Selected Topics in Adaptive Higher-Order FEM

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38th Woudschoten Conference of the Dutch & Flemish Numerical Analysis Communities, Woudschoten Conference Centre, Zeist, The Netherlands

Outline

- Team, collaborators, projects
- Higher-order finite elements
- Arbitrary-level hanging nodes
- *hp*-adaptivity and error estimation
- Adaptive multimesh *hp*-FEM
- Adaptive multimesh *hp*-FEM with dynamical meshes
- Time integration for adaptive *hp*-FEM
- Adaptive *hp*-FEM for eigenproblems
- NCLab - Cloud platform for LaTeX, GNU Octave, Python, FEM, and more

Acknowledgment

Team members:

- M. Balek, V. Cerny, B. Chaber, B. Filon, M. Hanus, P. Karban, L. Korous, V. Kotlan, P. Kus, F. Mach, M. Novak, D. Panek

Collaborators (alphabetical):

- V. de Almeida, G. Hansen (nuclear engineering)
- G. Bebis (computer vision)
- K. Bekris (robotics)
- M. Braun, P. Winkler (quantum chemistry)
- I. Dolezel (electrical engineering)
- C. Evrensel (mechanical engineering)
- D. Koracin, I. Zaliapin (atmospherical sciences)
- J. Kruis (civil engineering)
- M. Kuraz (hydrology)
- D. Kuzmin, M. Moeller (computational fluid dynamics)

+ around 50 contributors to the open source projects HERMES and AGROS2D

Main Funding Source

U.S. DOE Nuclear Engineering University Partnership (NEUP) program

http://hpfem.org/hermes

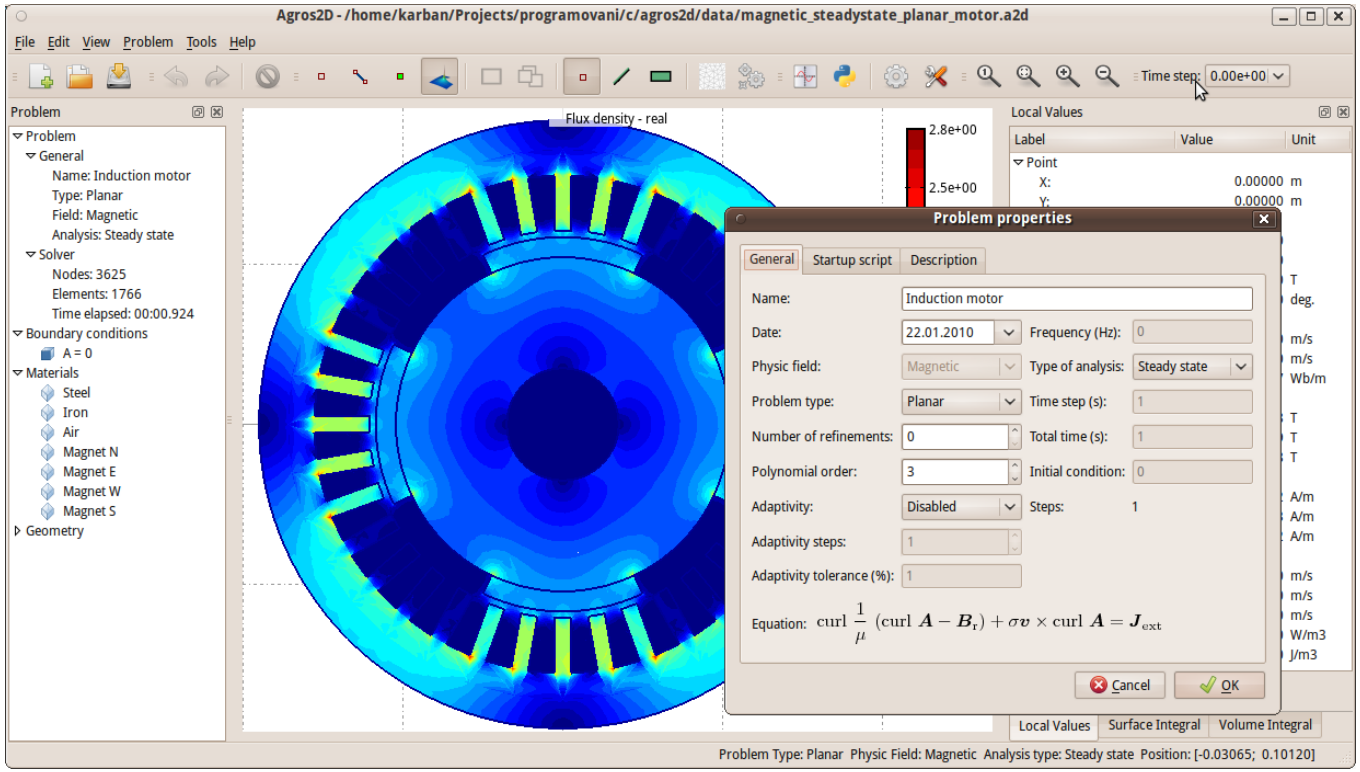
Hermes = HighER-order Modular finite Element System

Highlights:

- 8 types of adaptive *hp*-FEM (incl. transient problems).
- Spaces H^1, H(curl), H(div), L^2 + arbitrary combinations.
- Arbitrary-level hanging nodes in all spaces.
- 30 predefined time integration methods.
- Advanced methods for nonlinear problems.
- Monolithic multimesh *hp*-FEM for multiphysics coupled problems.
- Interface to major solver packages (MUMPS, PETSc, Trilinos, ...).

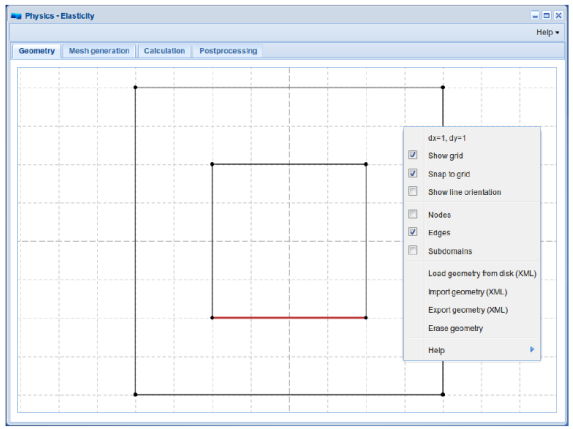
User documentation: http://hpfem.org/hermes/doc
Tutorial, examples, benchmarks: around 100 in total.

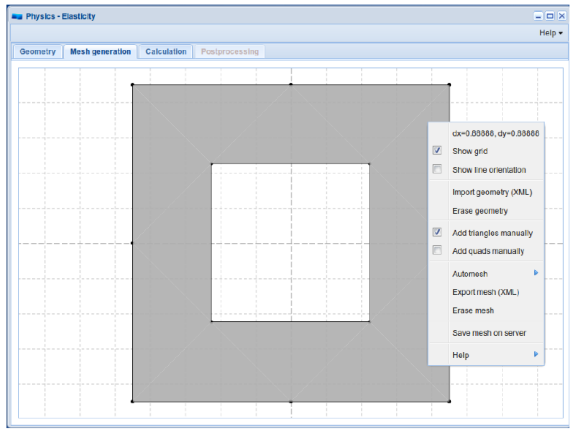
http://agros2d.org

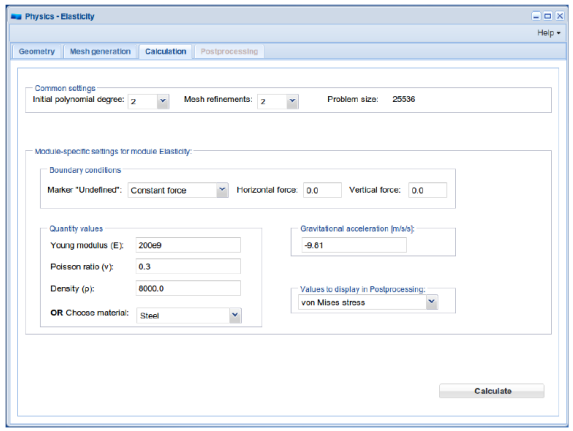


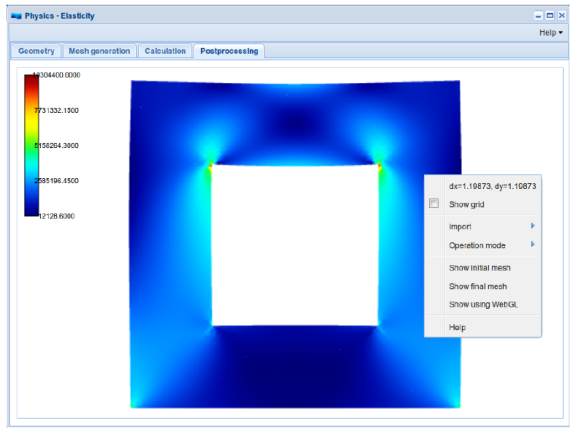
- Multiplatform application for the solution of engineering problems
- Based on Nokia QT, developed at the University of West Bohemia

http://nclab.com



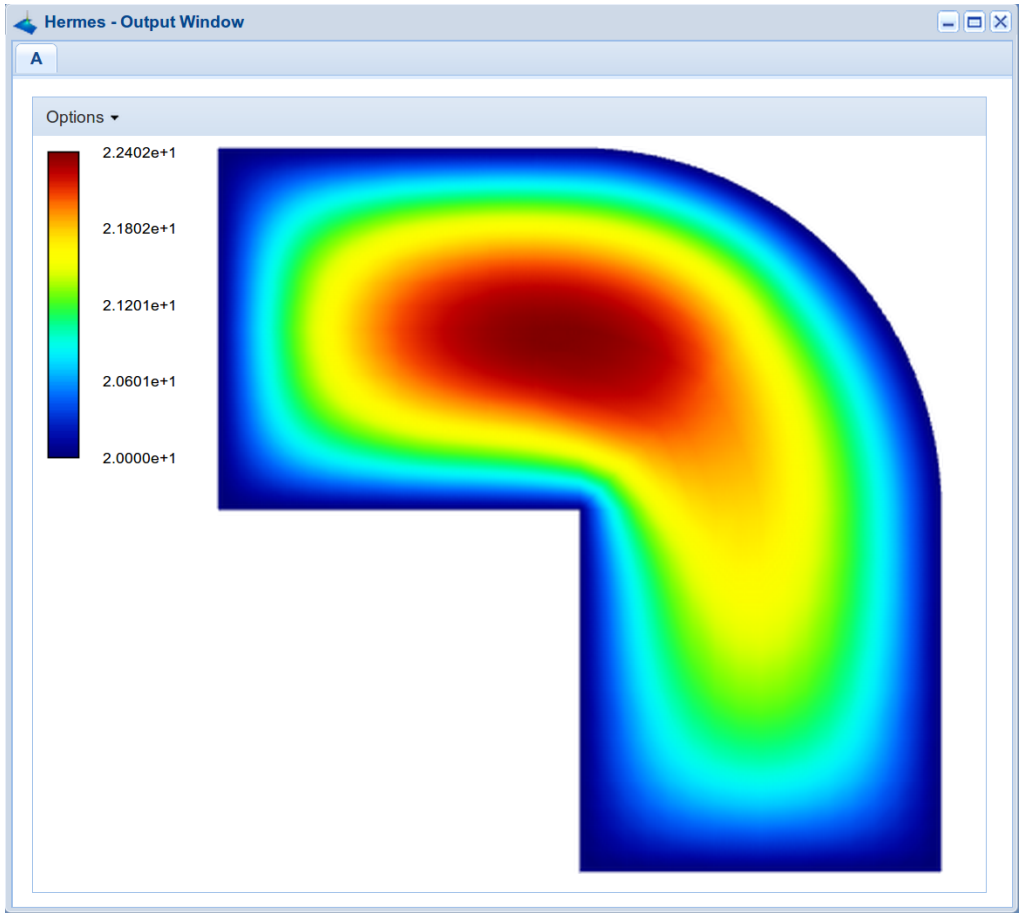




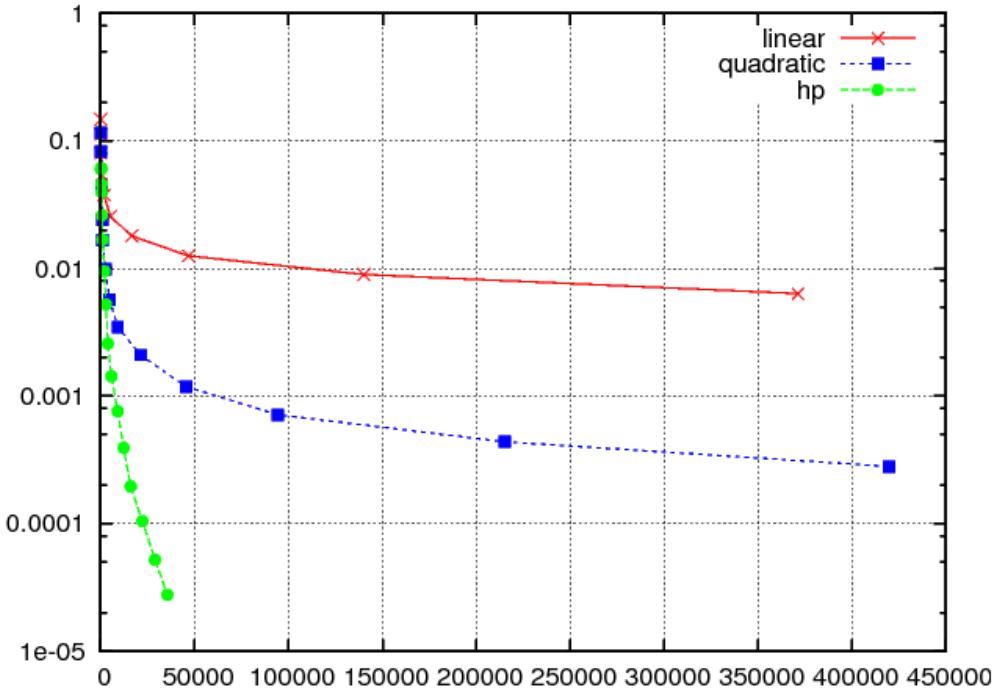


- Module = Geometry ed. + Mesh ed. + Problem ed. + FEM code + Postprocessor

Treating All Problems as Nonlinear



hp-FEM

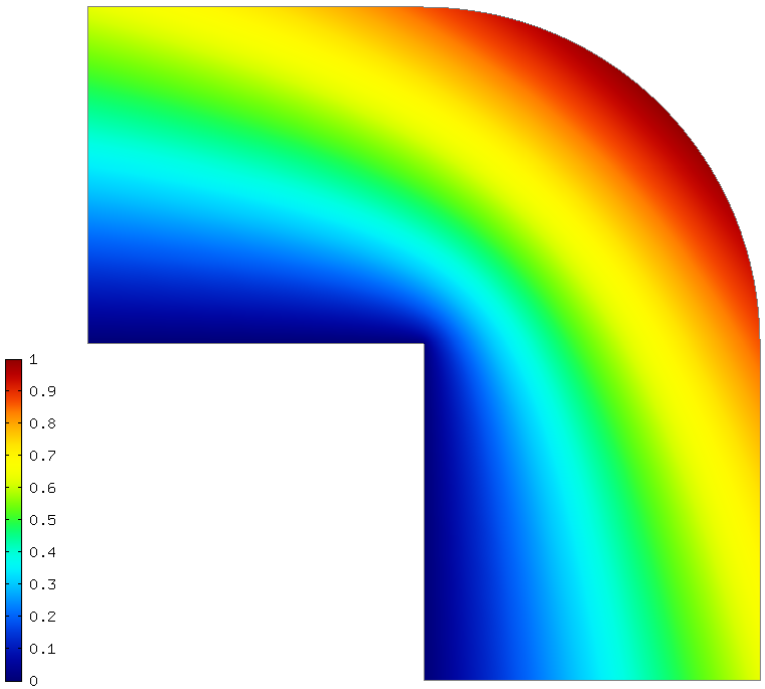

linear
quadratic
hp
1
0.1
0.01
0.001
0.0001
1e-05
0
50000
100000
150000
200000
250000
300000
350000
400000
450000

	Affine elements	*hp* elements
DOF	259393	6331
Error	1.617%	1.521%
Iterations	228	60
CPU time	**34 min**	**11.58 sec**

	Whitney edge elements	*hp* edge elements
DOF	2586540	4324
Error	0.6445%	0.6211%
CPU time	**21.2 min**	**2.49 sec**

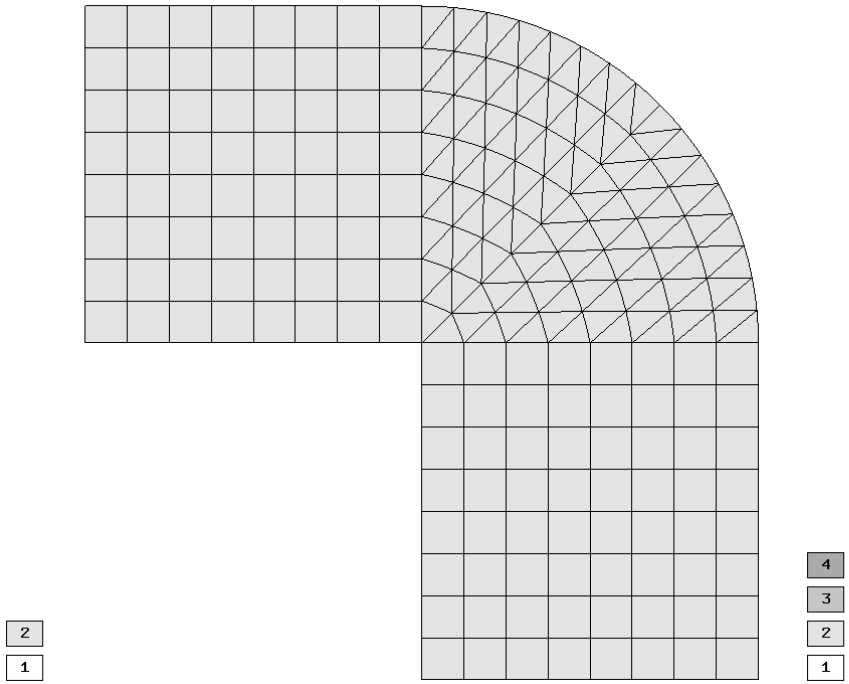
Reproducible – examples available in the Hermes public Git repository

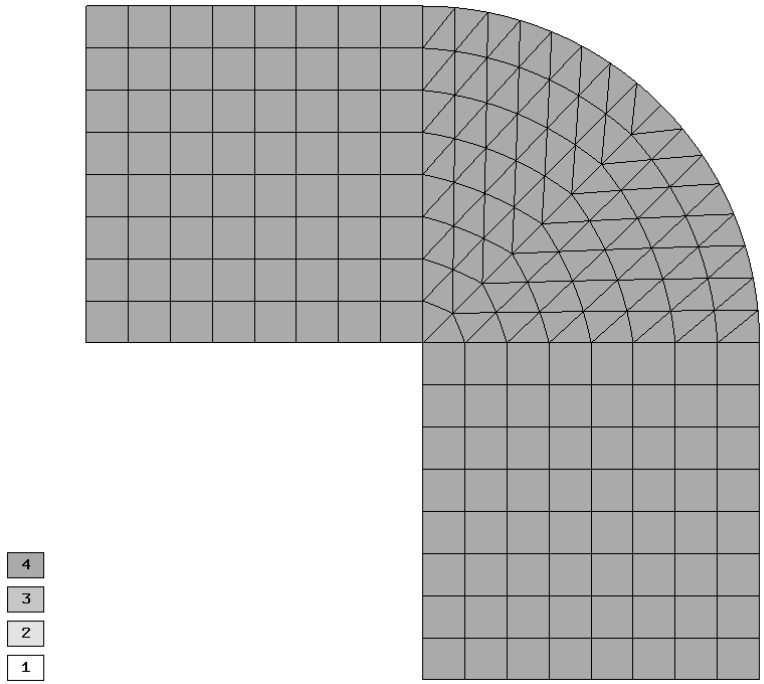
Let's do *hp*-FEM!



Sample problem with known exact solution.

Lesson one

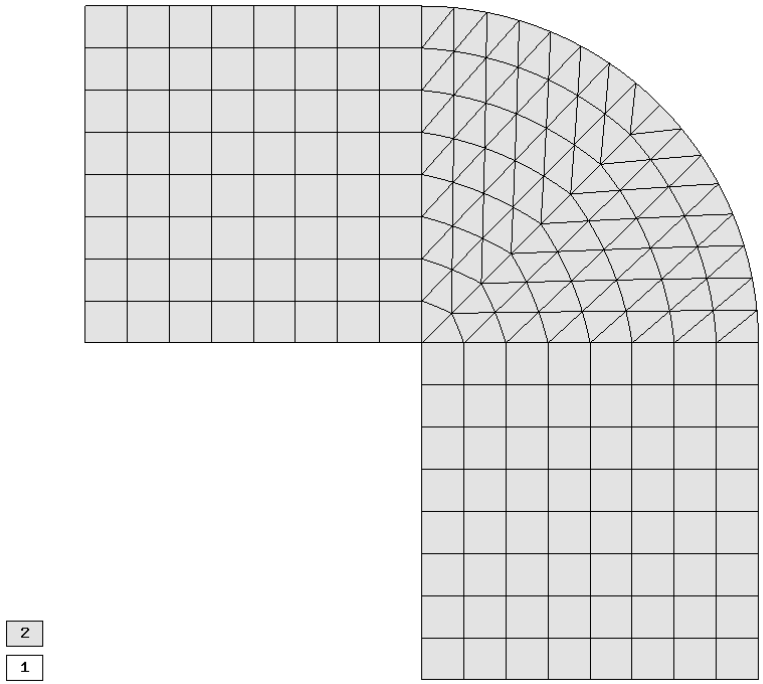


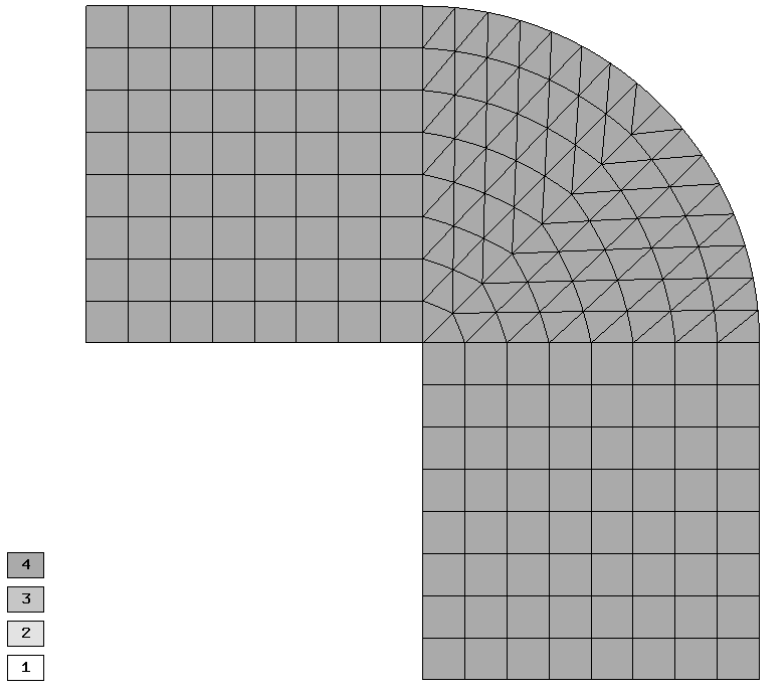


err = 2.7 %

err = 1.2 %

Lesson one





$err = 2.7\ \%$

$ndof = 705$

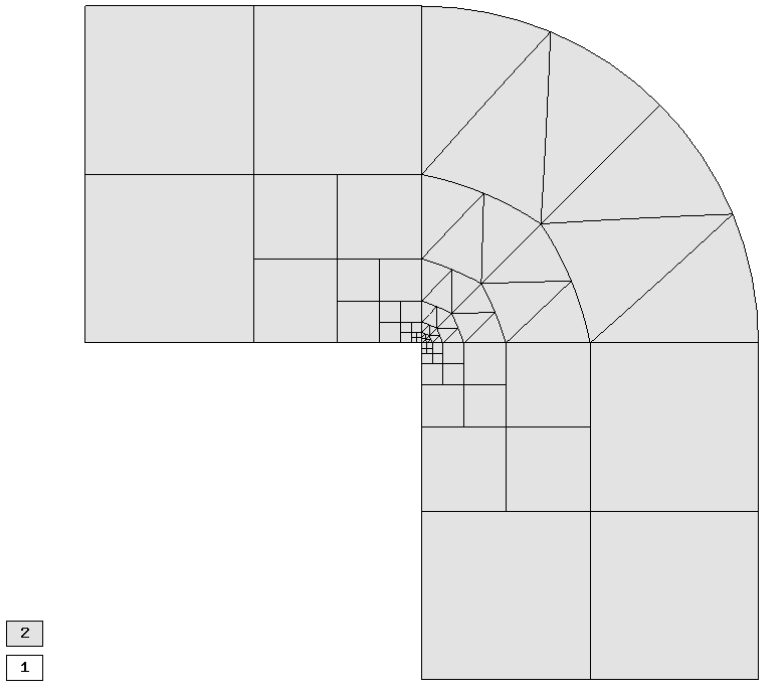
$err \cdot ndof = 1903$

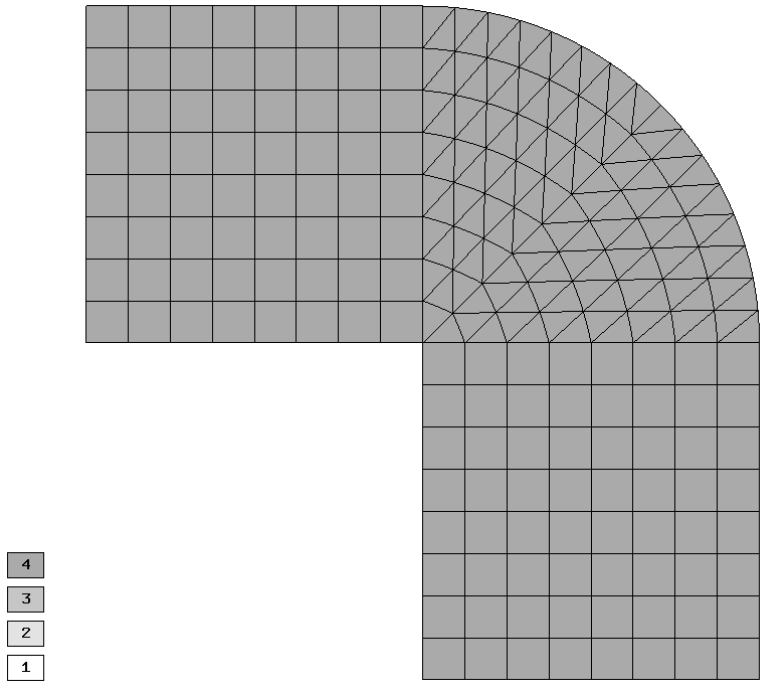
$err = 1.2\ \%$

$ndof = 2900$

$err \cdot ndof = 3480$

Lesson two





err = 0.92 %

$ndof$ = 173

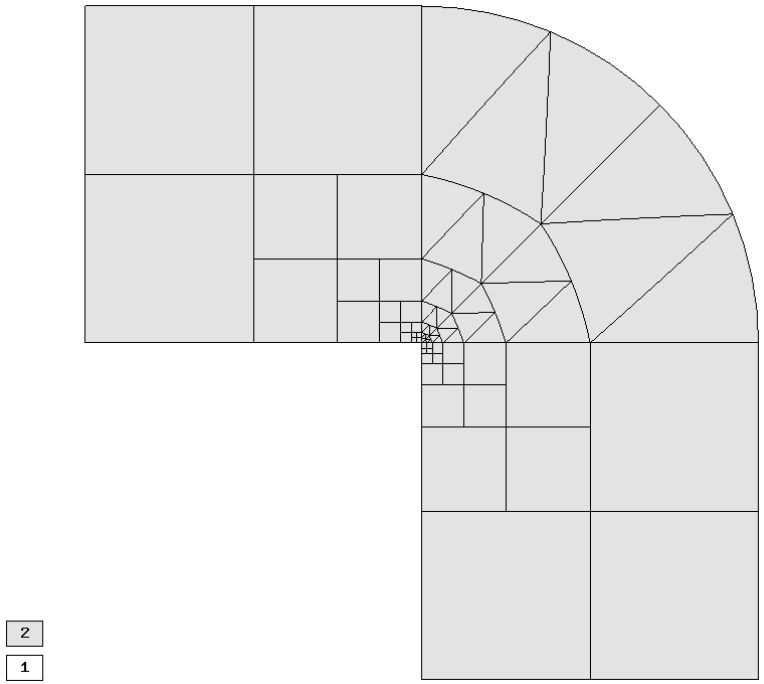
$err \cdot ndof$ = 159

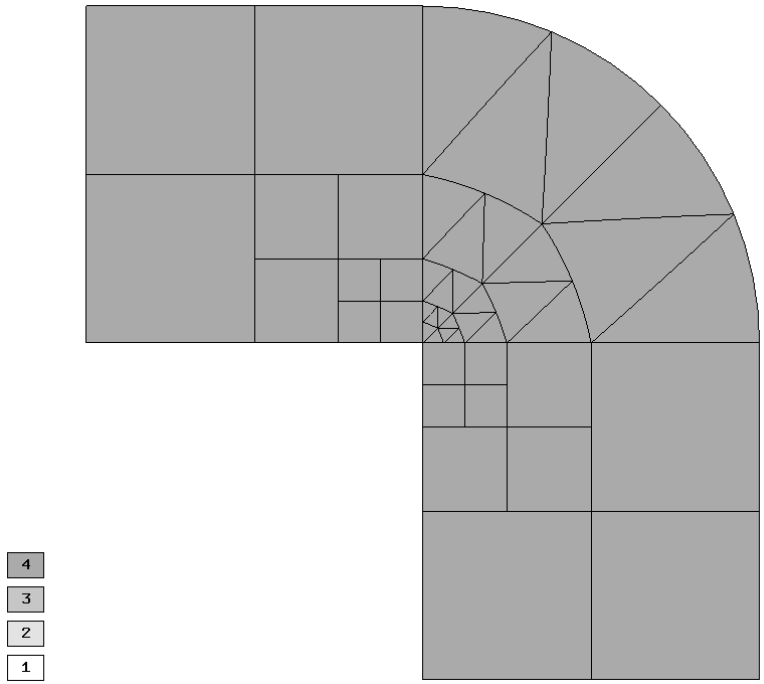
err = 1.2 %

$ndof$ = 2900

$err \cdot ndof$ = 3480

Lesson three





err = 0.92 %

$ndof$ = 173

$err \cdot ndof$ = 159

err = 1.02 %

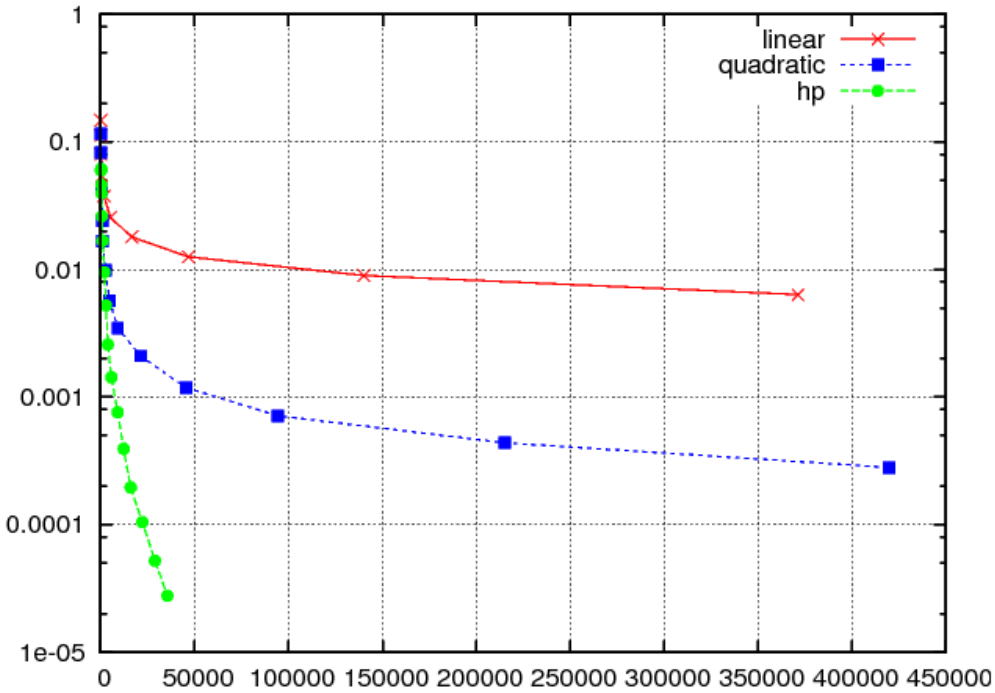
$ndof$ = 457

$err \cdot ndof$ = 466

Conclusion

The most powerful tools do not always give the best results.

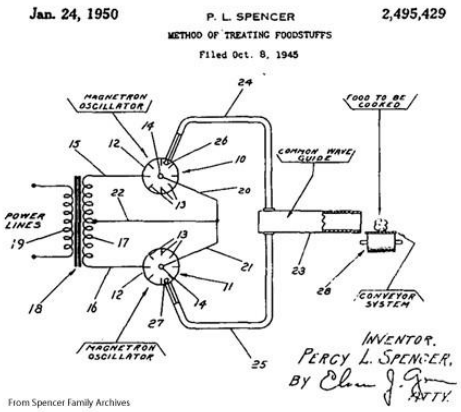
Remark #1



There is no higher-order method without adaptivity.

Remark #2 (for *hp*-FEM skeptics)

Things take time...



First microwave oven: 5.5 feet tall, 750 pounds, $3,000. Water cooled.
Average annual income in 1946: $2,600, average price of a new house: $5,150.