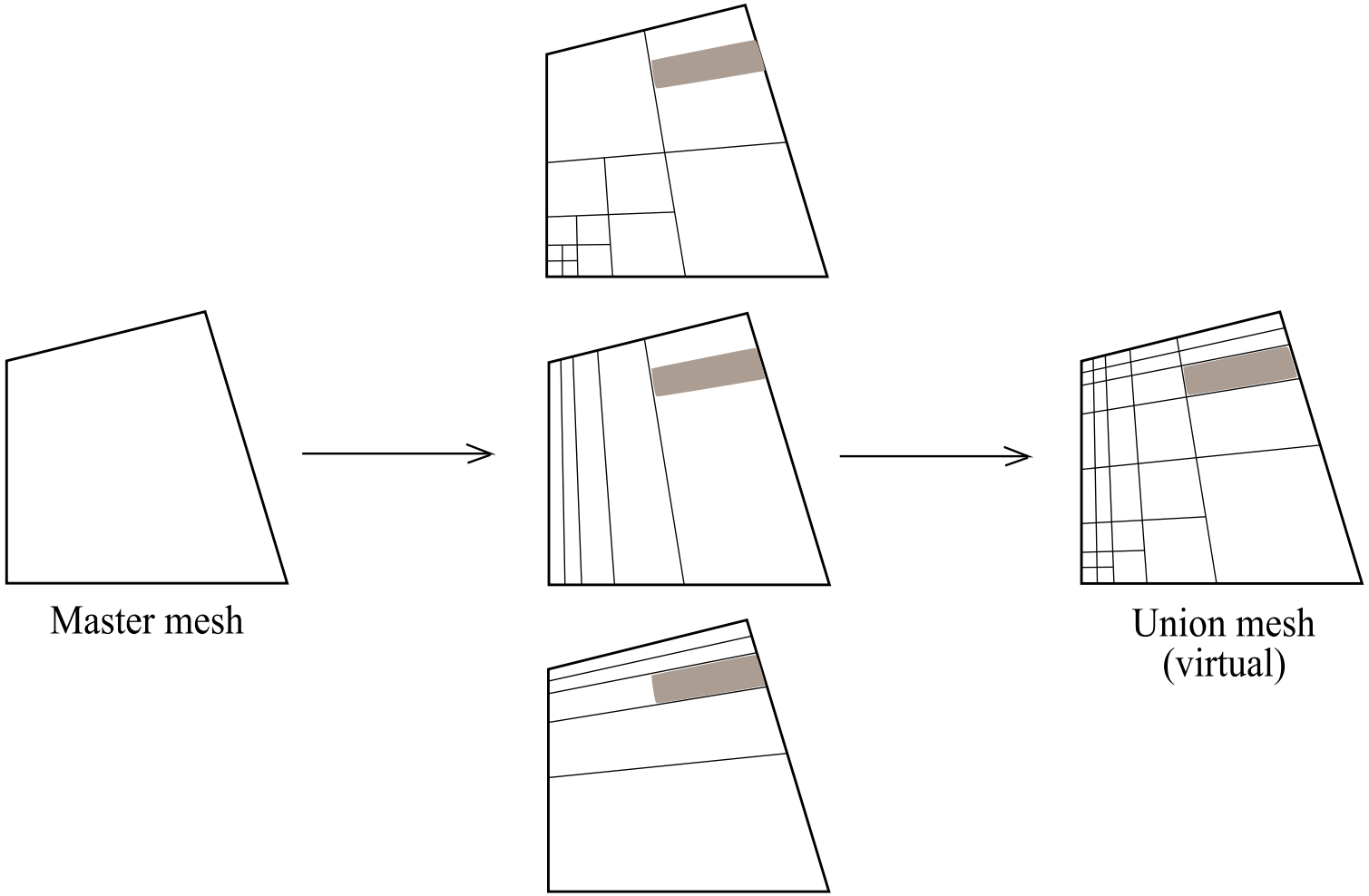

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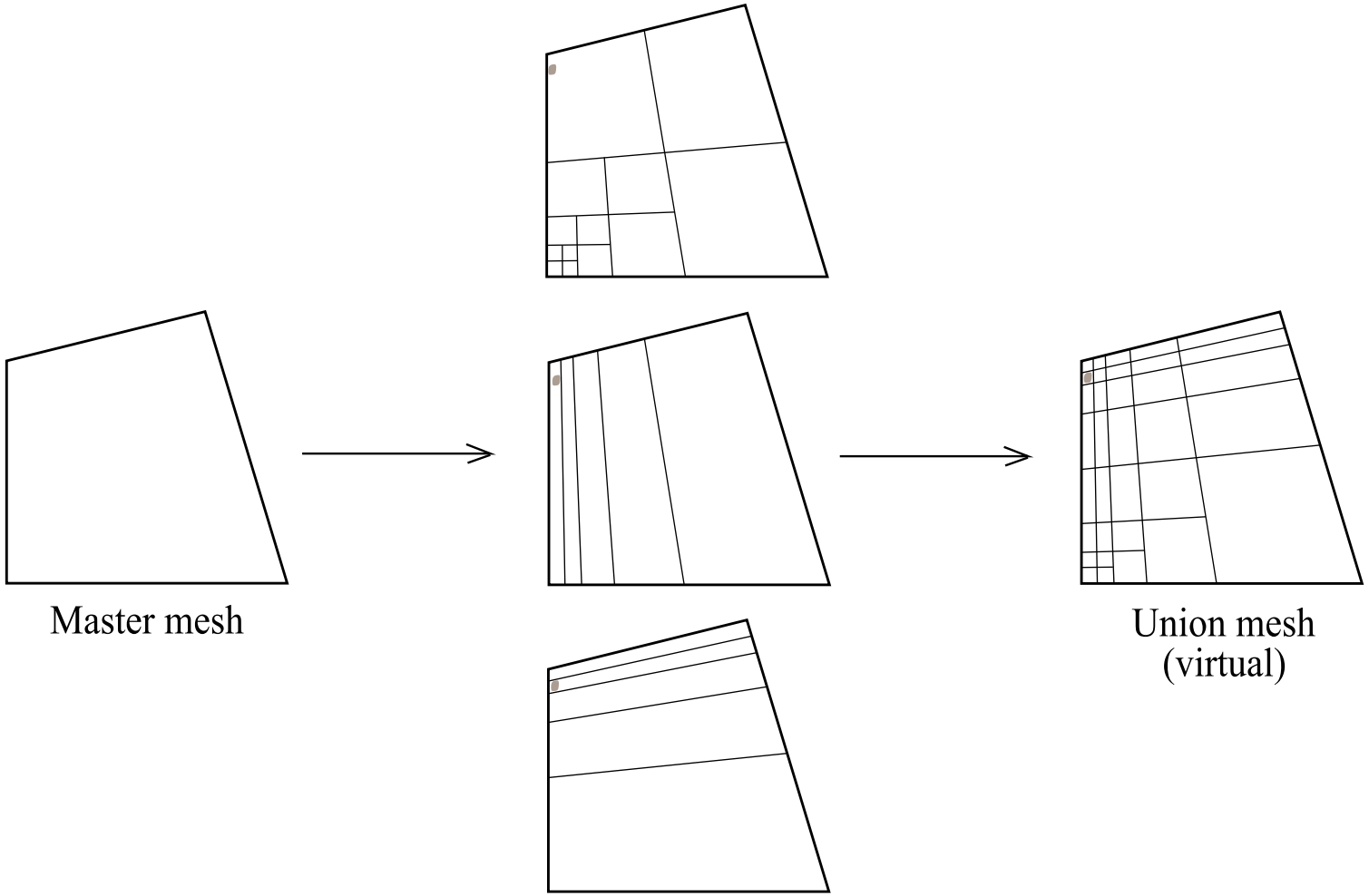

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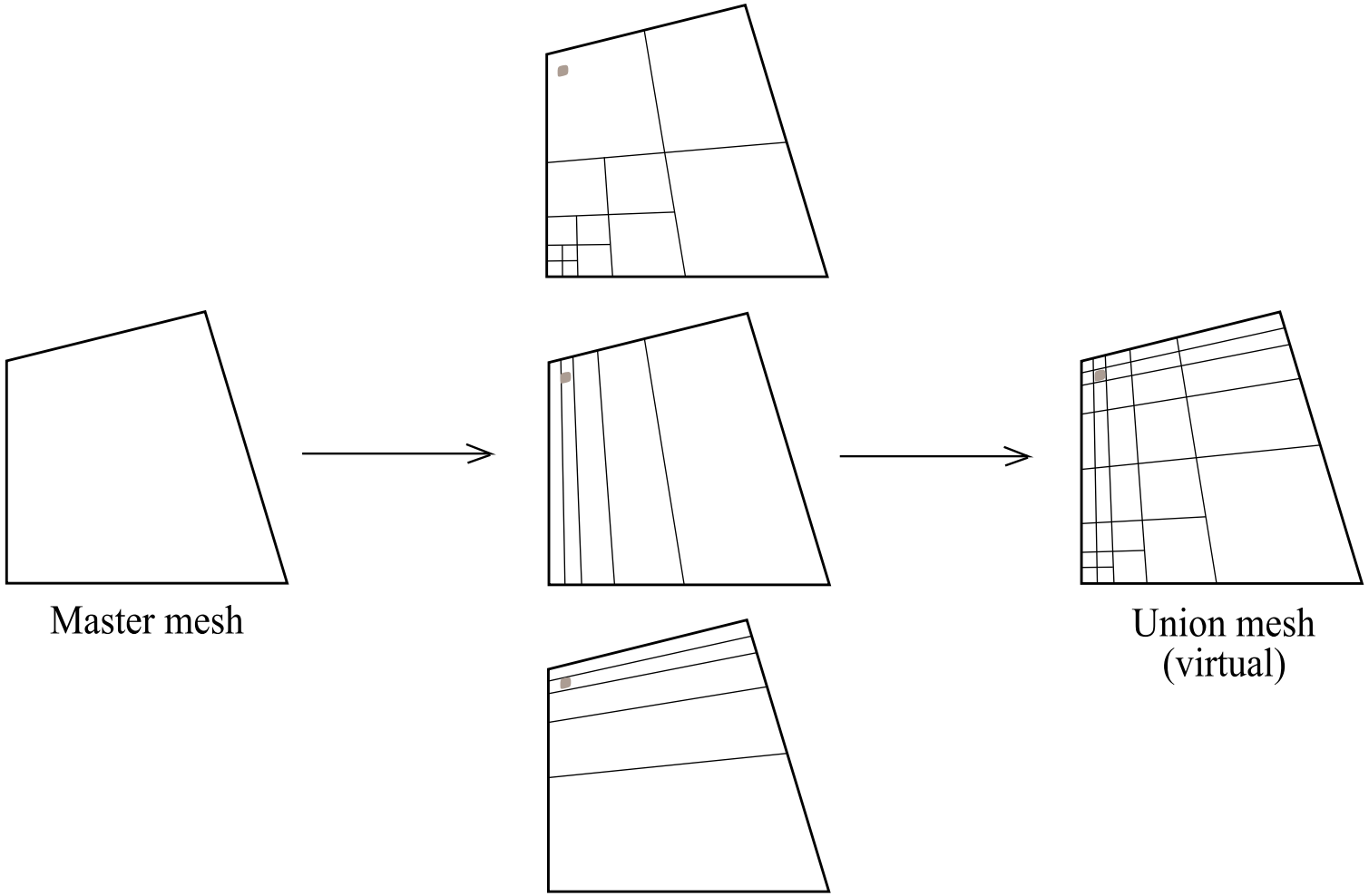

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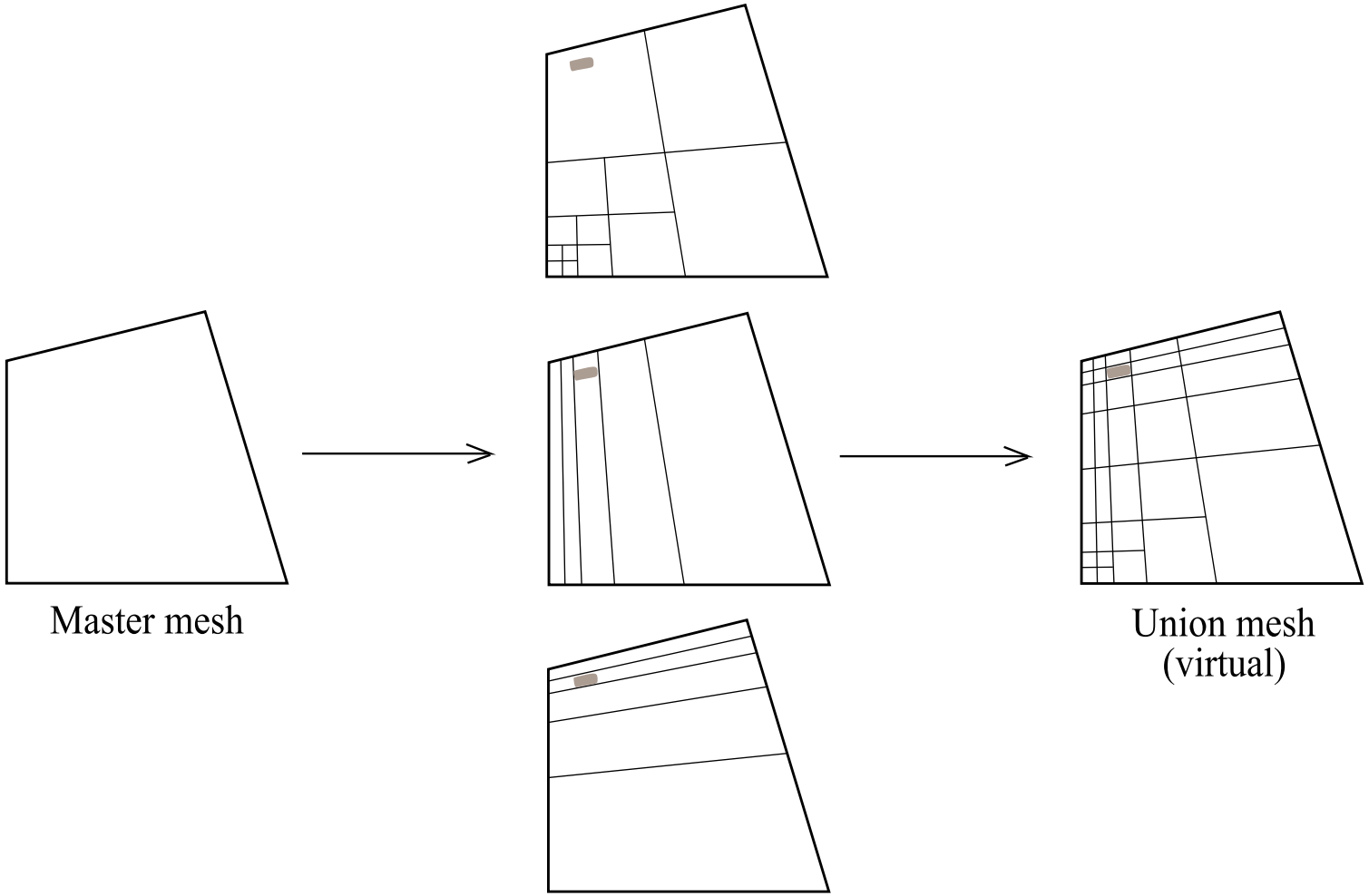

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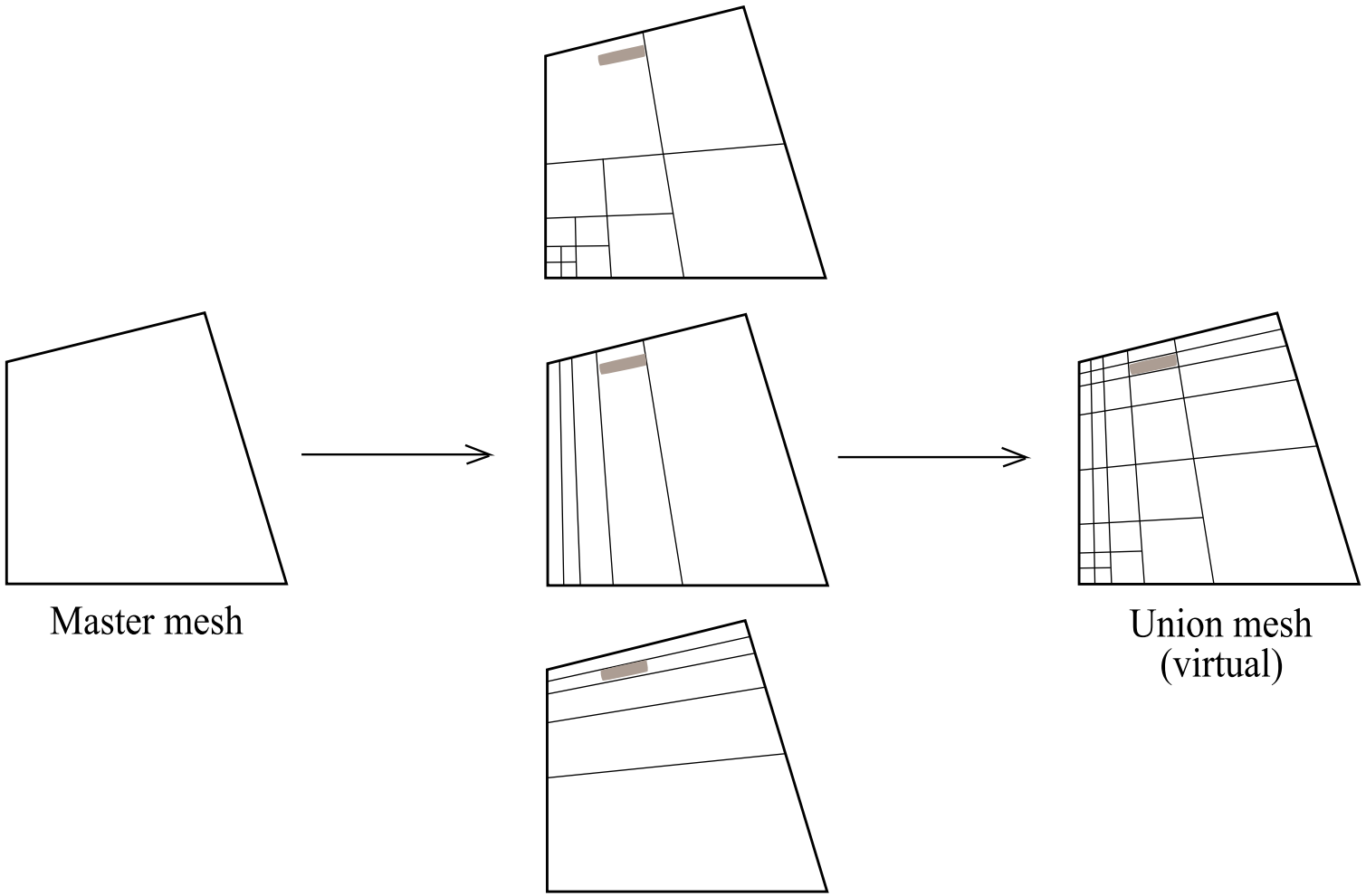

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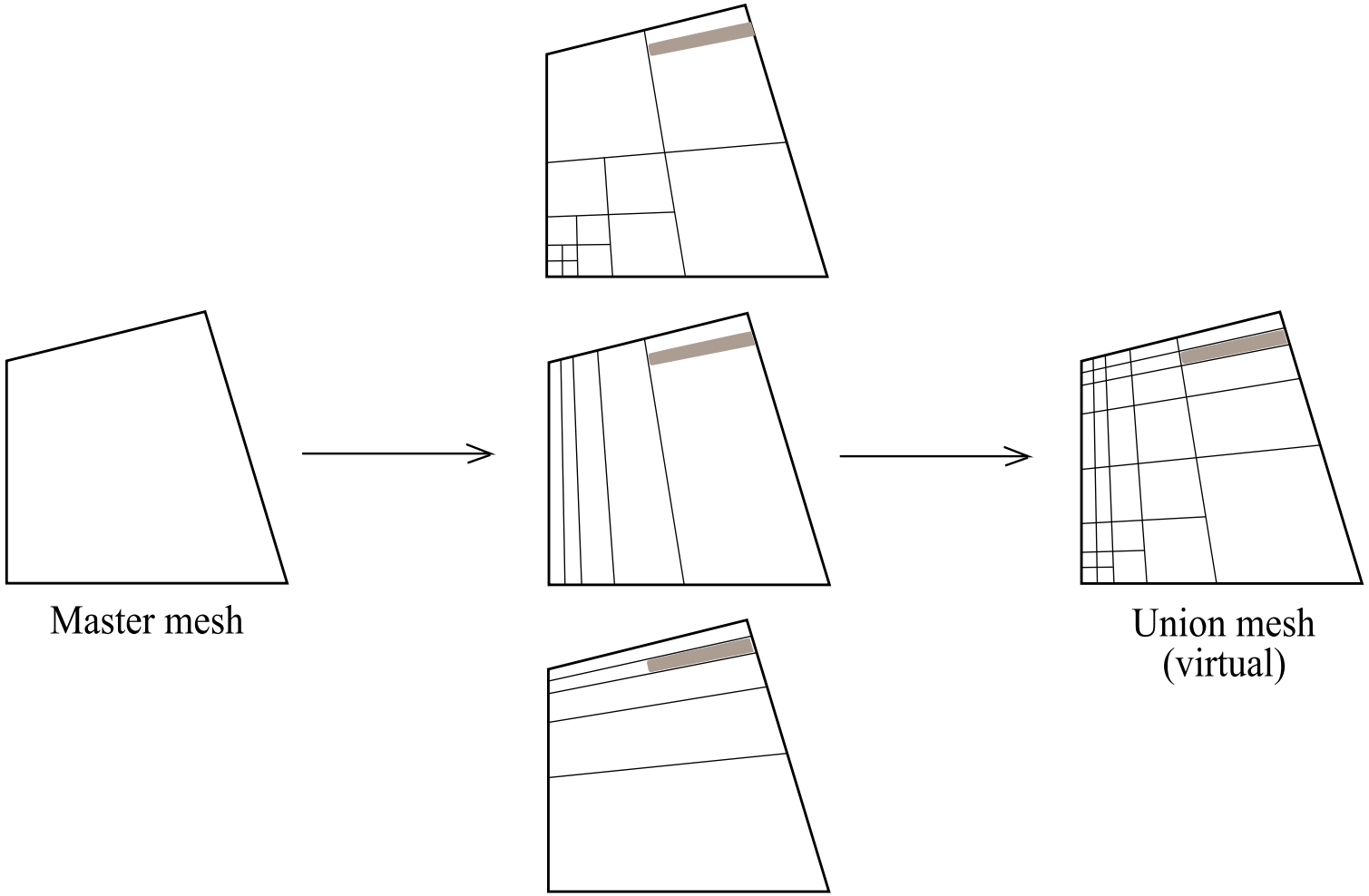

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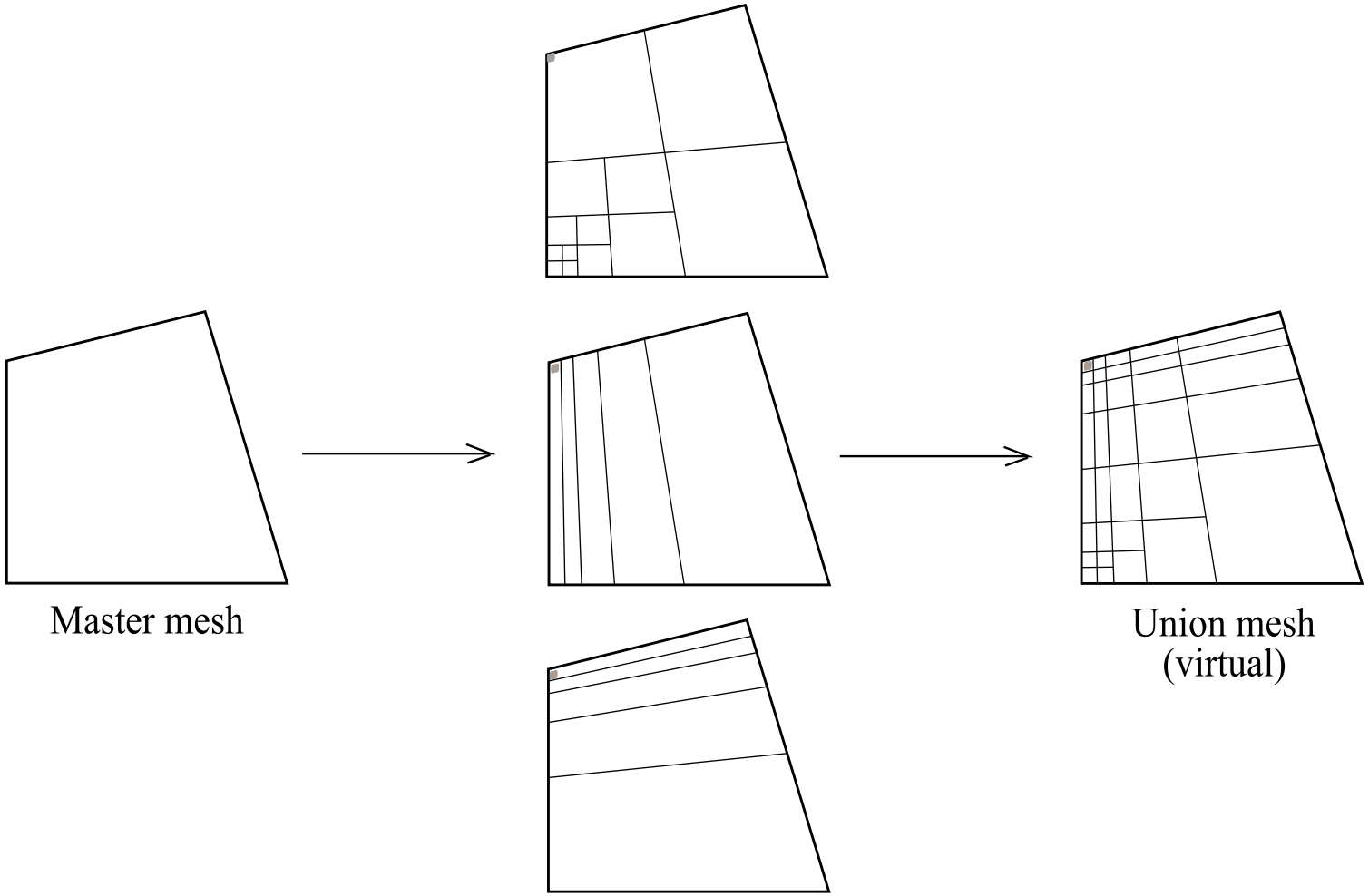

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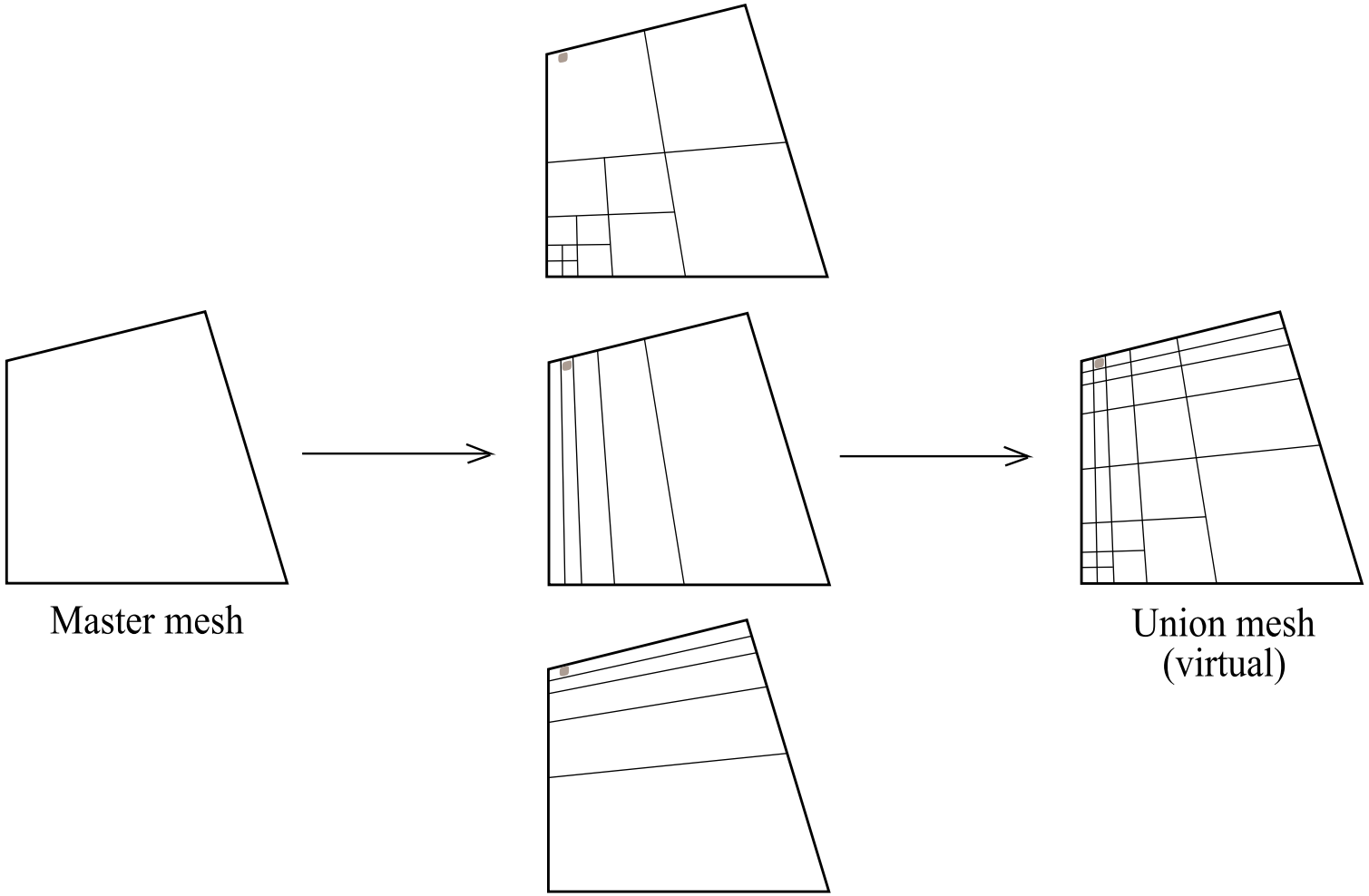

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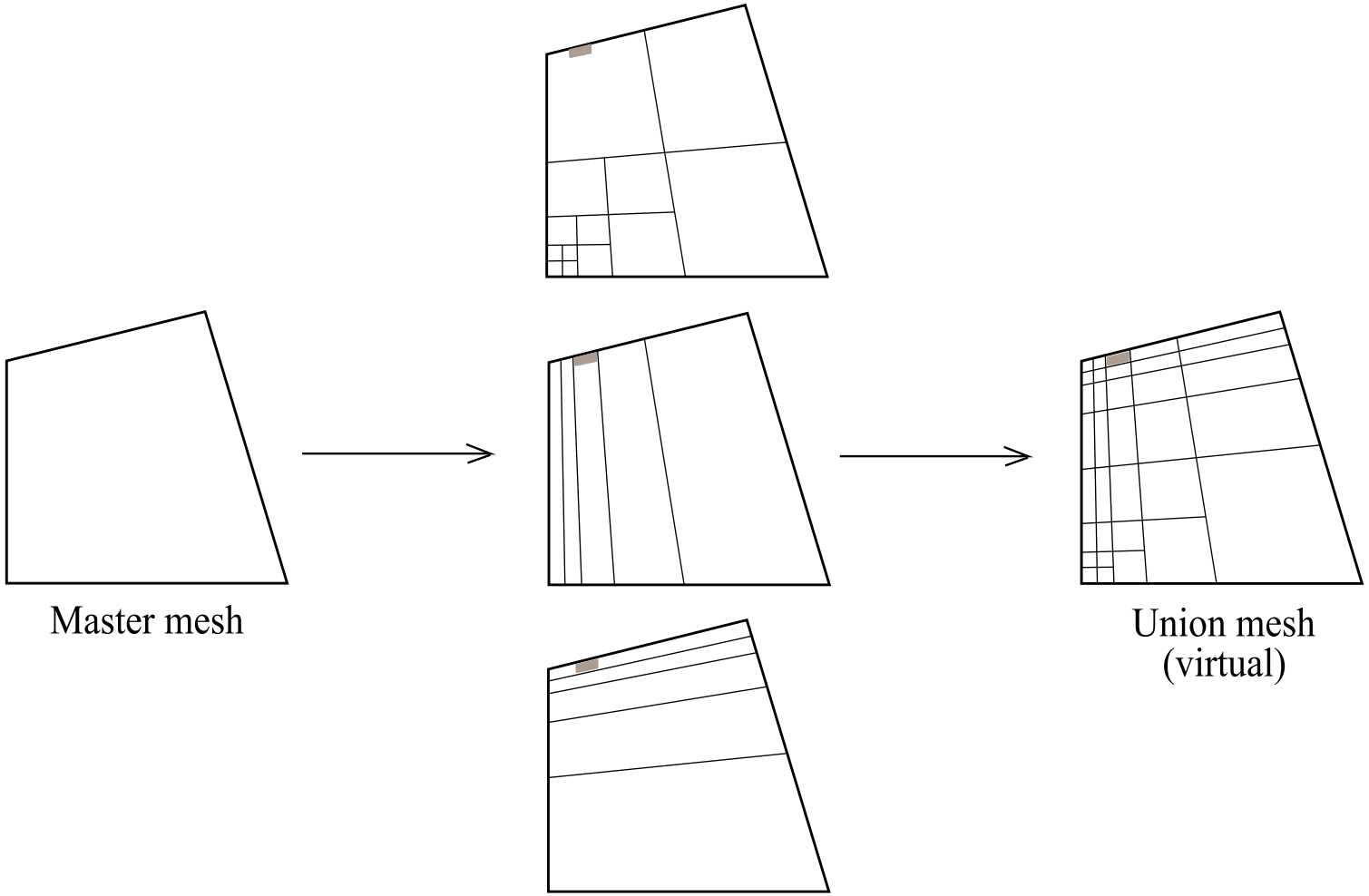

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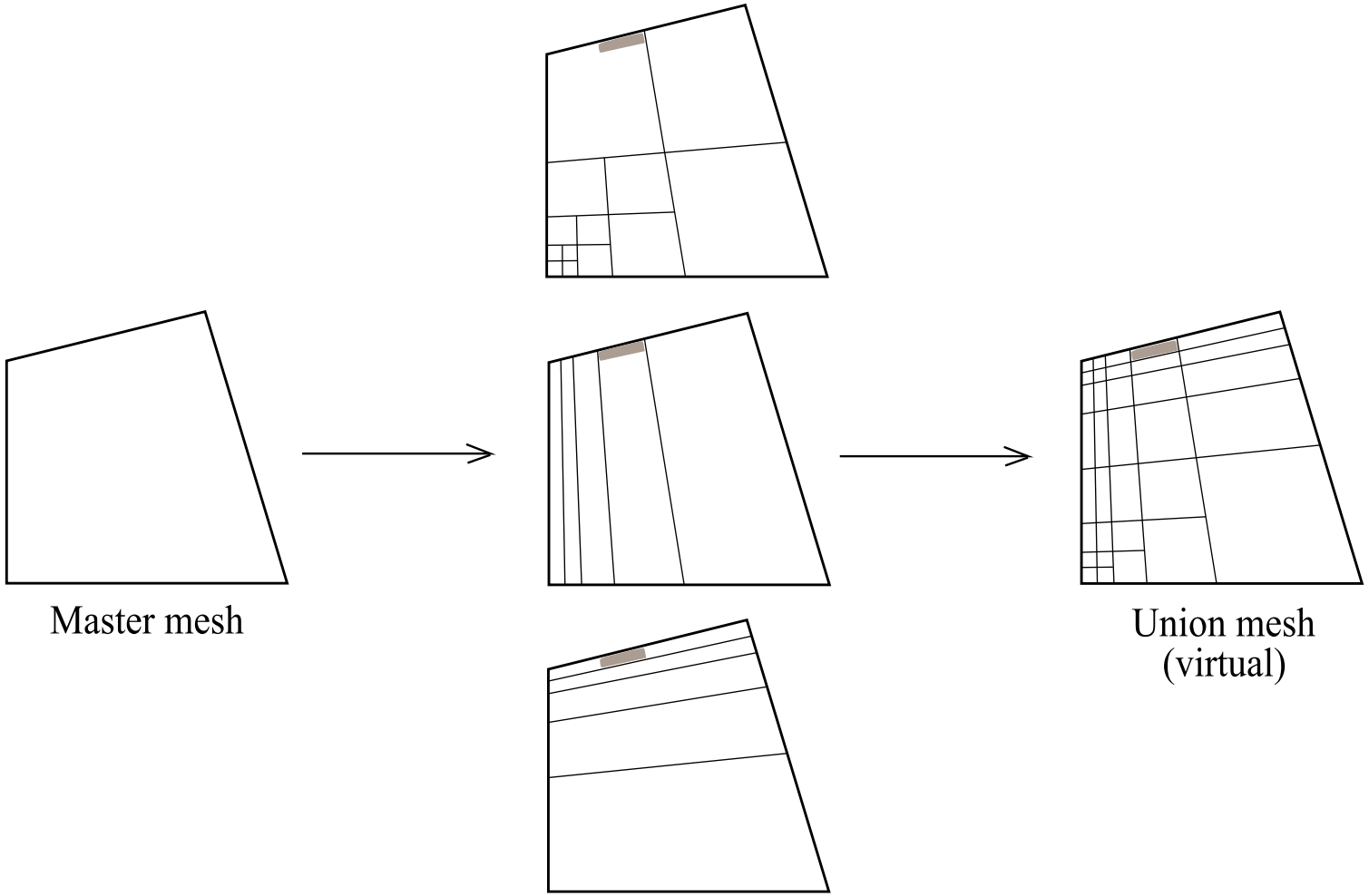

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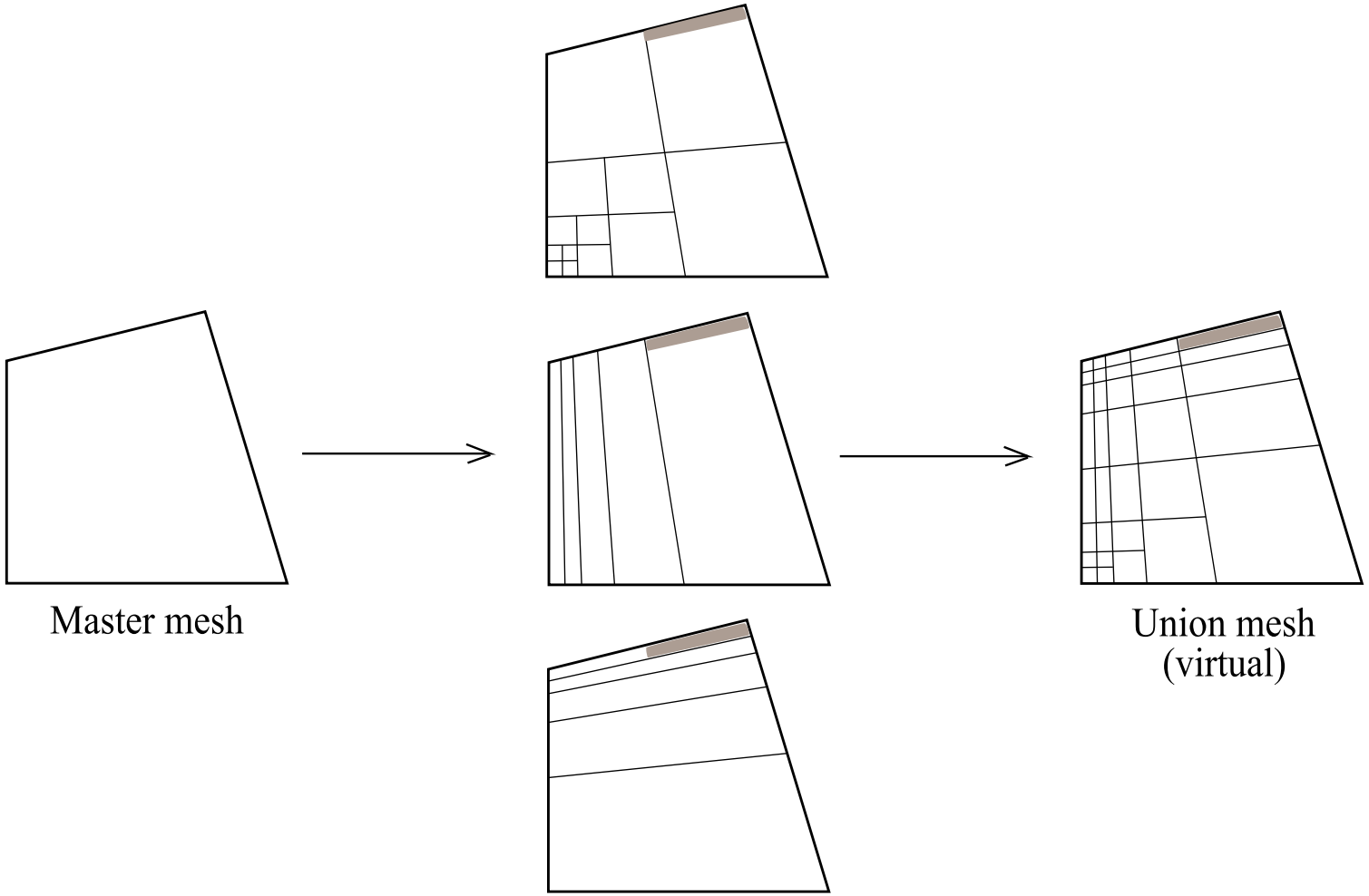
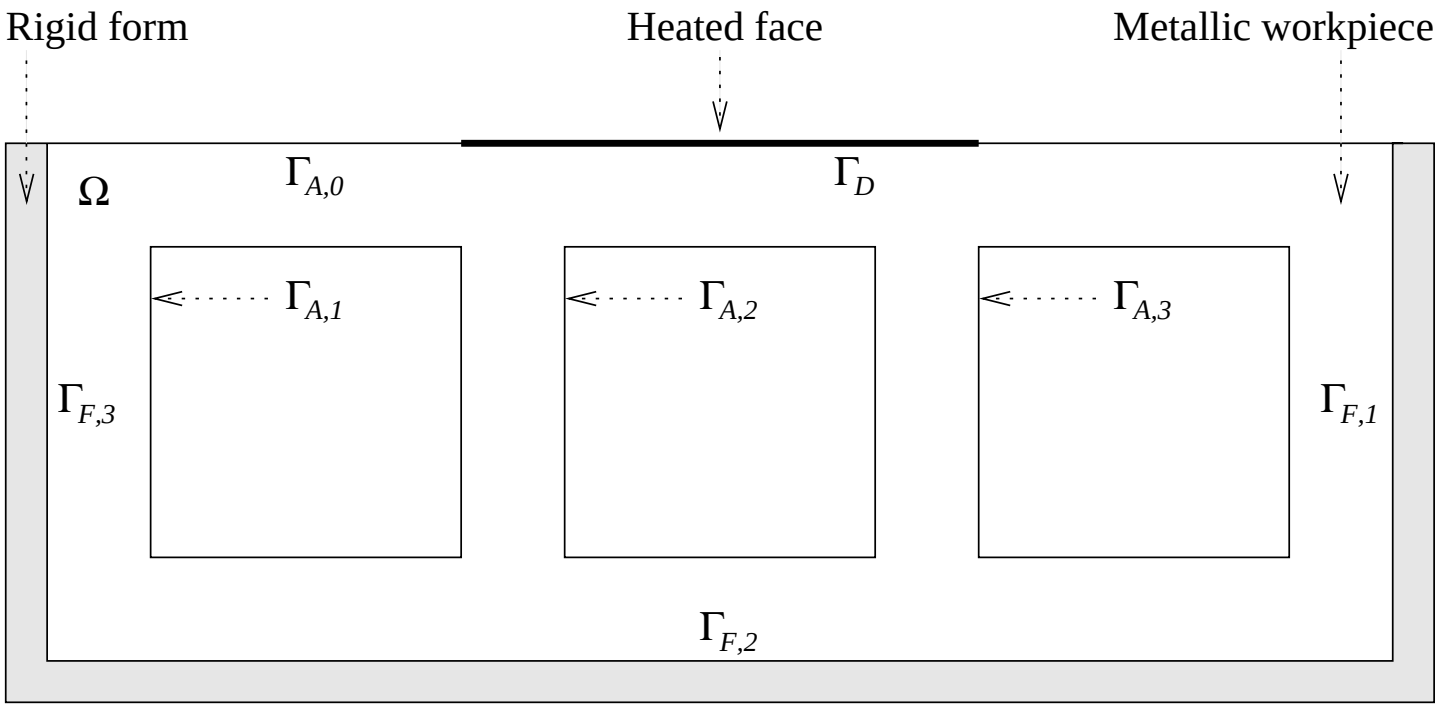


Adaptive Multimesh *hp*-FEM

Automatic adaptivity:

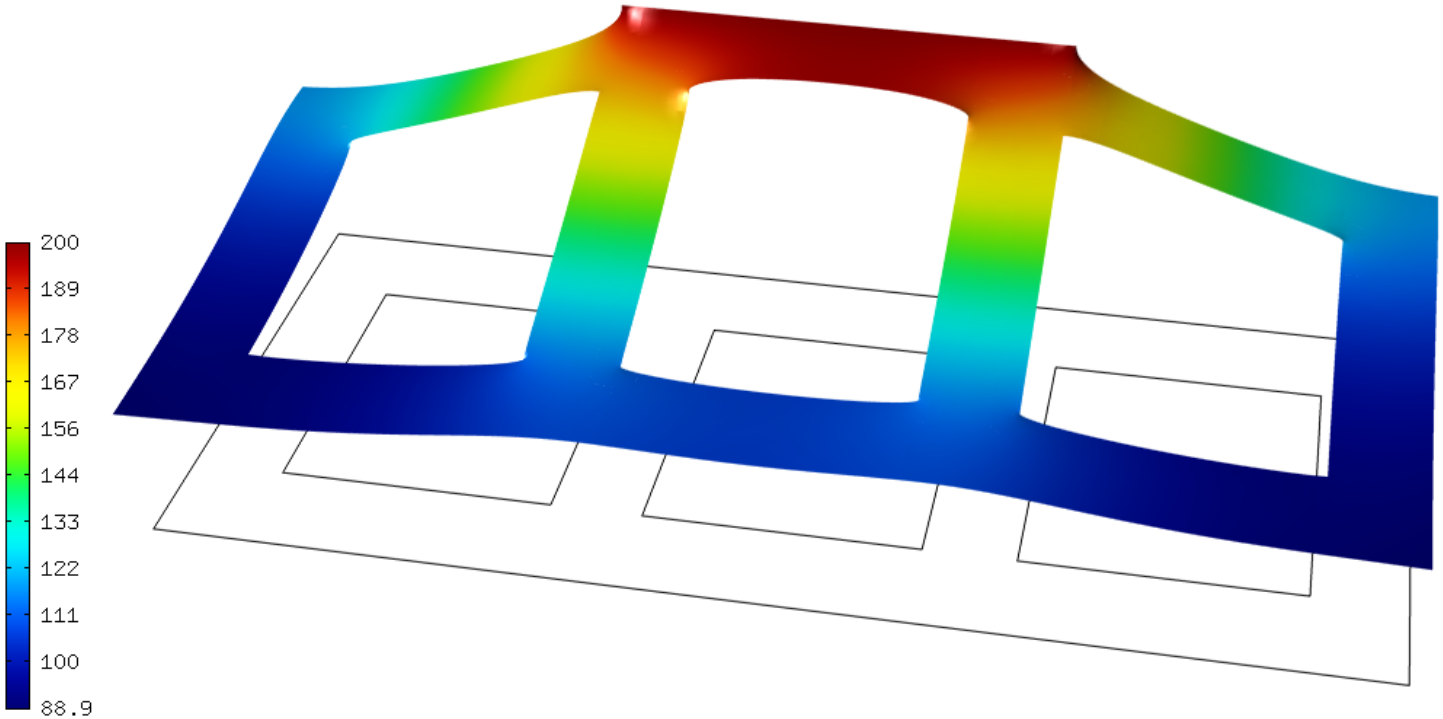
- Almost the same as in single-mesh *hp*-FEM.
- Put all elements of all meshes into one list.
- Sort according to their error estimates.
- Refine those with the largest errors.

Thermoelasticity



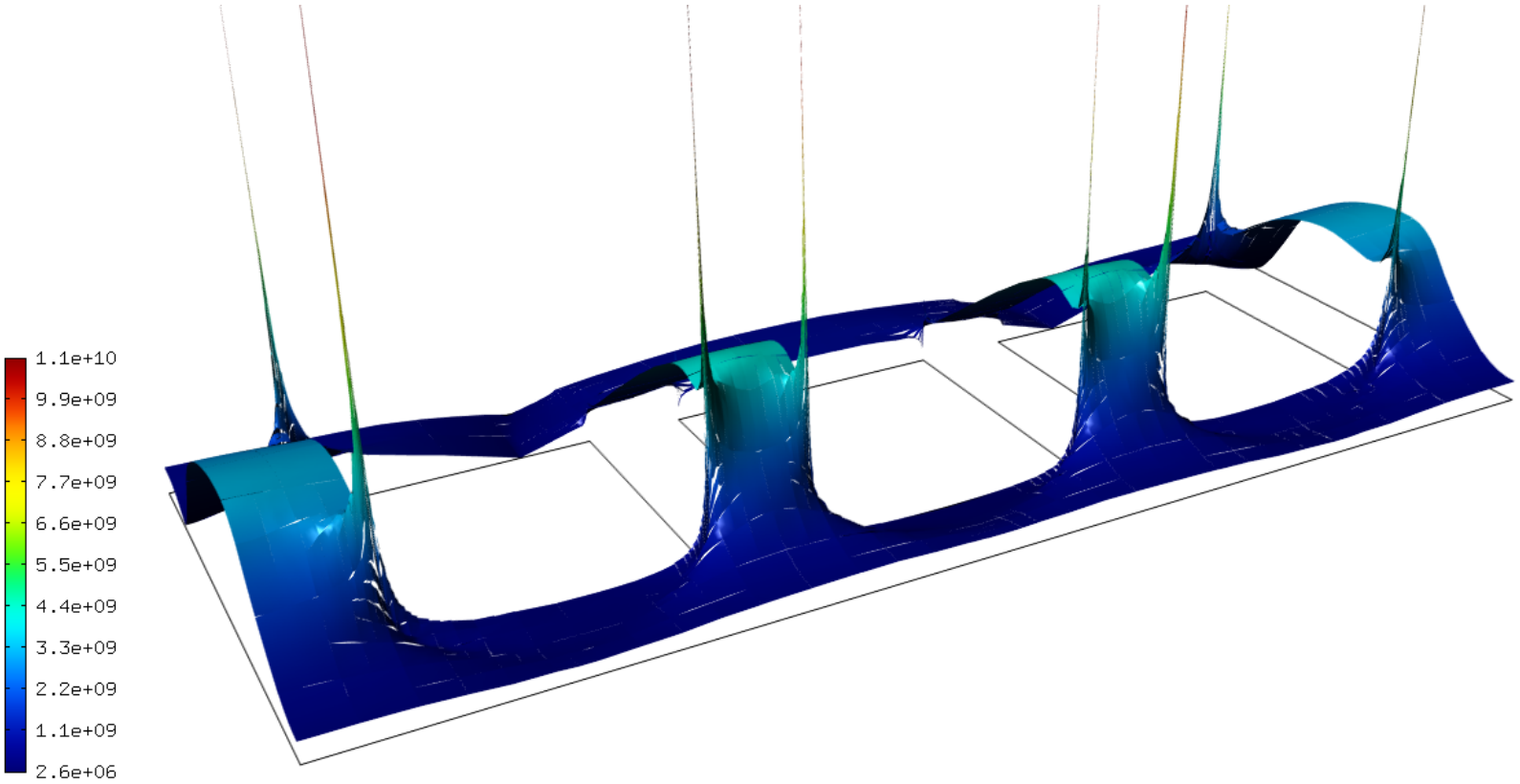
Thermoelasticity

- Solution: temperature

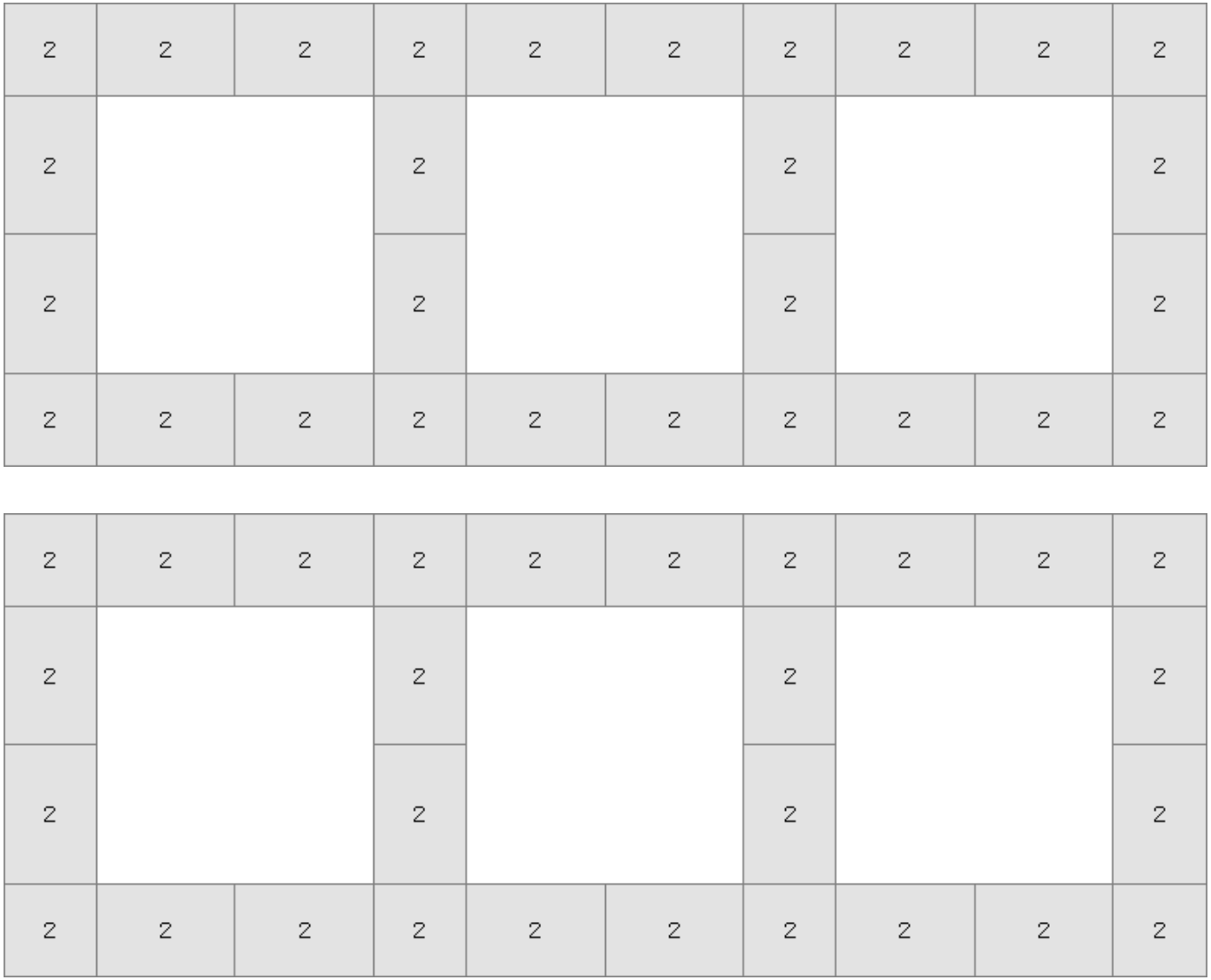


Thermoelasticity

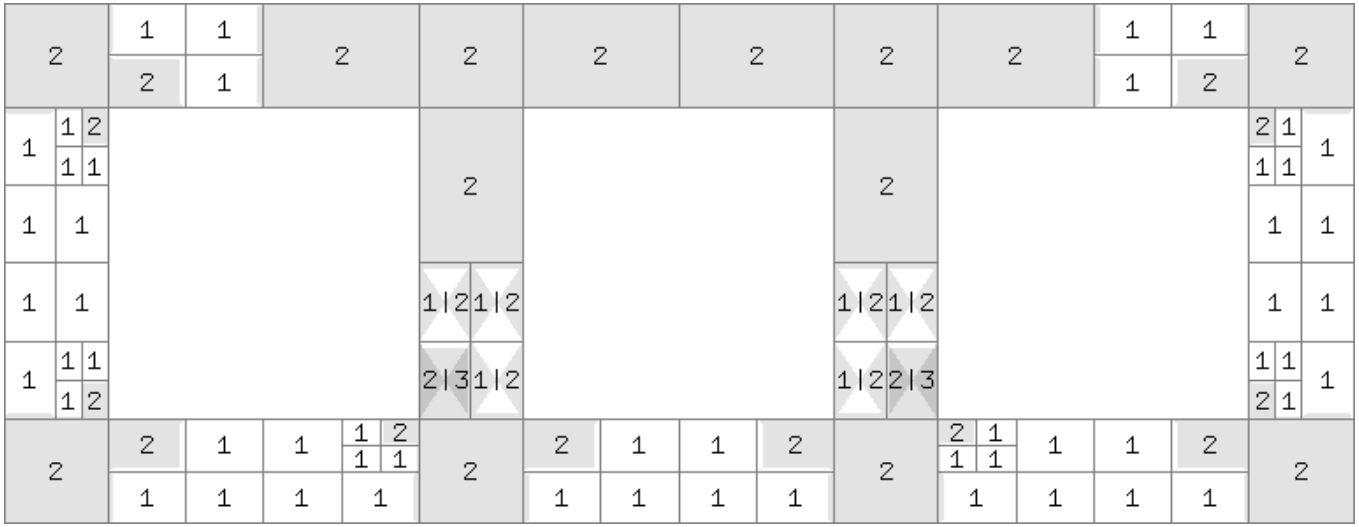
- Solution: stress

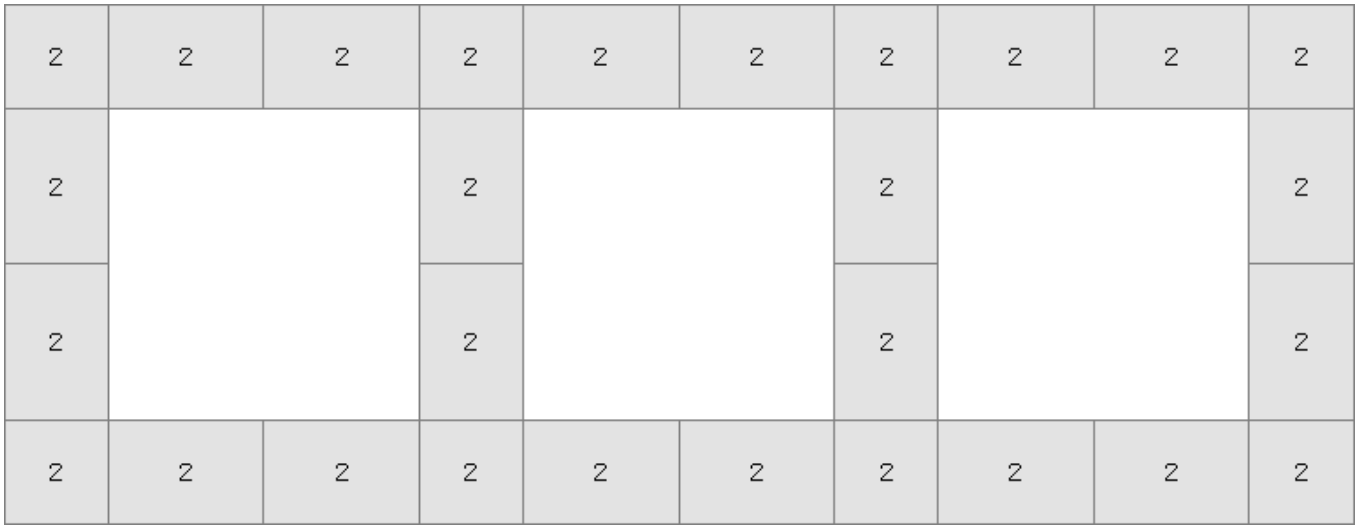


Thermoelasticity (step 1)

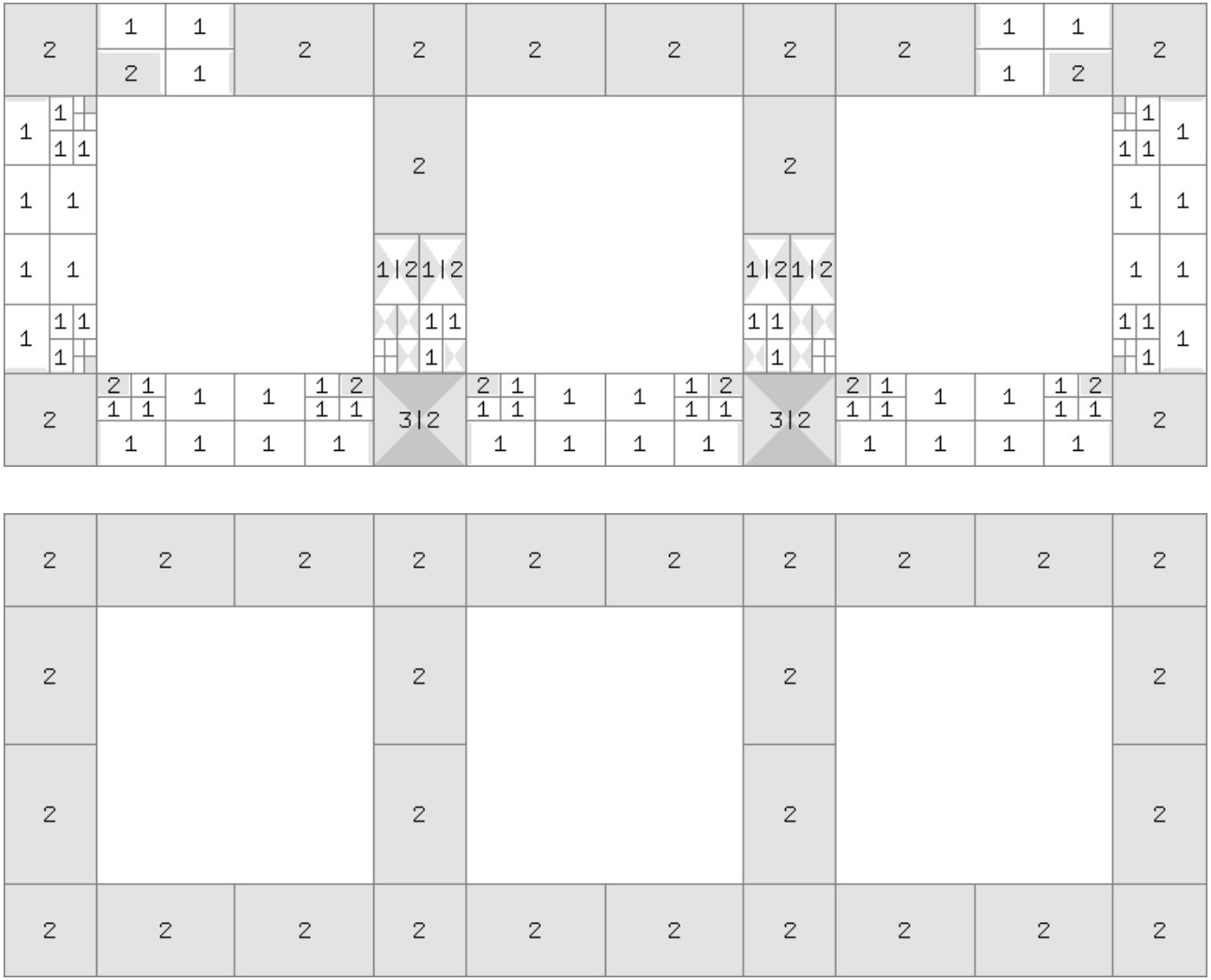


Thermoelasticity (step 2)

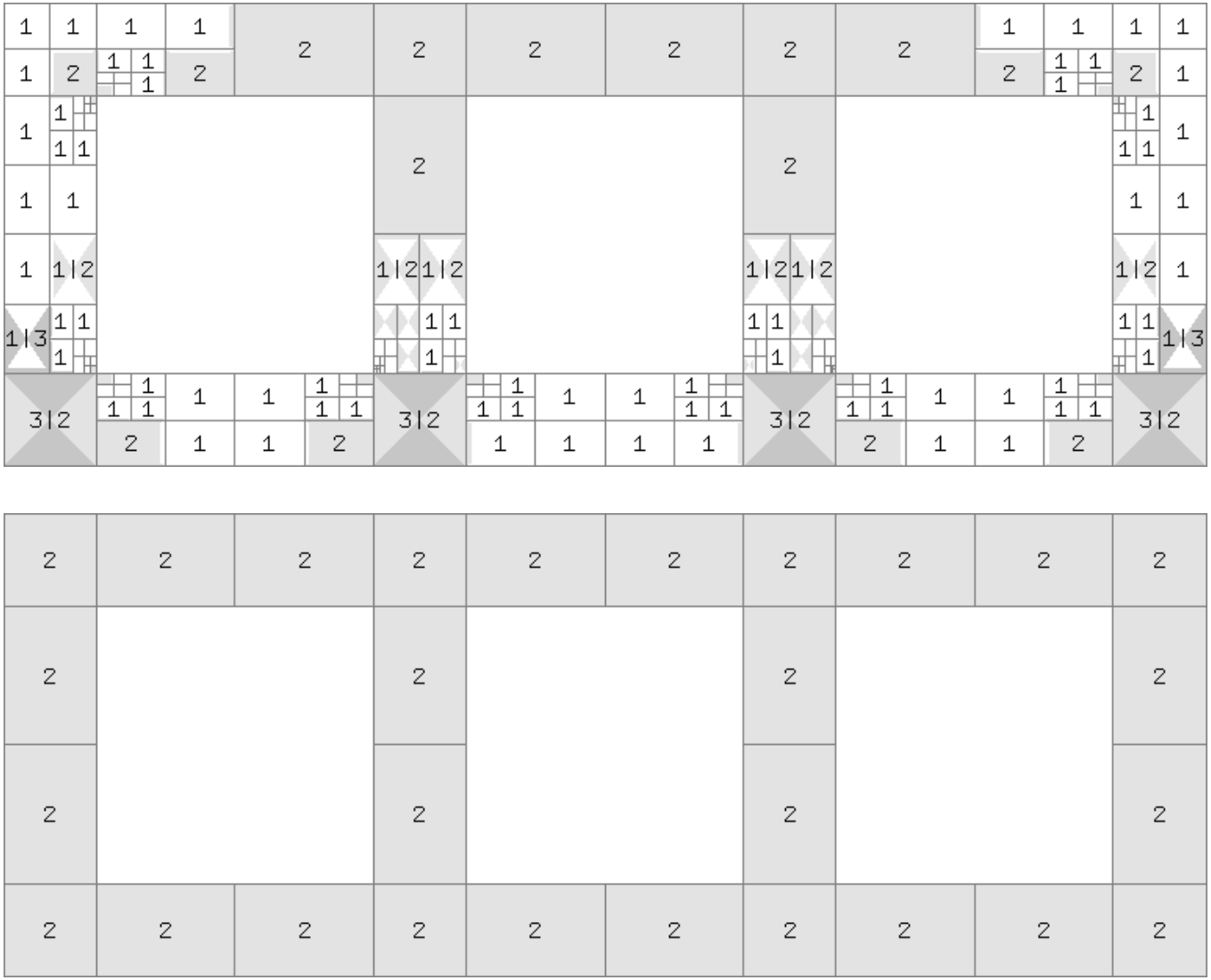




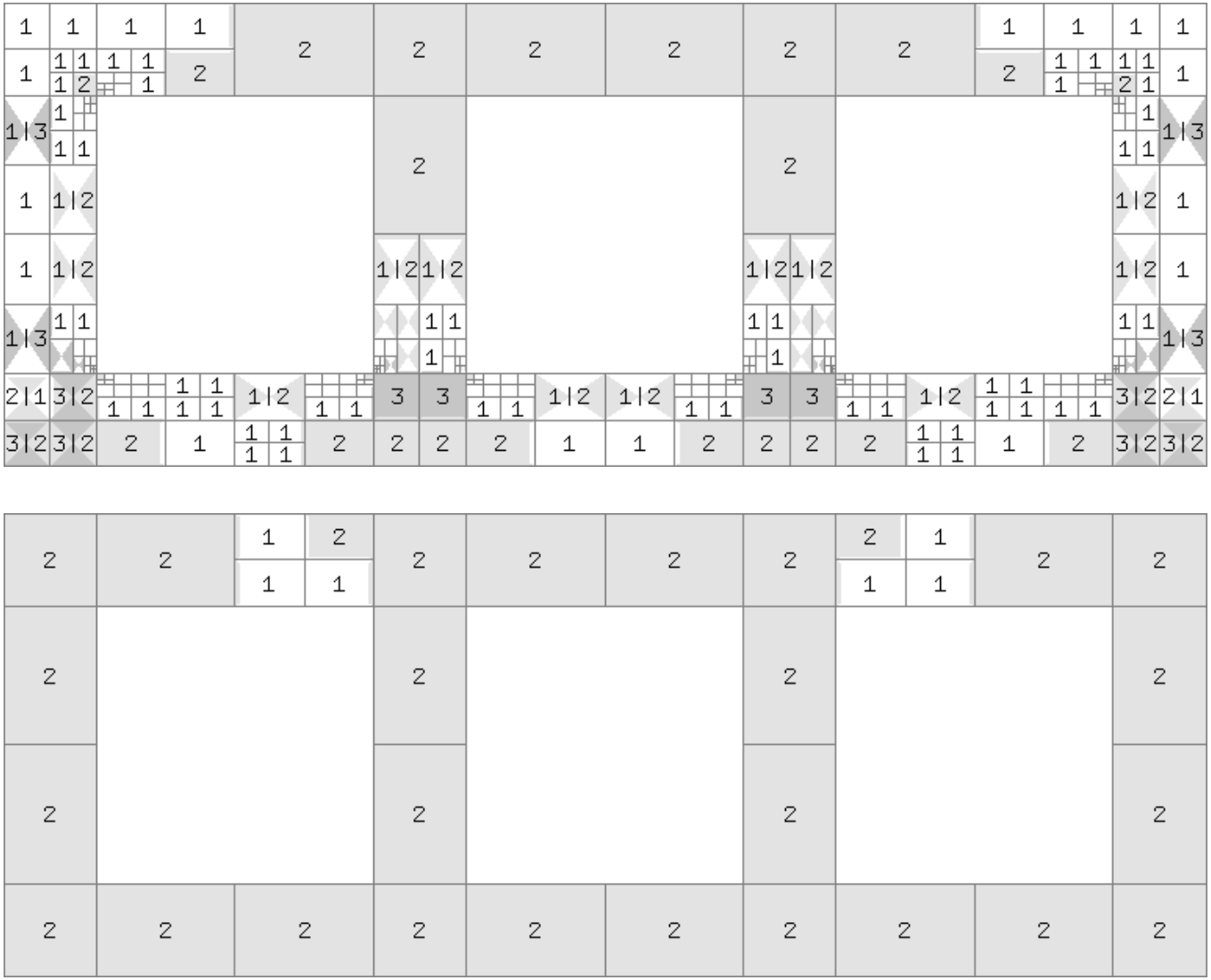
Thermoelasticity (step 3)



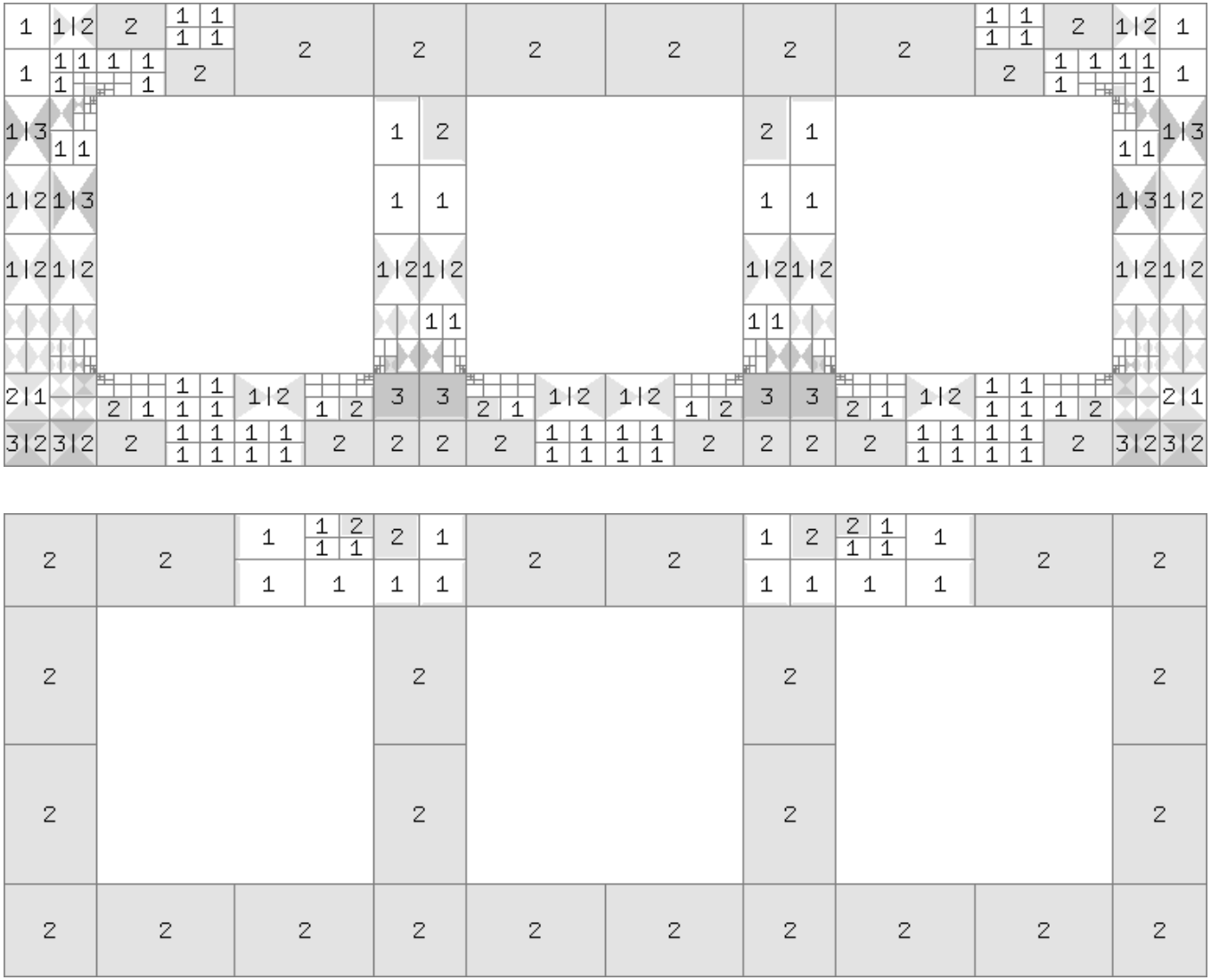
Thermoelasticity (step 4)



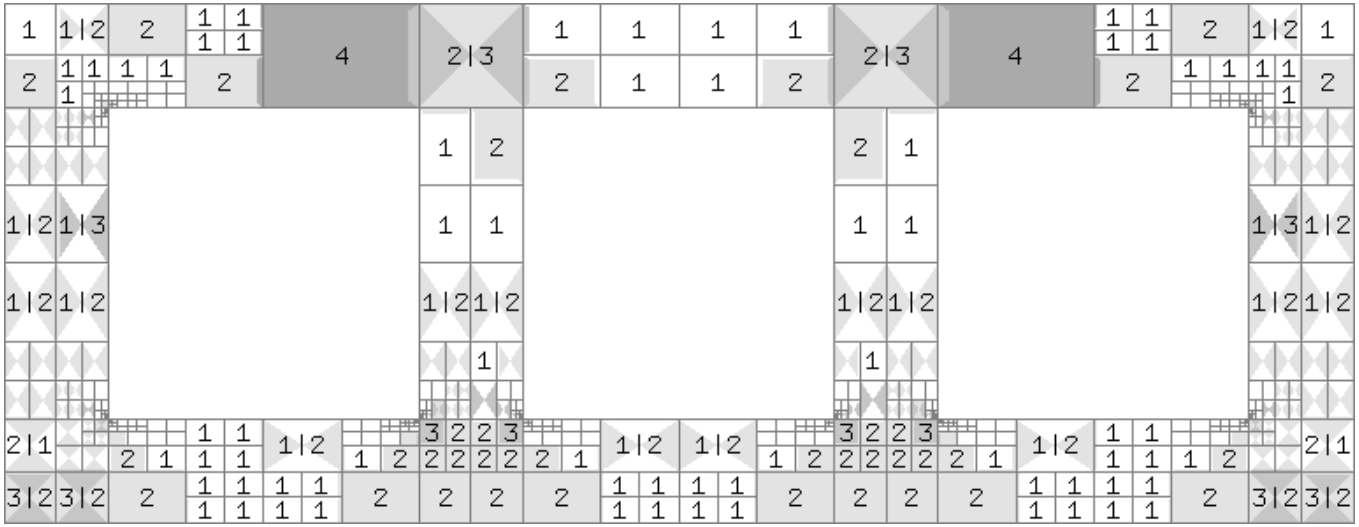
Thermoelasticity (step 5)

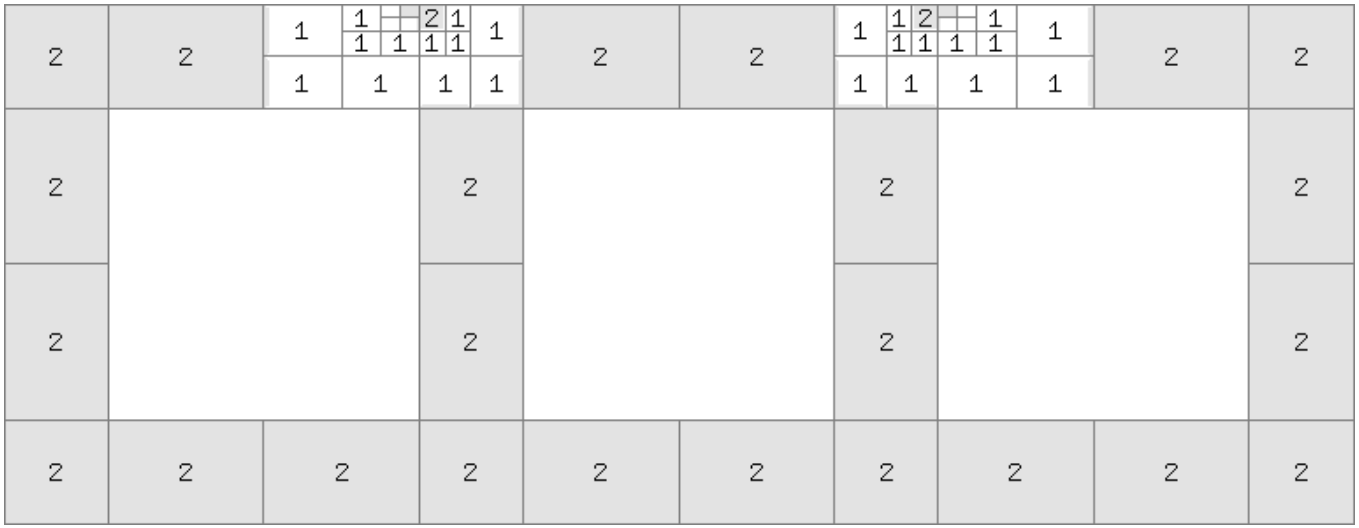


Thermoelasticity (step 6)

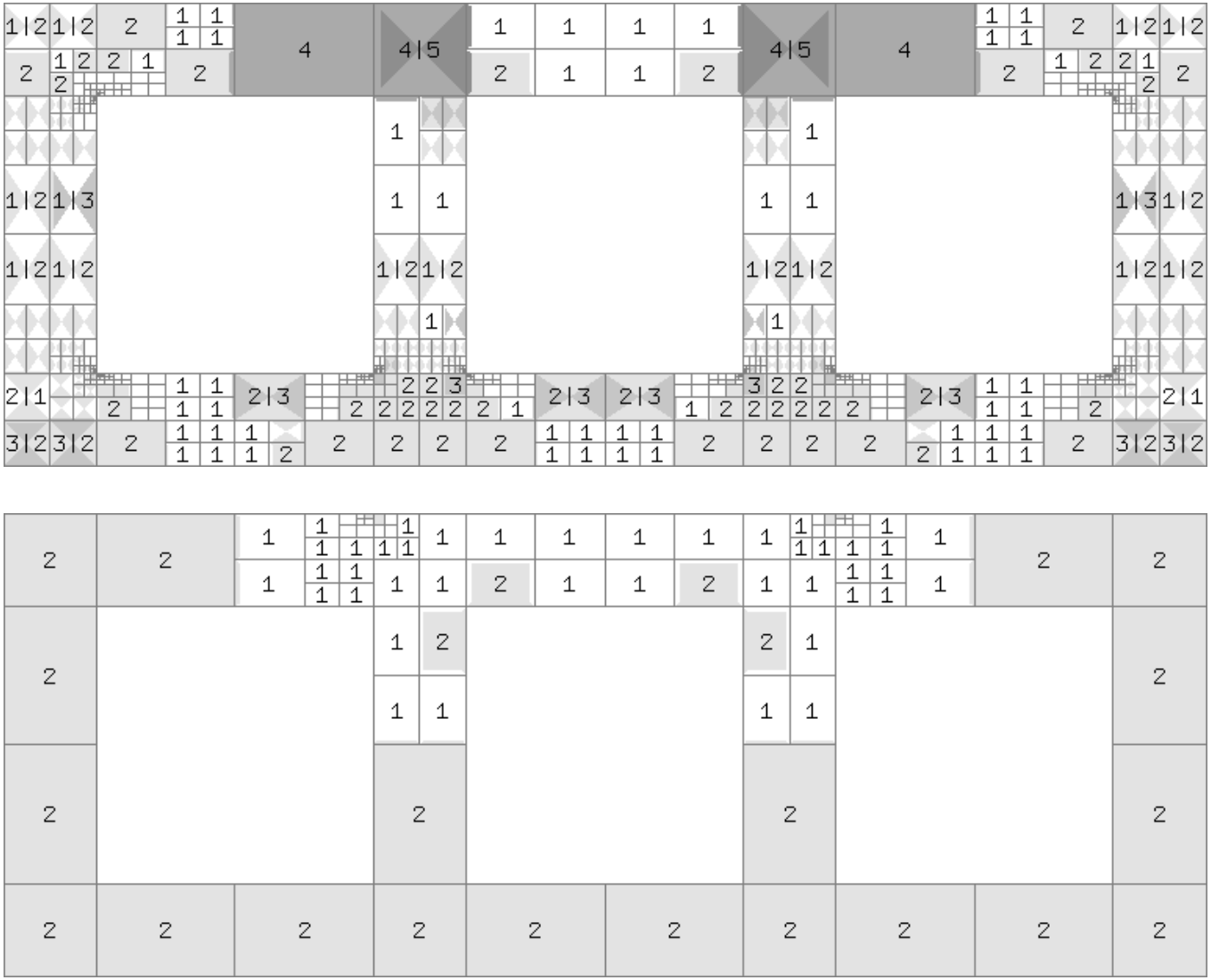


Thermoelasticity (step 7)

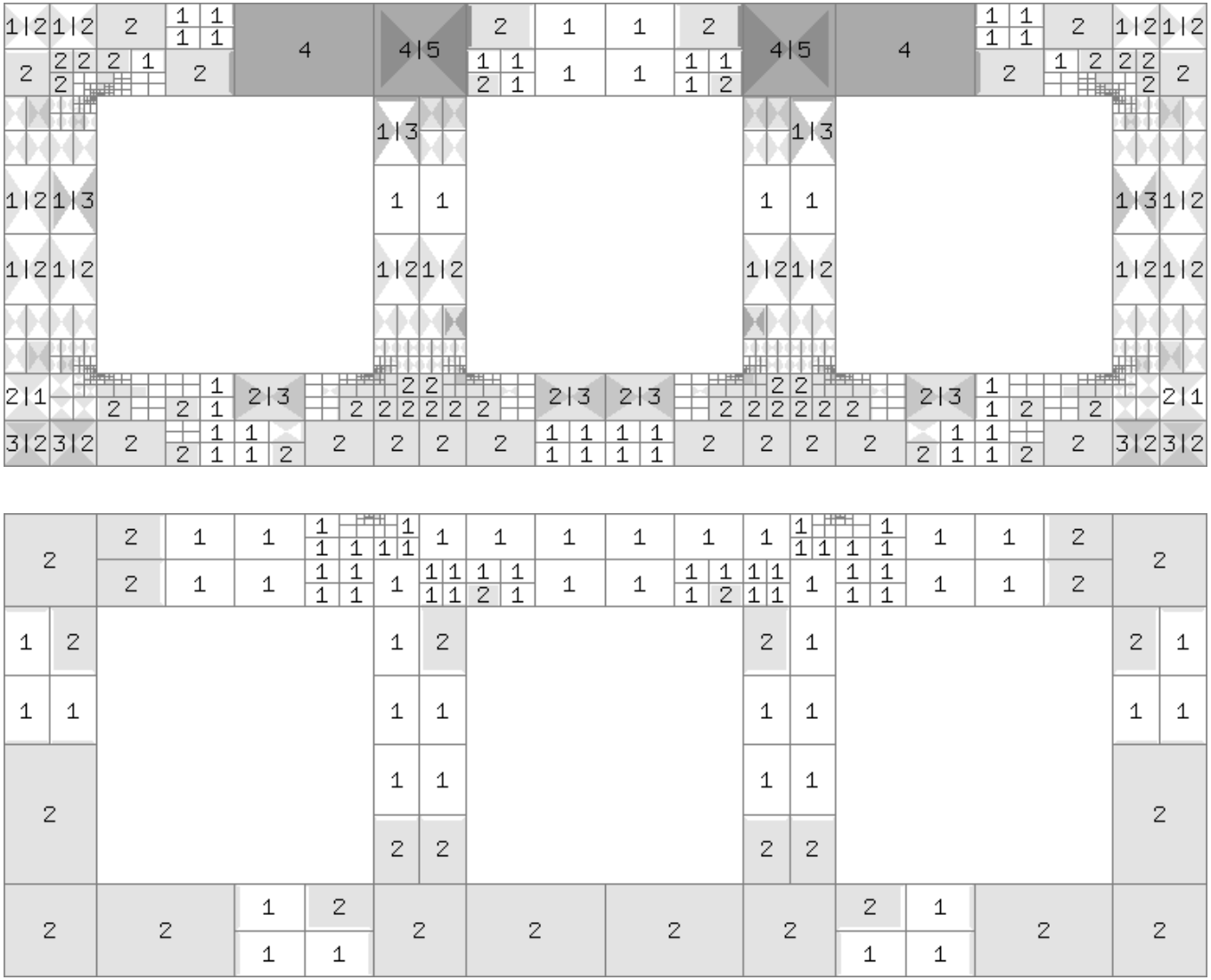




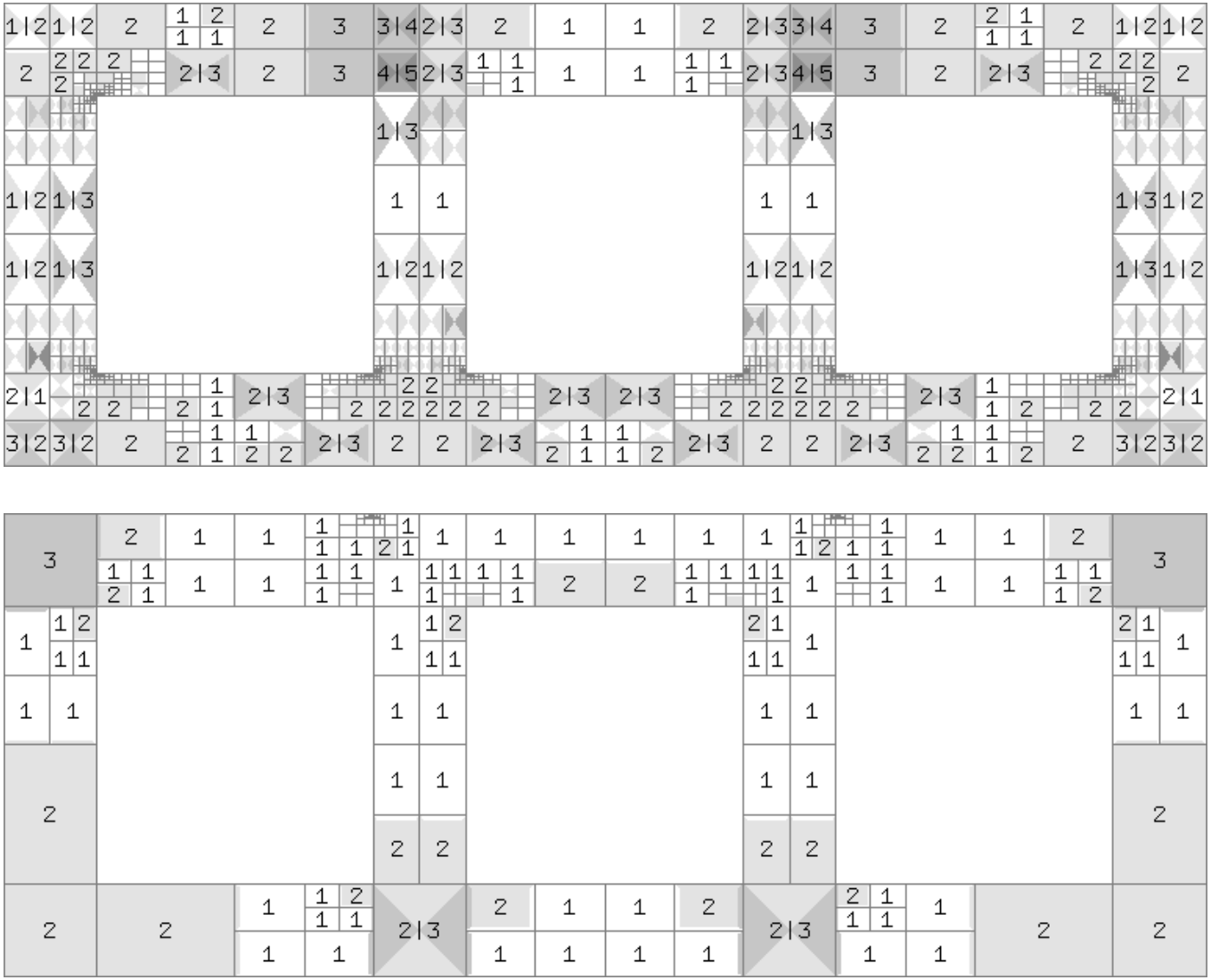
Thermoelasticity (step 8)



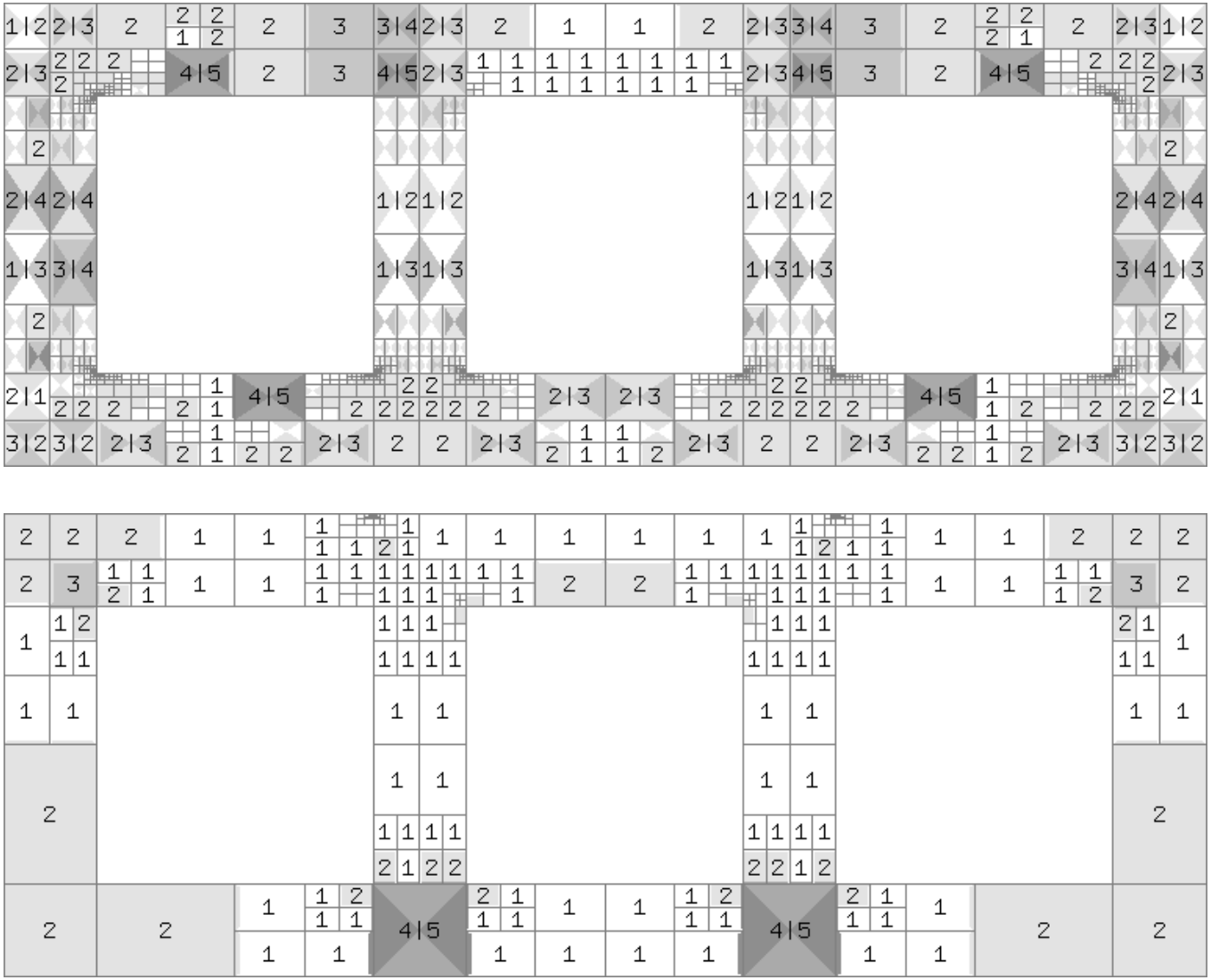
Thermoelasticity (step 9)



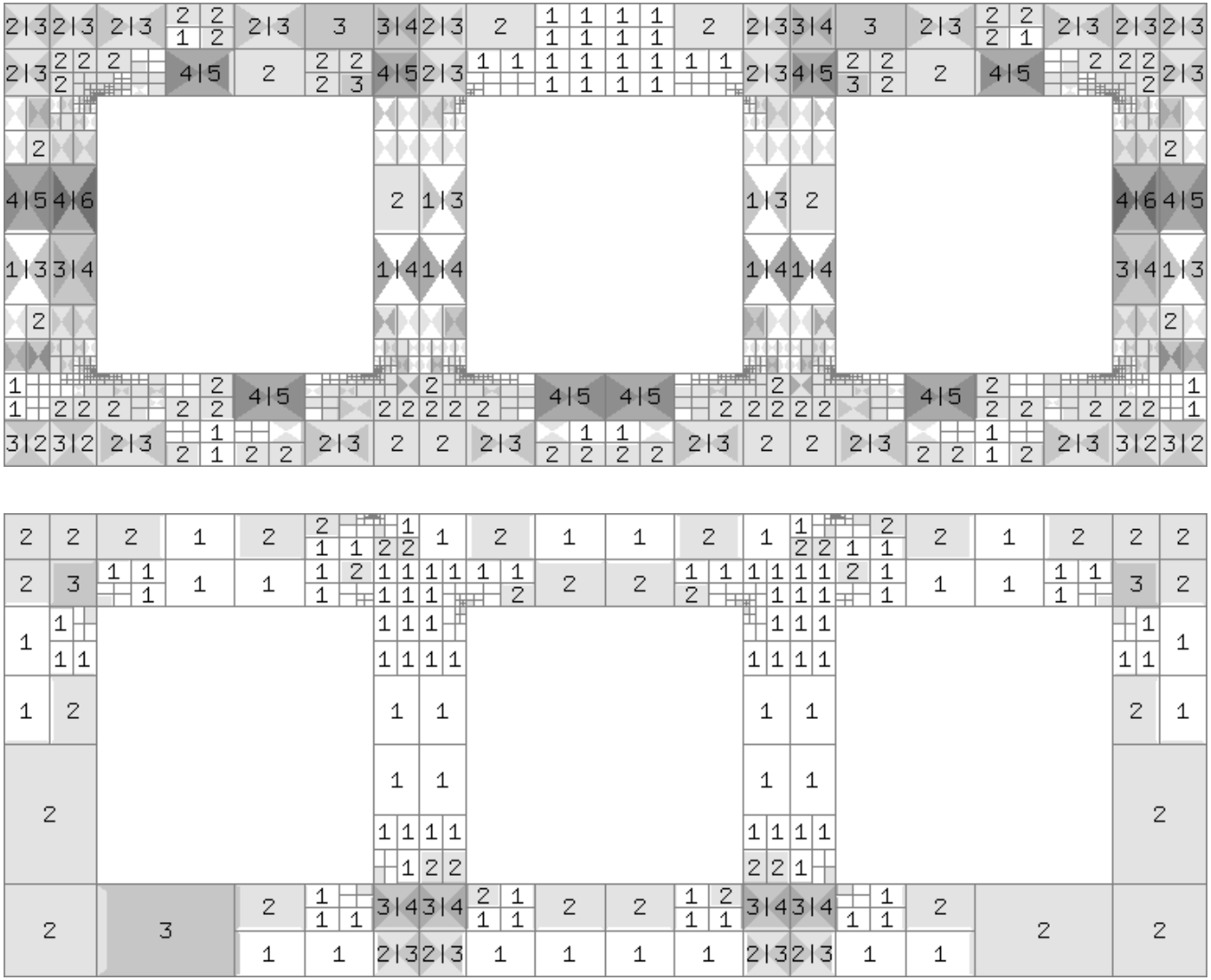
Thermoelasticity (step 10)



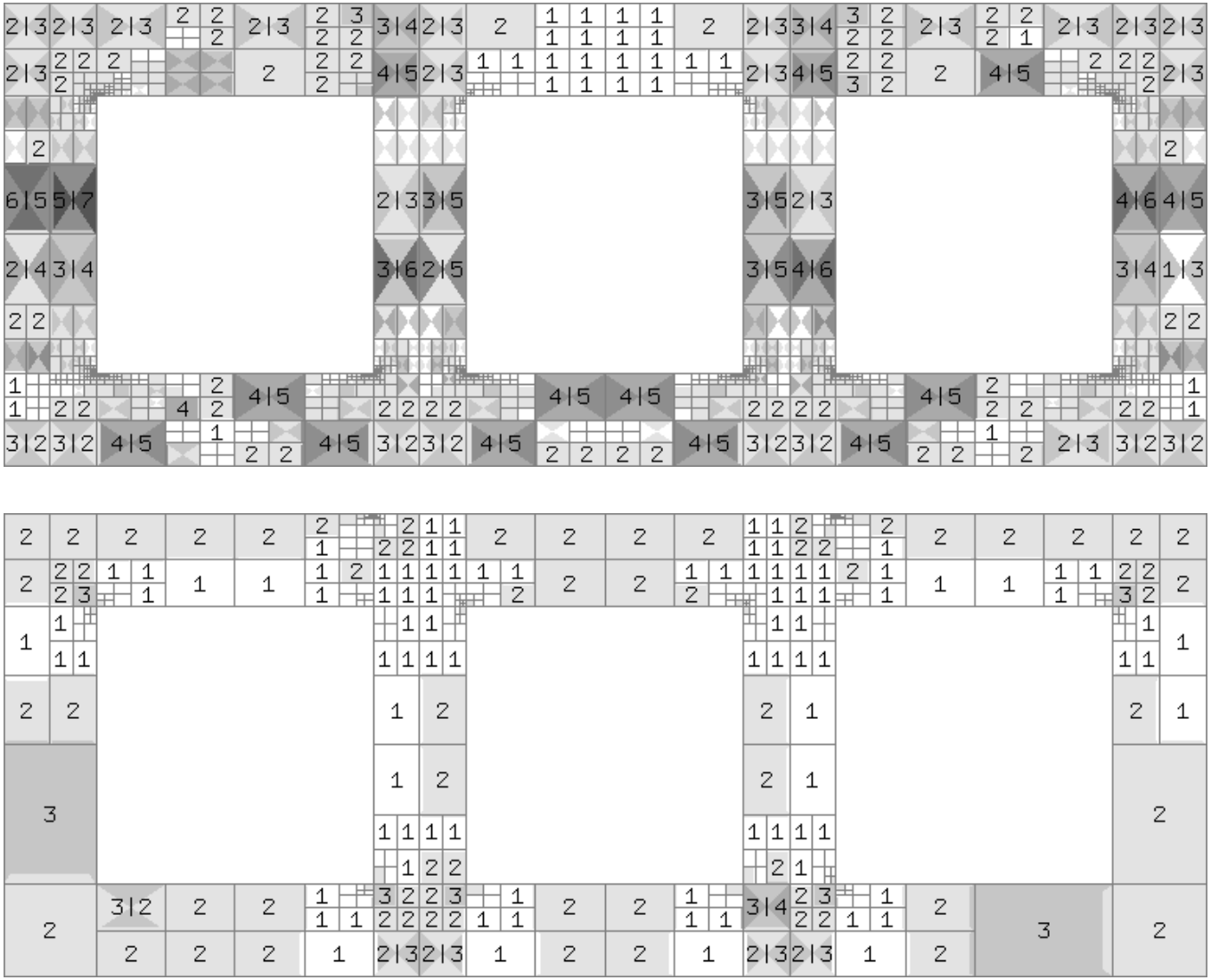
Thermoelasticity (step 11)



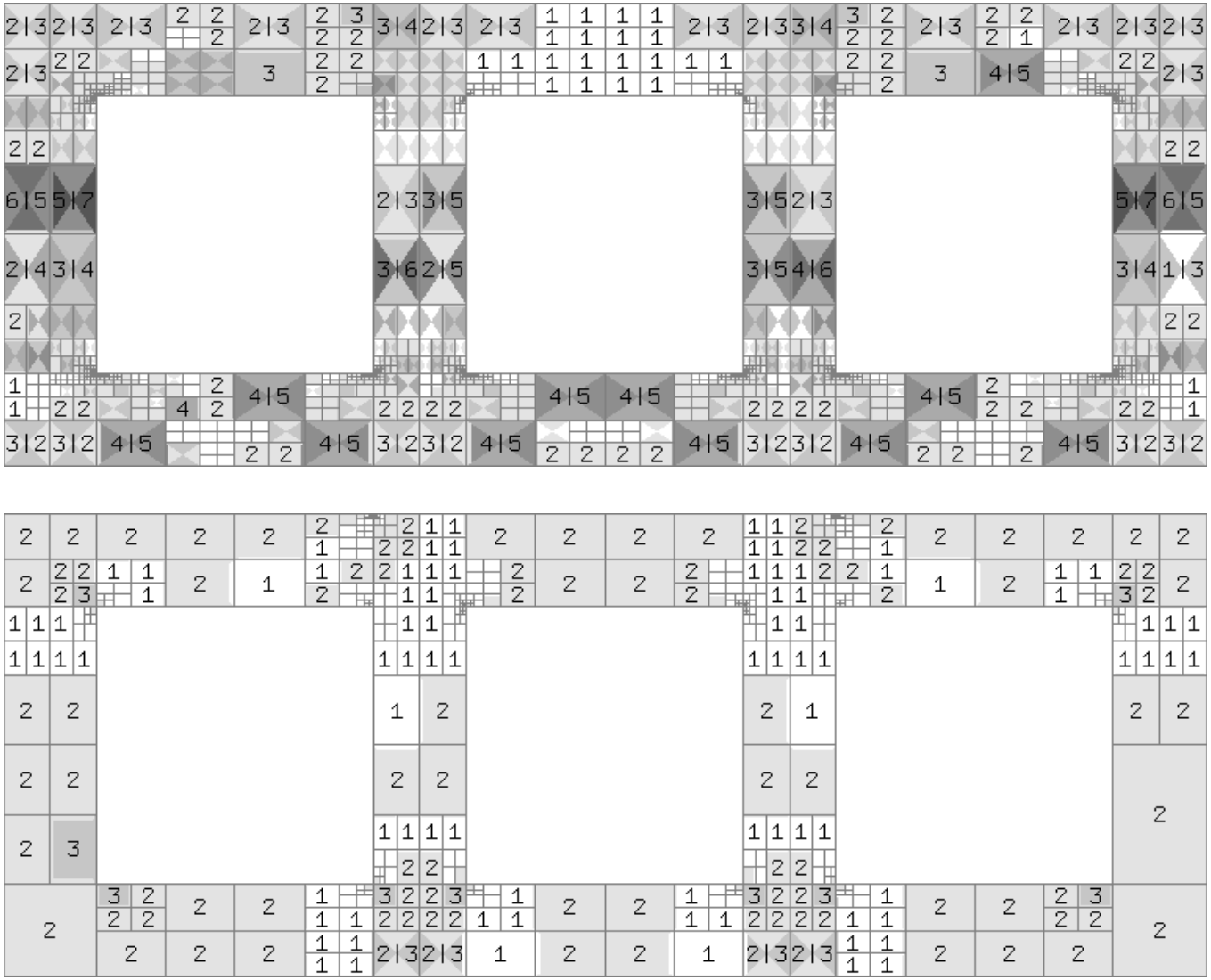
Thermoelasticity (step 12)



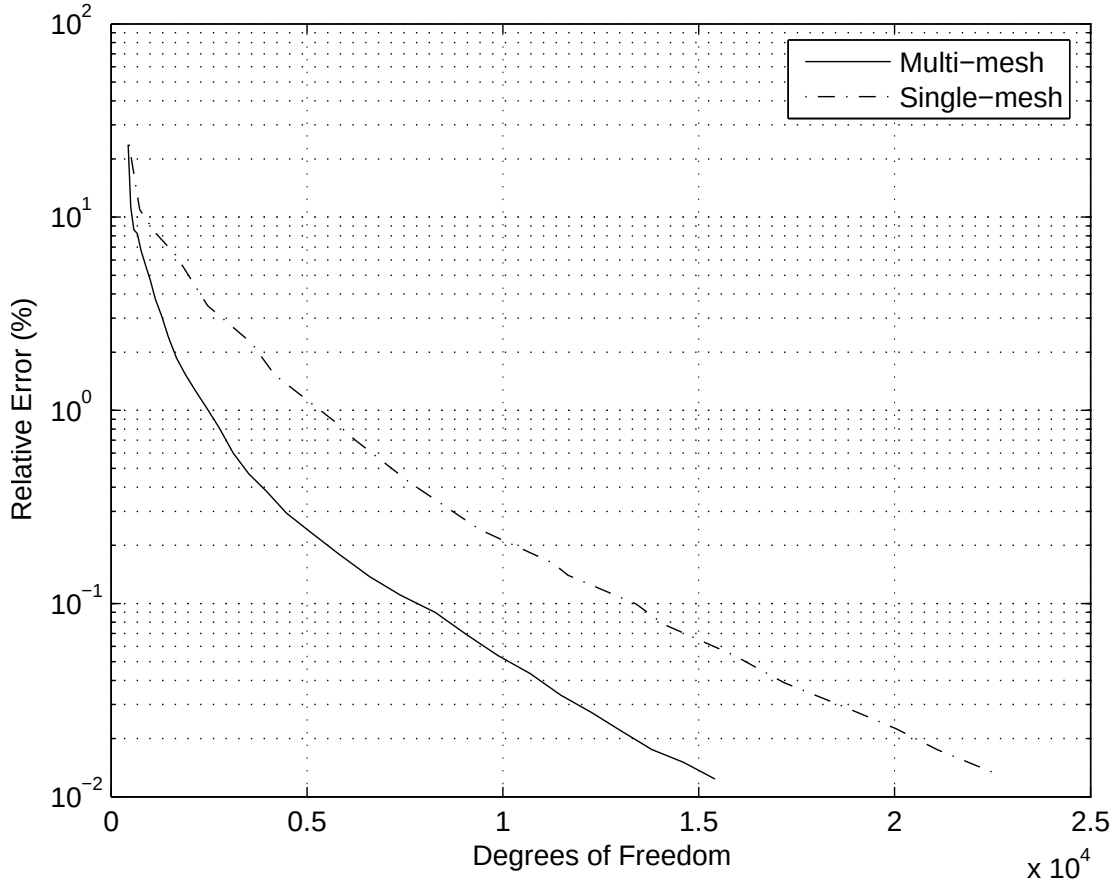
Thermoelasticity (step 13)



Thermoelasticity (step 14)



Convergence: multi-mesh vs. single-mesh



Adaptive *hp*-FEM with Dynamical Meshes

Sample time-dependent problem:

$$\frac{\partial u}{\partial t} - \Delta u = f$$

Implicit Euler:

$$\frac{u_{n+1} - u_n}{h} - \Delta u_{n+1} = f_{n+1}$$

Multimesh *hp*-FEM:

$$\underbrace{\frac{u_{n+1}}{h} - \Delta u_{n+1}}_{\text{solve using adaptivity}} = \underbrace{f_{n+1} - \frac{u_n}{h}}_{\text{last refined mesh}}$$

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