Refinement of regular mesh



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Refinement of regular mesh





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Refinement of regular mesh







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Refinement of regular mesh







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Using one-level hanging nodes (1-irregular mesh)



Refinement of regular mesh







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Using one-level hanging nodes (1-irregular mesh)





Refinement of regular mesh







Using one-level hanging nodes (1-irregular mesh)







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#### **Forced refinements**

- Create unnecessary DOF
- Spoil element shapes
- Cannot use just quads
- Have non-local, recursive nature
- Make hp-adaptivity difficult
- $\Rightarrow$  introduce arbitrary-level hanging nodes.

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#### Arbitrary-level hanging nodes



- Make element refinement a local operation.
- hp-adaptivity becomes way easier.
- P.S., J. Cerveny, I. Dolezel: Arbitrary-Level Hanging Nodes and Automatic Adaptivity in the hp-FEM, Math. Comput. Simul. 77 (2008), 117 - 132.

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#### Waveguide problem



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### Waveguide problem (step 1)



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### Waveguide problem (step 2)



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### Waveguide problem (step 3)



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### Waveguide problem (step 4)



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### Waveguide problem (step 5)



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### Waveguide problem (step 6)



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### Waveguide problem (step 7)



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### Waveguide problem (step 8)



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### Waveguide problem (step 9)



#### Waveguide problem (step 10)



#### Waveguide problem (step 11)



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### Waveguide problem (step 12)



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### Waveguide problem (step 13)



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#### Waveguide problem (step 14)



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#### Waveguide problem (step 15)



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#### Waveguide problem (step 16)



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#### Mesh regularity vs. performance



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