A Sharp Interface Immersed Boundary Method for Flow and Aeroacoustics with Complex Moving Boundaries-I: Methodology and Implementation

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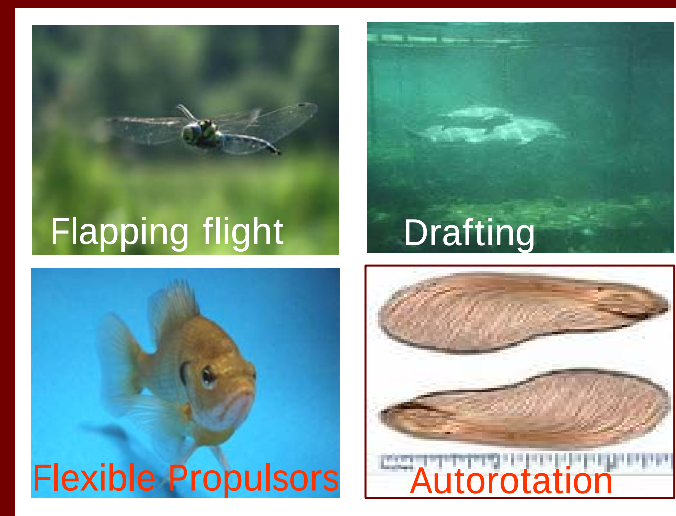
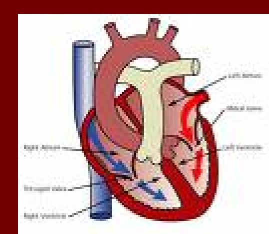
Biological Flows

- Biomimetics and Bioinspired Engineering
  - – What can we learn from Nature ?
  - – How can we adapt Nature's solutions into engineered devices/machines ?

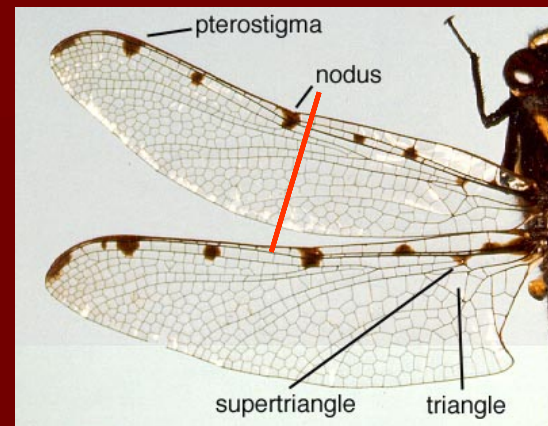


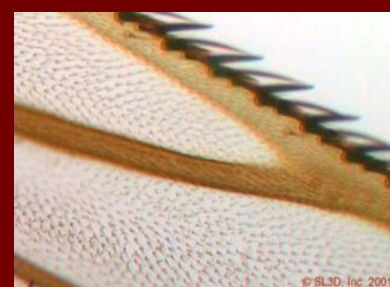
- Biomedical Engineering
  - – Cardiovascular flows
  - – Respiratory flows
  - – Phonatory/Speech Mechanisms
  - – Biomedical Devices

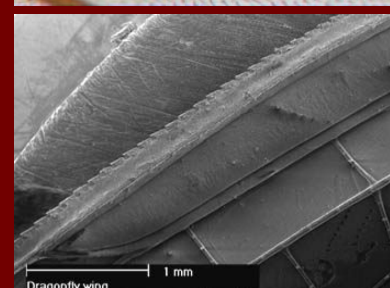


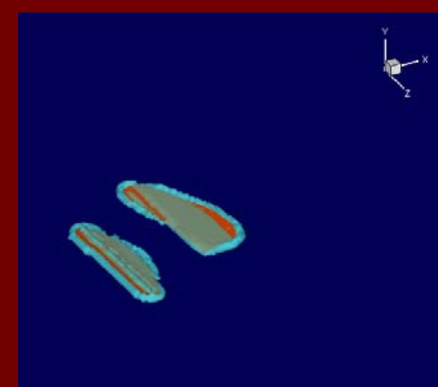
Inspiration from Dragonflies

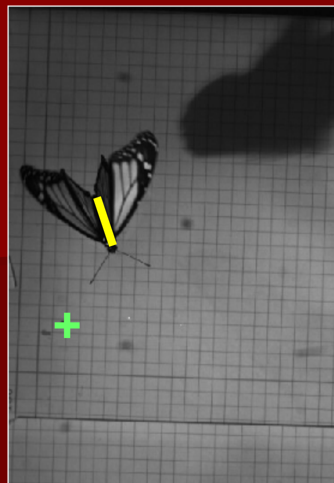
- Dragonflies
  - Existed for 350 million years
  - Wingspan from 2 – 80 cm
  - Fast and agile
- Wing Design
  - Thin, lightweight
  - Vein reinforced
  - Pleated along chord
  - Pterostigma
  - Microstructure
- Wing Configuration
  - Wing-wing interaction?
- Wing Flexion?

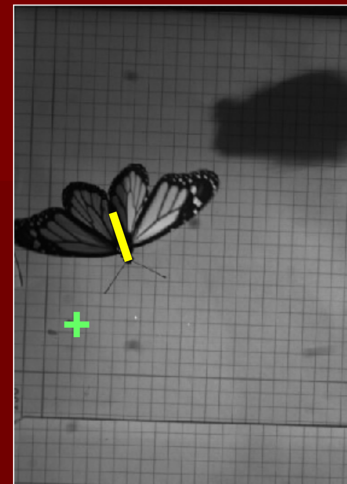


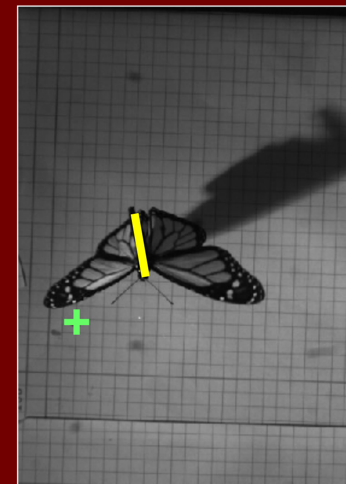


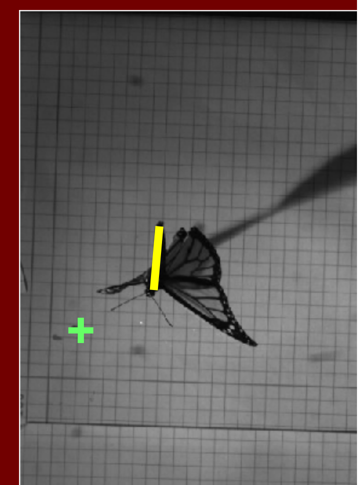




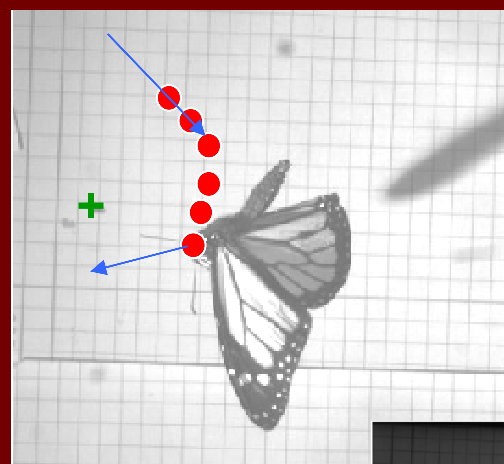


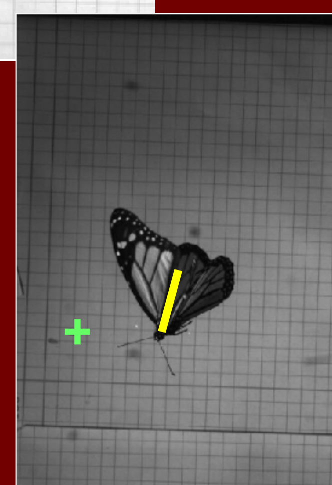






- **Turning in a Monarch Butterfly**
- Sequence shows 1.5 flaps
- >90^{o} change in heading !
- Turning distance < body size
- Turn on a dime!





Biophysics of Phonation

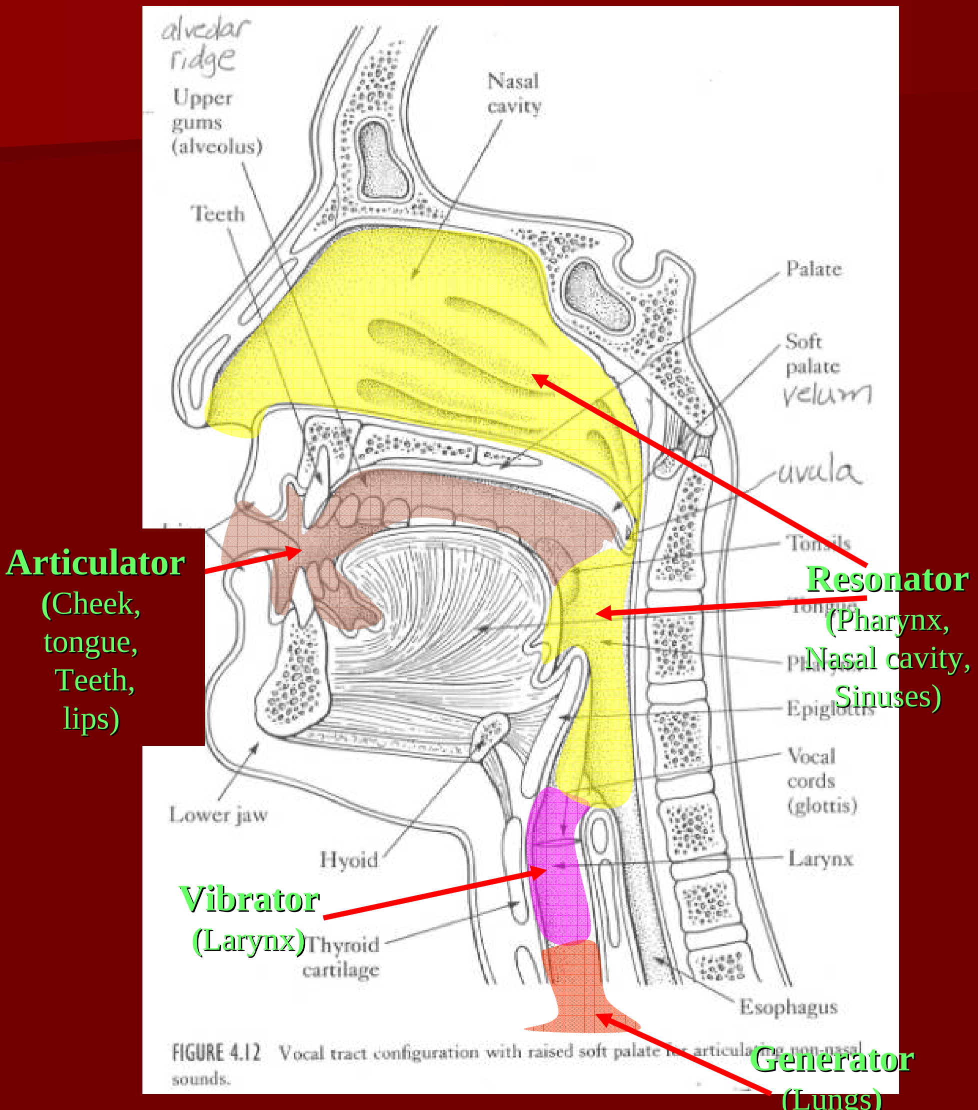
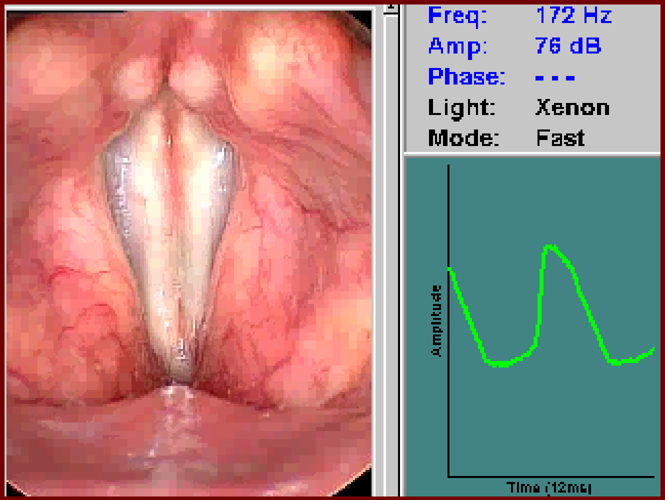
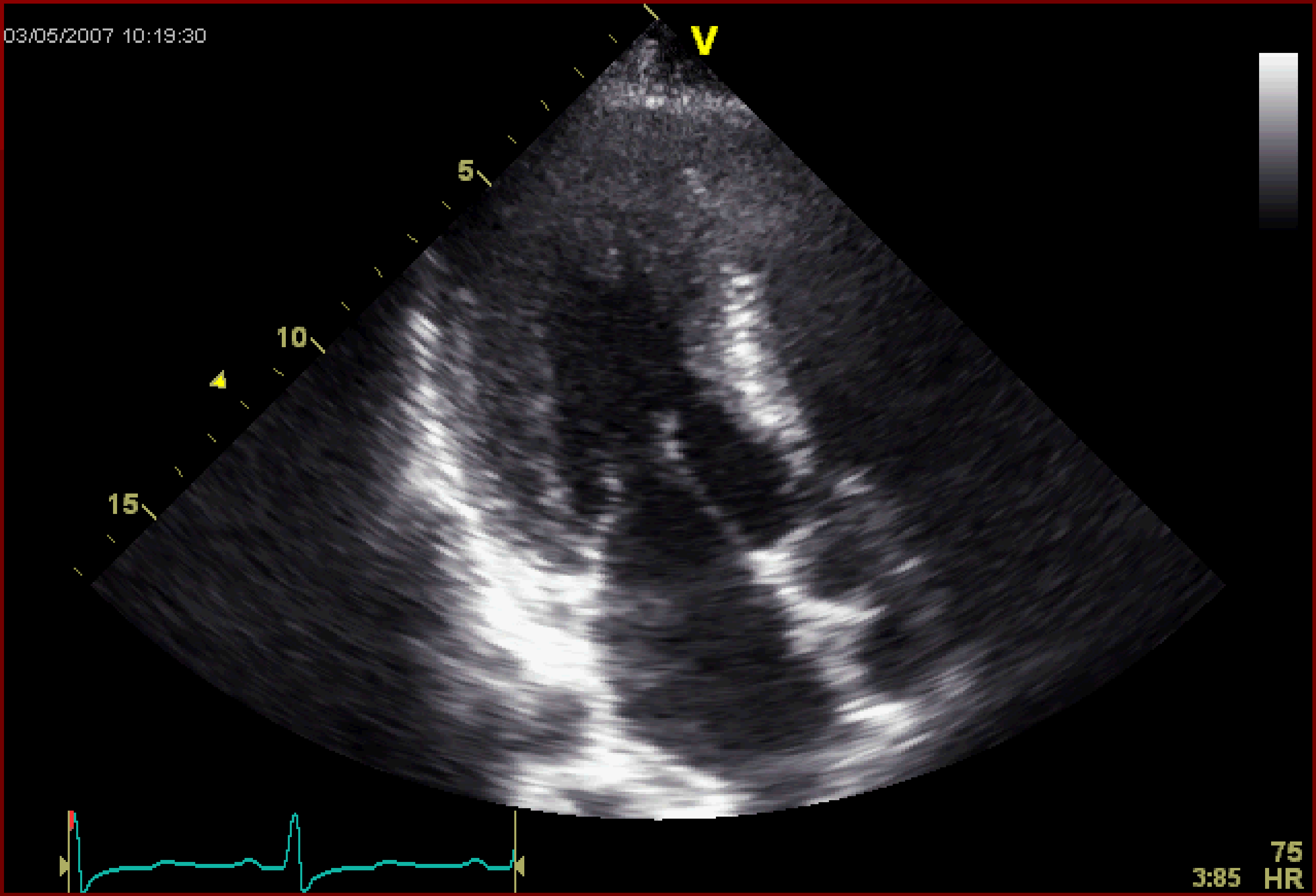


FIGURE 4.12 Vocal tract configuration with raised soft palate for articulating non-nasal sounds.



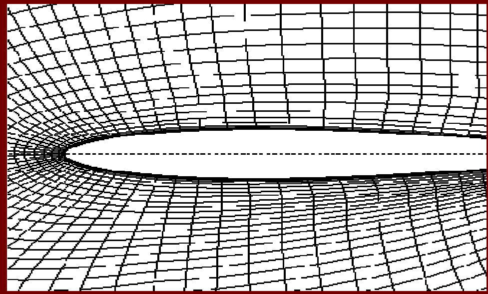
- Fluid-structure interaction between airflow and vocal folds is key to phonation.
- Re~3000
- M ~ 0.1

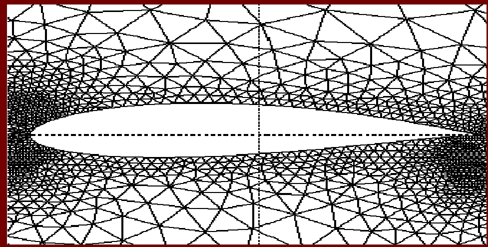
Cardiac Hemodynamics



Computational Modeling

- Need to tackle
  - – Complex 3D geometries
  - – Moving boundaries
  - – Fluid-Structure Interaction
  - – Vortex dynamics
  - – Relatively low Reynolds numbers



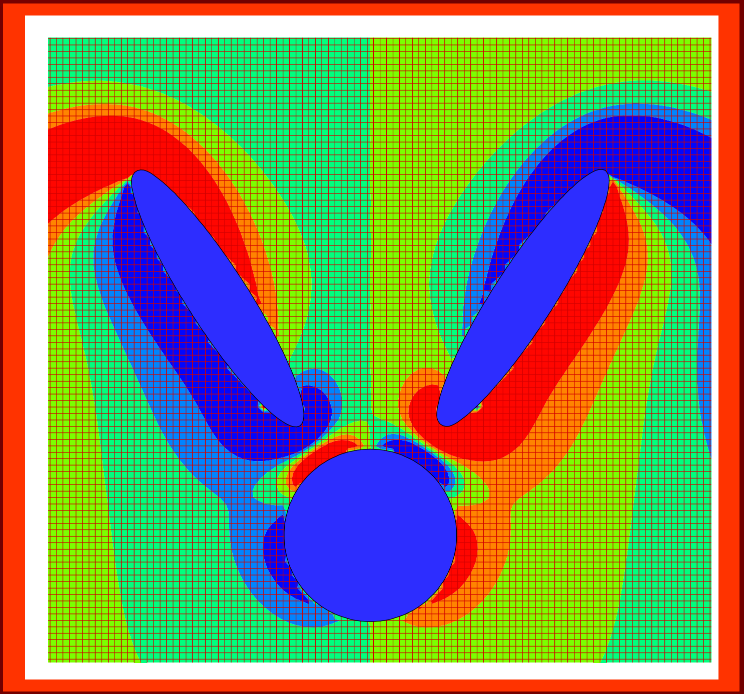


- Very challenging for conventional body fitted methods.

- Immersed Boundary Methods
  - – handle these problems in all their complexity.

Body Non-Conformal (Immersed Boundary) Methods

- Simulations performed on Cartesian grids that do not conform to the shape of the boundaries.



- Advantages
  - mesh generation is simple even with very complicated geometries
  - boundary motion does not affect the mesh
  - simple grid connectivity
- Challenges
  - need special techniques to apply BC
  - maintain accuracy and conservation
  - difficult to provide enhanced resolution in localized regions
  - therefore not appropriate for high Re applications unless some local-refinement techniques are used

Continuous Forcing Approach

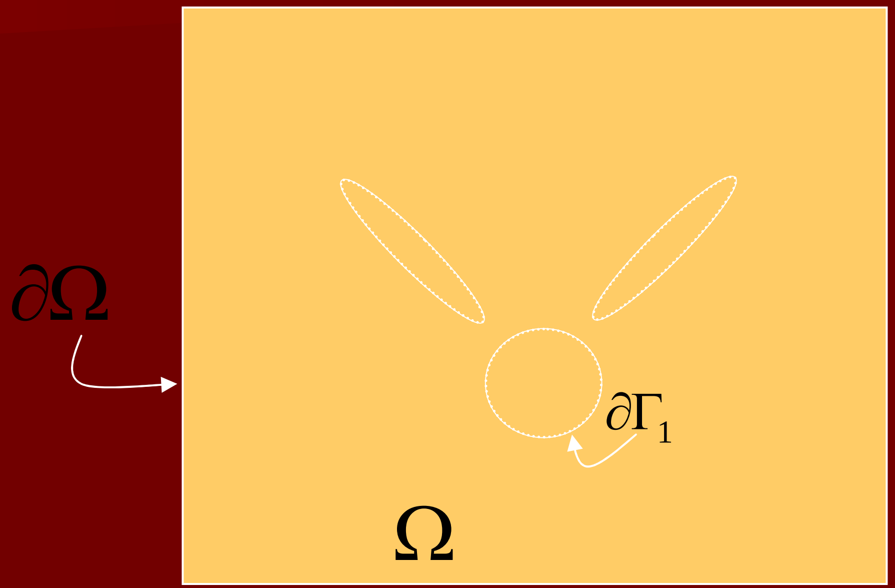
$$L(u) = f \quad \text{in} \quad \Omega - \sum \Gamma_k$$
$$u = g \quad \text{on} \quad \partial\Omega$$
$$u = h_k \quad \text{on} \quad \partial\Gamma_k$$

$\Big\Downarrow$ BC Removal

$$L(u) = f + \delta_k H_k \quad \text{in} \quad \Omega$$
$$u = g \quad \text{on} \quad \partial\Omega$$

$\Big\Downarrow$ Discretization on Cartesian Mesh

$$\tilde{L}(\tilde{u}) = \tilde{f} + \tilde{\delta}_k H_k \quad \text{in} \quad \Omega$$
$$\tilde{u} = \tilde{g} \quad \text{on} \quad \partial\Omega$$

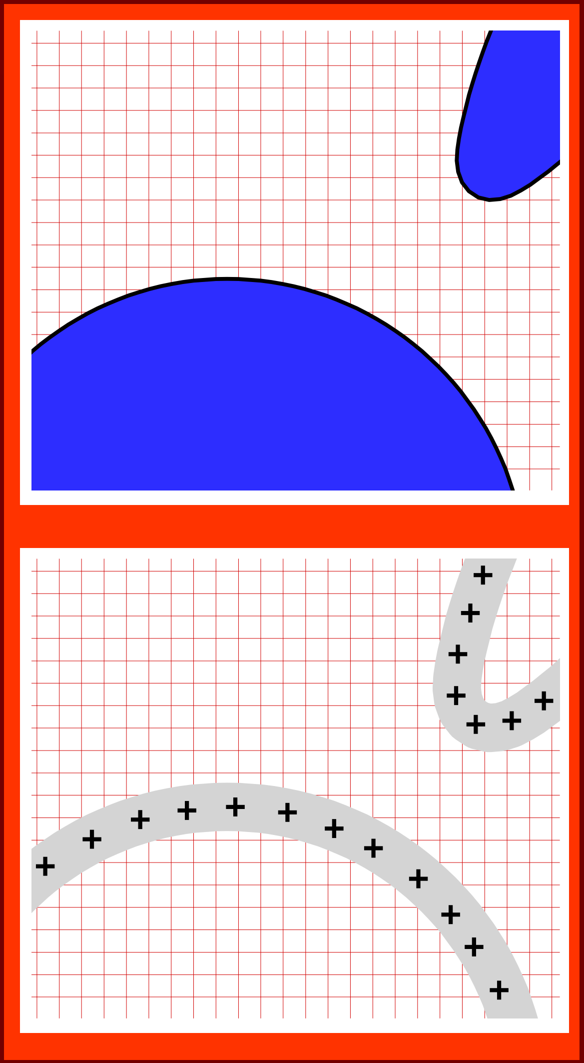


H_k determined such that

$u \approx h_k$ on $\partial\Gamma_k$

Pros and Cons

- Distributed force used to model the boundary
- Effect of boundary has to be spread over a number of cells
  - "Diffuse" boundary
- Lose accuracy precisely where it might be most important !
- Large and localized source term leads to a stiff system of equations.
- Advantage:- technique relatively independent of discretization scheme.
- Used extensively
  - Peskin (1972)
  - Goldstein et al. (1993)
  - Penalization methods



Discrete Forcing Approach

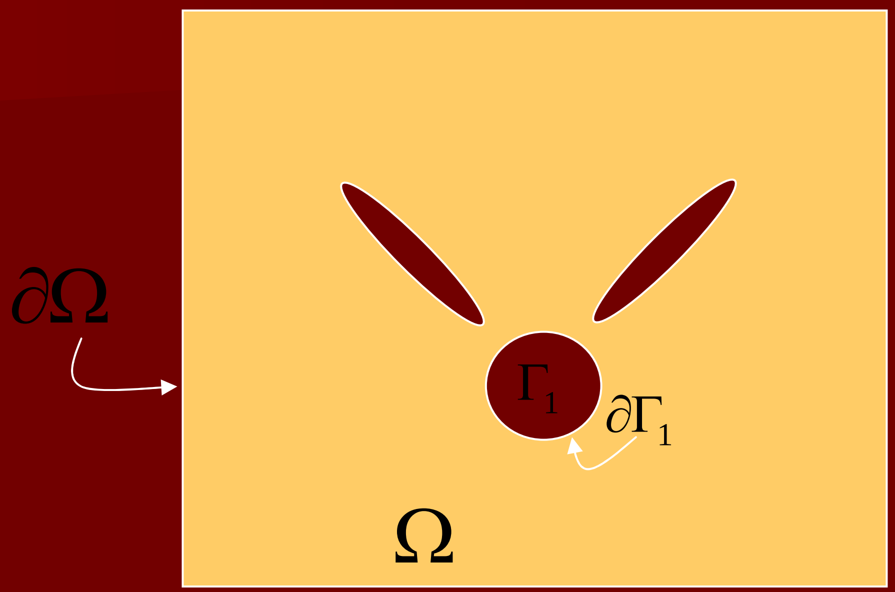
$$L(u) = f \quad \text{in} \quad \Omega - \sum \Gamma_k$$
$$u = g \quad \text{on} \quad \partial\Omega$$
$$u = h_k \quad \text{on} \quad \partial\Gamma_k$$

Discretization on Cartesian Mesh

$$\tilde{L}(\tilde{u}) = \tilde{f} \quad \text{in} \quad \Omega$$
$$\tilde{u} = \tilde{g} \quad \text{on} \quad \partial\Omega$$

BC Removal

$$\tilde{L}(\tilde{u}) = \tilde{f} + J_k \quad \text{in} \quad \Omega$$
$$\tilde{u} = \tilde{g} \quad \text{on} \quad \partial\Omega$$

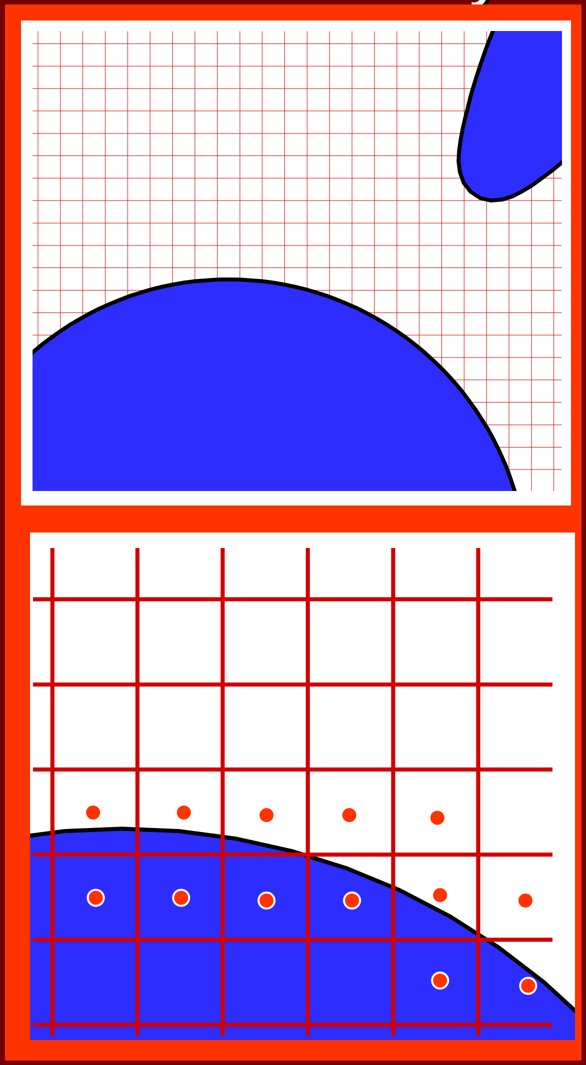


J_k determined such that

$$\tilde{u} \approx \tilde{h}_k \quad \text{on} \quad \partial\Gamma_k$$

Pros and Cons

- Modification of discretization used to model the boundary
  - Can be viewed as a forcing in the discrete system



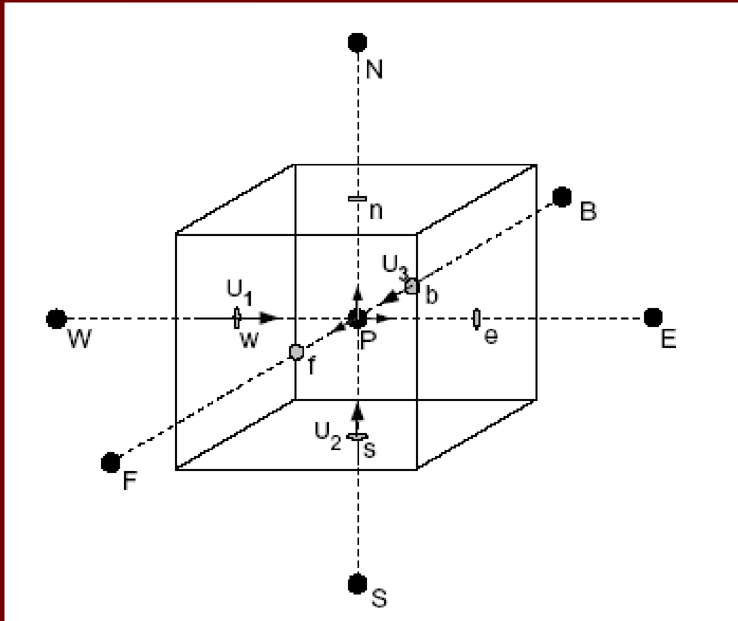
- "Sharp" boundary representation
- Maintain accuracy near boundary.
- Do not solve for a spurious flow inside the solid.
- No stiffness introduced in the discrete equations.
- Technique is obviously dependent on discretization scheme.
- Discontinuity in space leads to "fresh" & "dead" cells as boundary moves.

Formulation: Governing Equations

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right)$$

Incompressible Navier-Stokes



- **Fractional-Step Method**
- **Second-order central scheme in space**

$$\frac{u_i^* - u_i^n}{\Delta t} + \frac{1}{2}\left[3N_i^n - N_i^{n-1}\right] = -\frac{1}{\rho}\frac{\delta p^n}{\delta x_i} + \frac{1}{2}\left(D_i^* + D_i^n\right)$$

$$\frac{u_i^{n+1} - u_i^*}{\Delta t} = -\frac{1}{\rho}\frac{\delta p'}{\delta x_i}$$

$$\frac{1}{\rho}\frac{\delta}{\delta x_i}\left(\frac{\delta p'}{\delta x_i}\right) = \frac{1}{\Delta t}\frac{\delta U_i^*}{\delta x_i}$$

$$p^{n+1} = p^n + p'$$

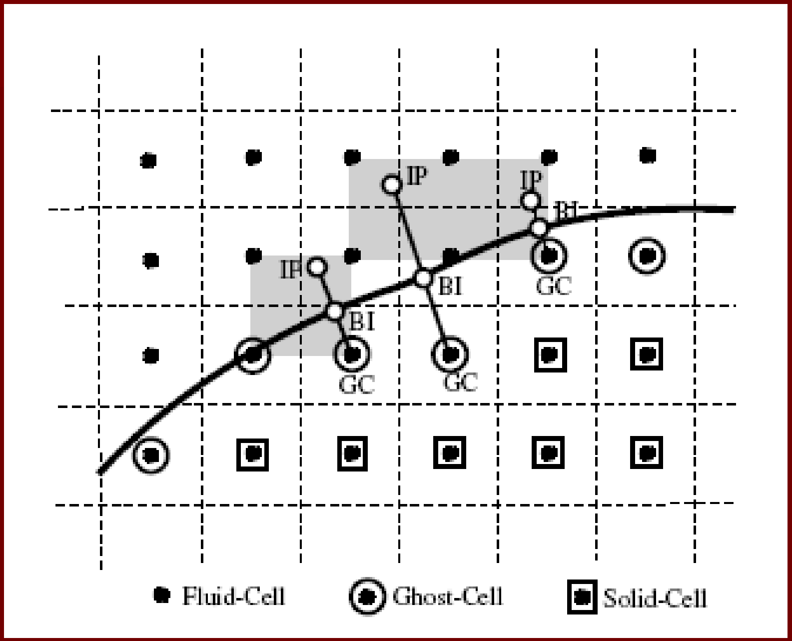
$$u_i^{n+1} = u_i^* - \Delta t \frac{1}{\rho}\left(\frac{\delta p'}{\delta x_i}\right)_{cc}$$

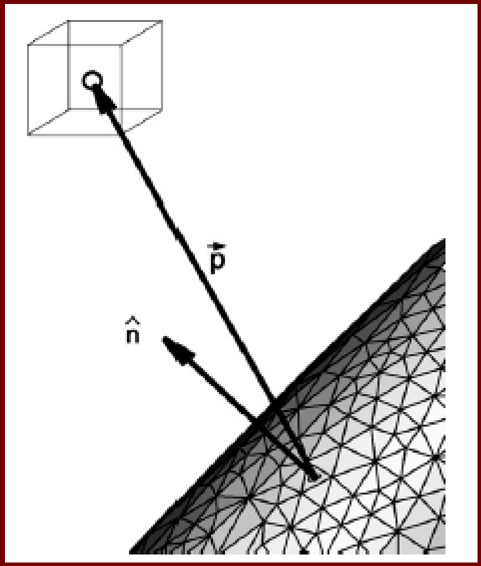
$$U_i^{n+1} = U_i^* - \Delta t \frac{1}{\rho}\left(\frac{\delta p'}{\delta x_i}\right)_{fc}$$

ViCar3D

Viscous Cartesian Grid Solver for 3D Immersed Boundaries

- Multi-dimensional ghost-cell methodology
- Immersed surfaces represented
  by triangular element mesh





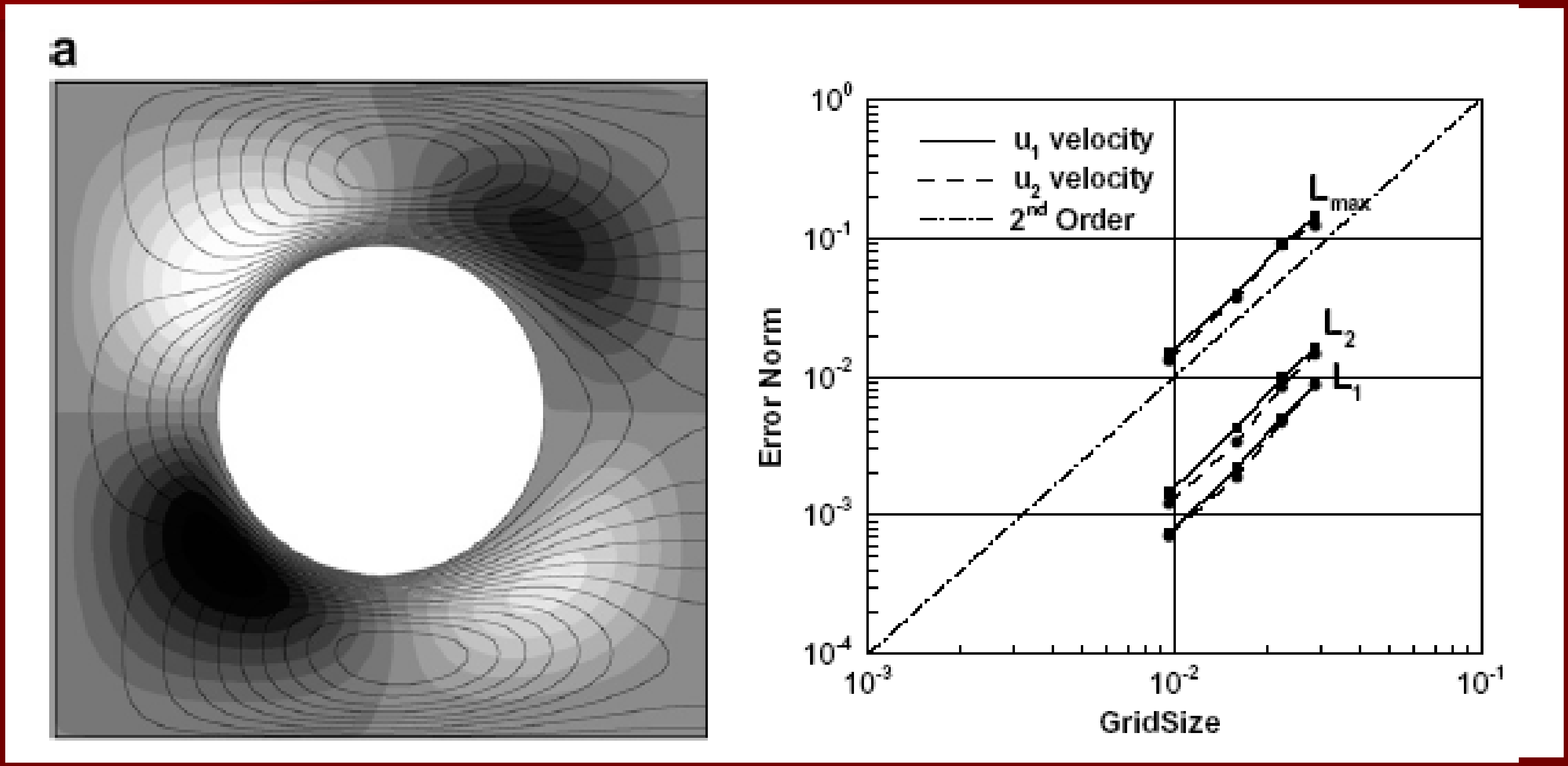
$$\phi(x_1,x_2,x_3) = C_1x_1x_2x_3 + C_2x_1x_2 + C_3x_2x_3 + C_4x_1x_3 + C_5x_1 + C_6x_2 + C_7x_3 + C_8$$

$$\phi_{\mathrm{IP}} = \sum_{i=1}^{8} \beta_i \phi_i + \mathrm{T.E.}$$

$$\phi_{\mathrm{BI}} = \frac{1}{2}(\phi_{\mathrm{IP}} + \phi_{\mathrm{GC}}) + \mathrm{O}(\Delta l^2) = \frac{1}{2}\left(\sum_{i=1}^{8} \beta_i \phi_i + \phi_{\mathrm{GC}}\right) + \mathrm{O}(\Delta^2) + \mathrm{O}(\Delta l^2)$$

$$\left(\frac{\delta\phi}{\delta n}\right)_{\mathrm{BI}} = \frac{\phi_{\mathrm{IP}} - \phi_{\mathrm{GC}}}{\Delta l} + \mathrm{O}(\Delta l^2) = \frac{1}{\Delta l}\left(\sum_{i=1}^{8} \beta_i \phi_i - \phi_{\mathrm{GC}}\right) + \mathrm{O}(\Delta^2/\Delta l) + \mathrm{O}(\Delta l^2)$$

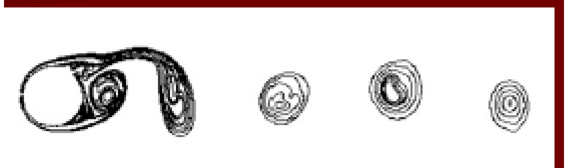
ViCar3D: Accuracy

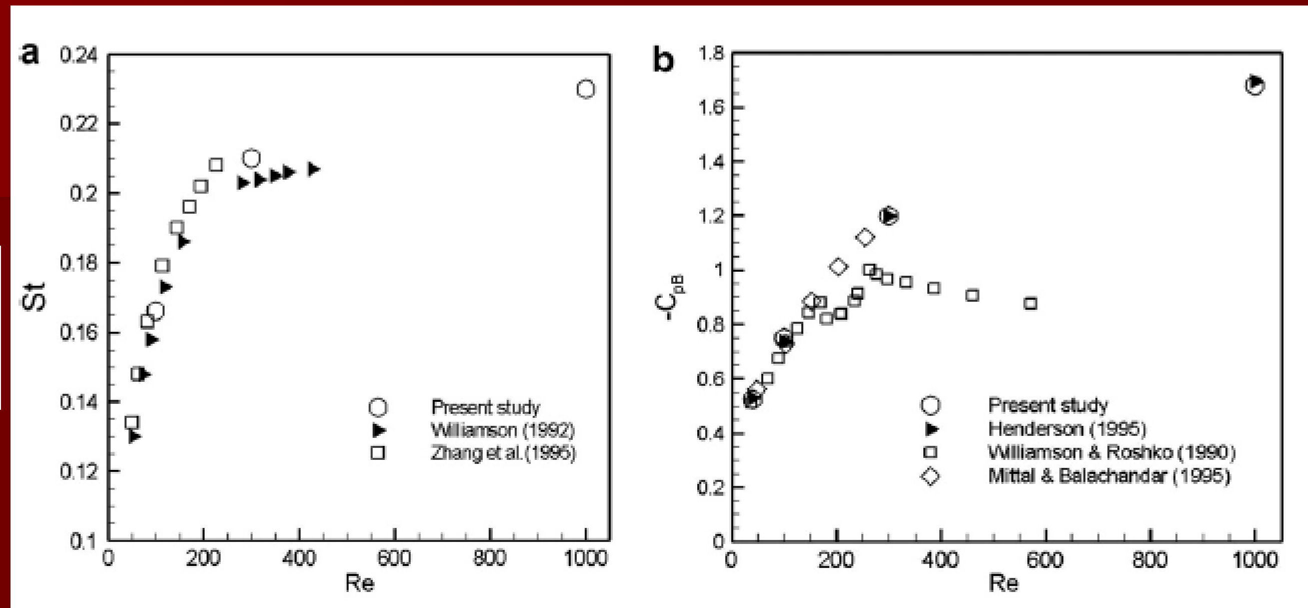


Flow past a Circular Cylinder

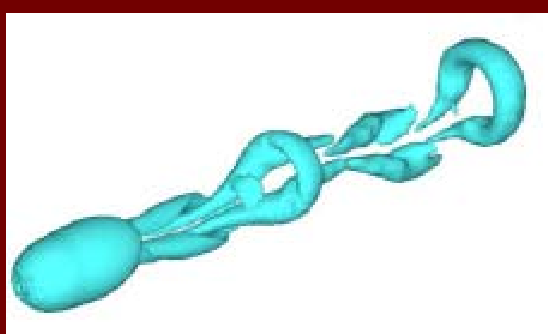
ViCar3D: Validation

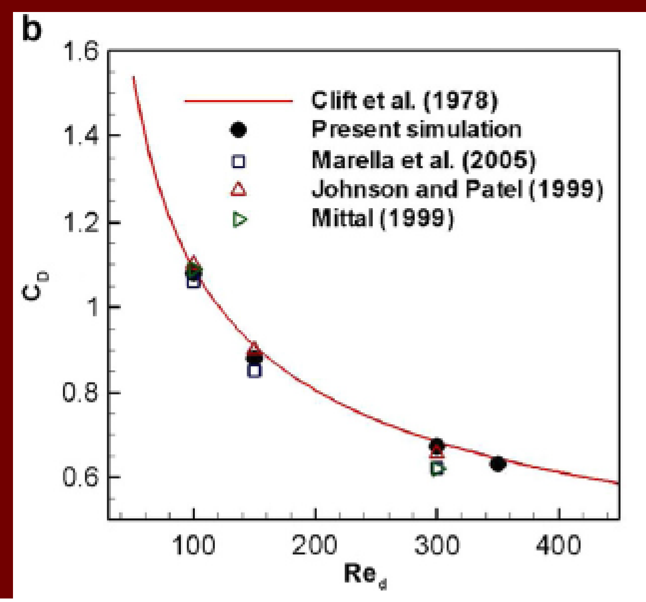
Flow past a circular cylinder





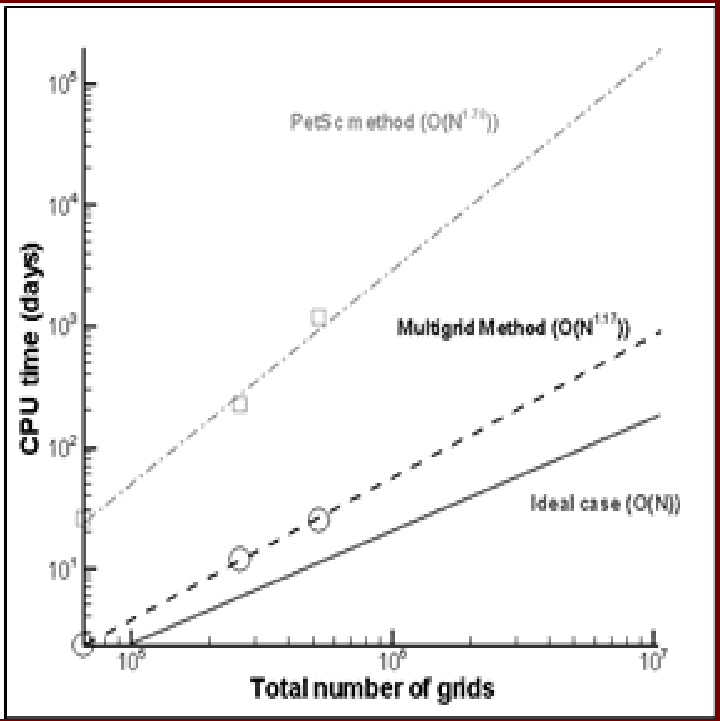
Flow past a sphere

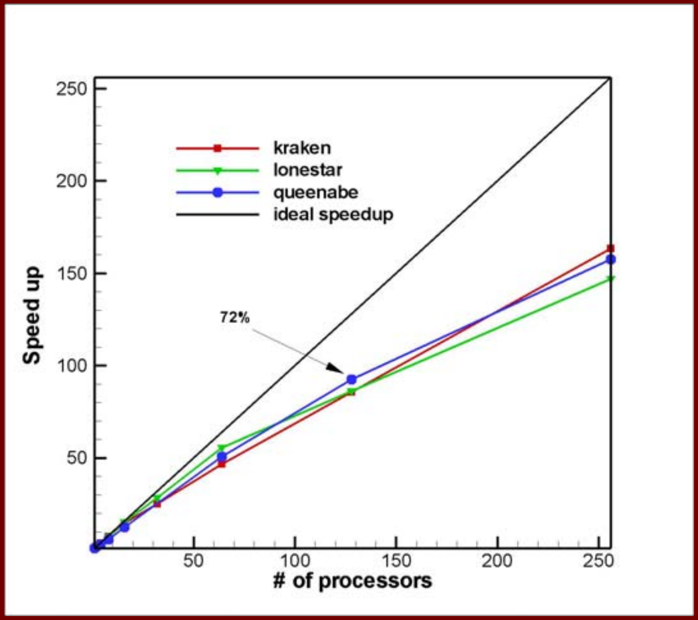




ViCar3D: Performance

- Pressure Poisson
  - > 80% of CPU time
  - Geometric multigrid method.
  - Semi-coarsening + LSOR
  - Approx. reconstruction of IB on coarse levels.





- MPI based parallelization
  - 2D domain decomposition

Modeling Fluid-Tissue Interaction

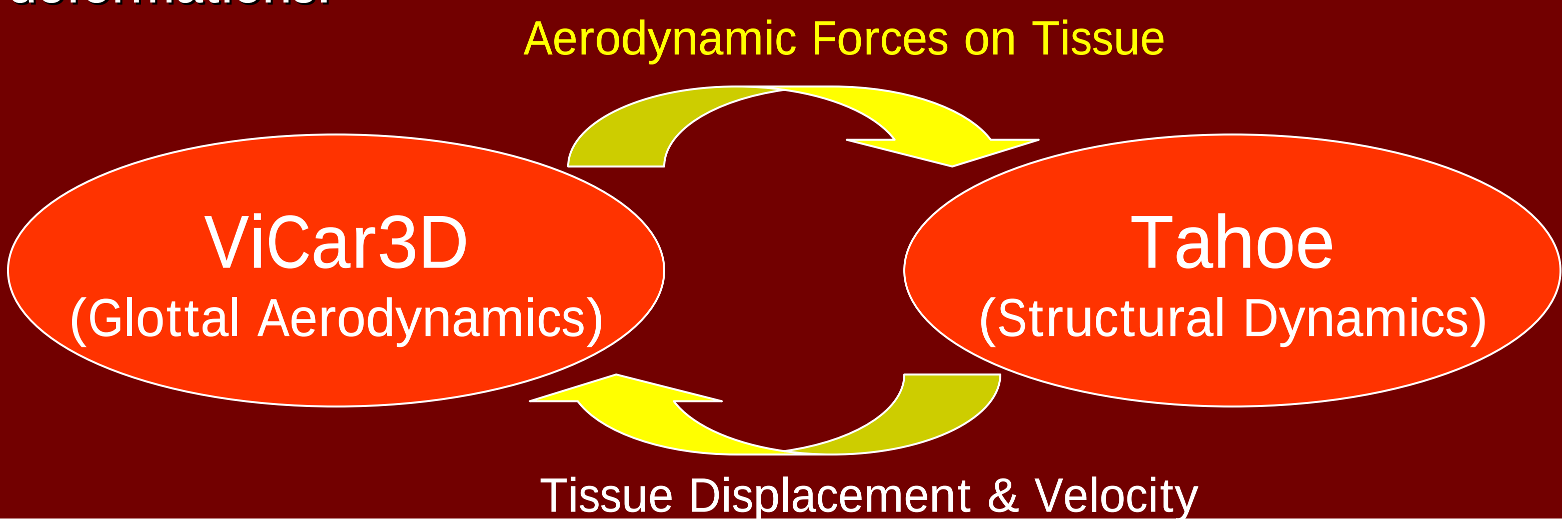
- ViCar3D coupled to another solver that computes deformation of elastic structures
- Tahoe
  - Open source C++ FEM based solid mechanics solver.
  - Research-oriented, parallel, modularized and highly flexible code.
  - developed at Sandia National Lab.
  - Variety of constitutive models and can handle large deformations.

Navier-Cauchy equations

$$\nabla \cdot \sigma + \mathbf{F} = \rho \ddot{\mathbf{u}}$$

$$\varepsilon = \frac{1}{2}\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathrm{T}}\right]$$

$$\sigma = \mathbf{C} : \varepsilon$$



Tahoe: Constitutive Models and Solvers

Small Strain (linear)

<u>Anisotropic</u>
- Hookean
- Cubic

<u>Isotropic</u>
- St-Venant
- St-Venant J2

<u>Viscoelastic</u>
- Linear viscoelastic (Standard linear material)
- Linear viscoelastic prony series

Large strain (non-linear)

- Hookean
- Cubic
- St-Venant
- Simo isotropic
- Quad log
- Quad log Ogden
- Simo J2
- Quad log J2
- Resee-Govindjee nonlinear viscoelastic model

Solvers

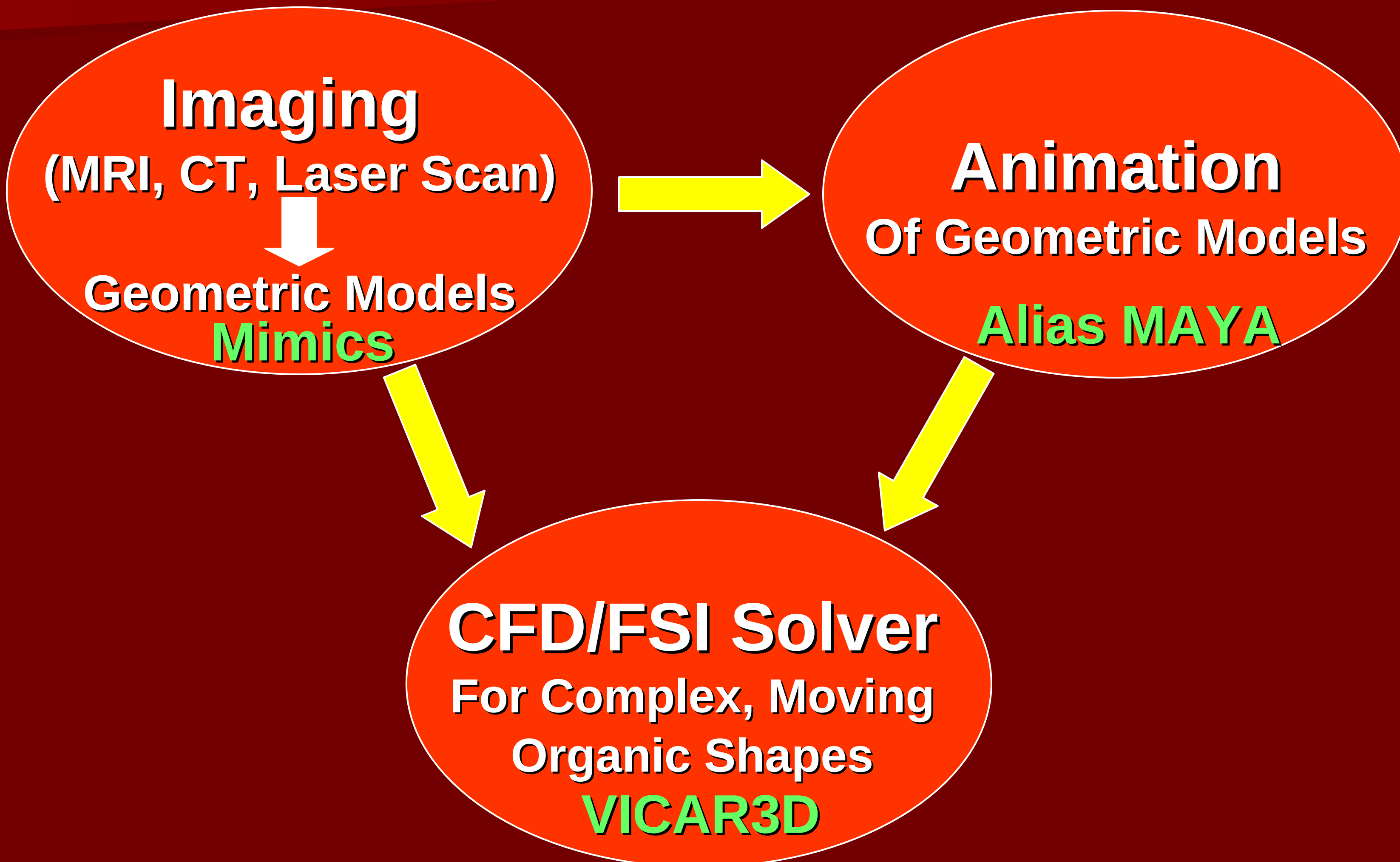
- Linear solver
- Non linear solver
- PCG solver
- Non linear solver

Matrix options

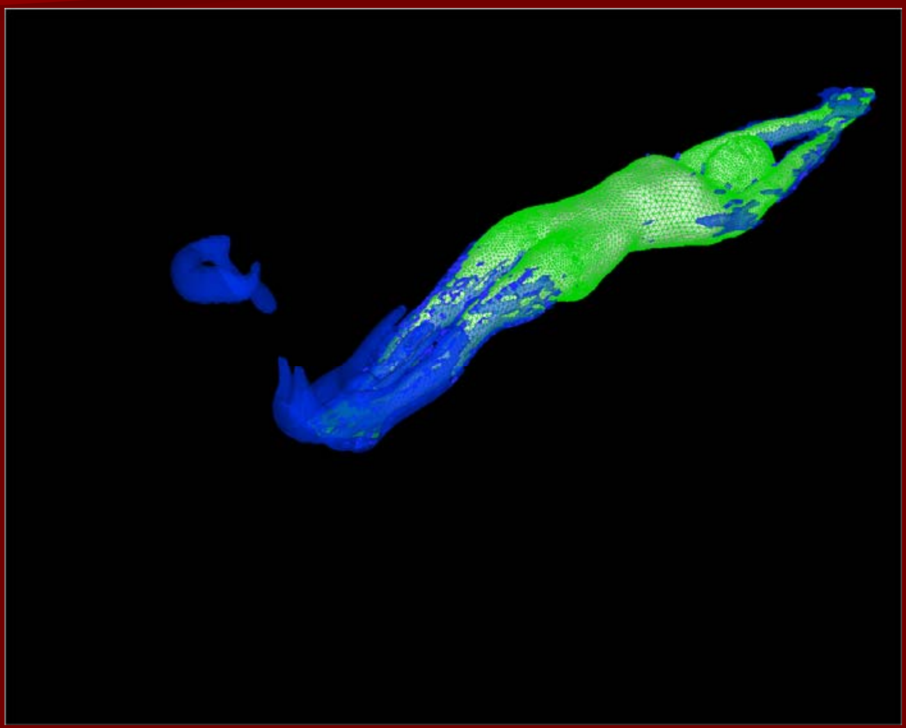
- Full
- Profile
- Diagonal
- SPOOLES — **FASTEST**

SPOOLES 2.2 : SParse Object Oriented Linear Equations Solver, developed at Boeing, 1999
Non linear solver: NOX package based on Newton's method, developed at Sandia

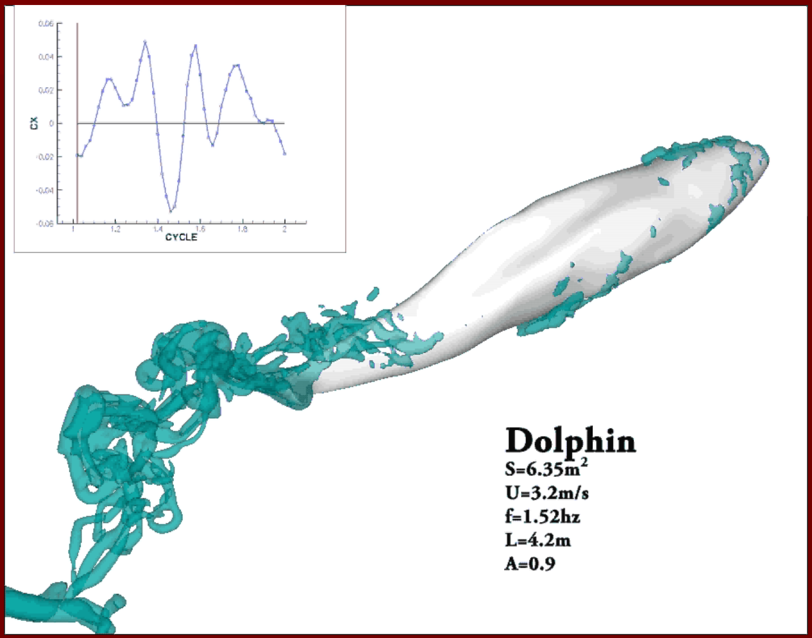
Closing the Loop for CFD in Biology/Biomedical Engineering



ViCar3D-Capabilities



CFD of the dolphin kick

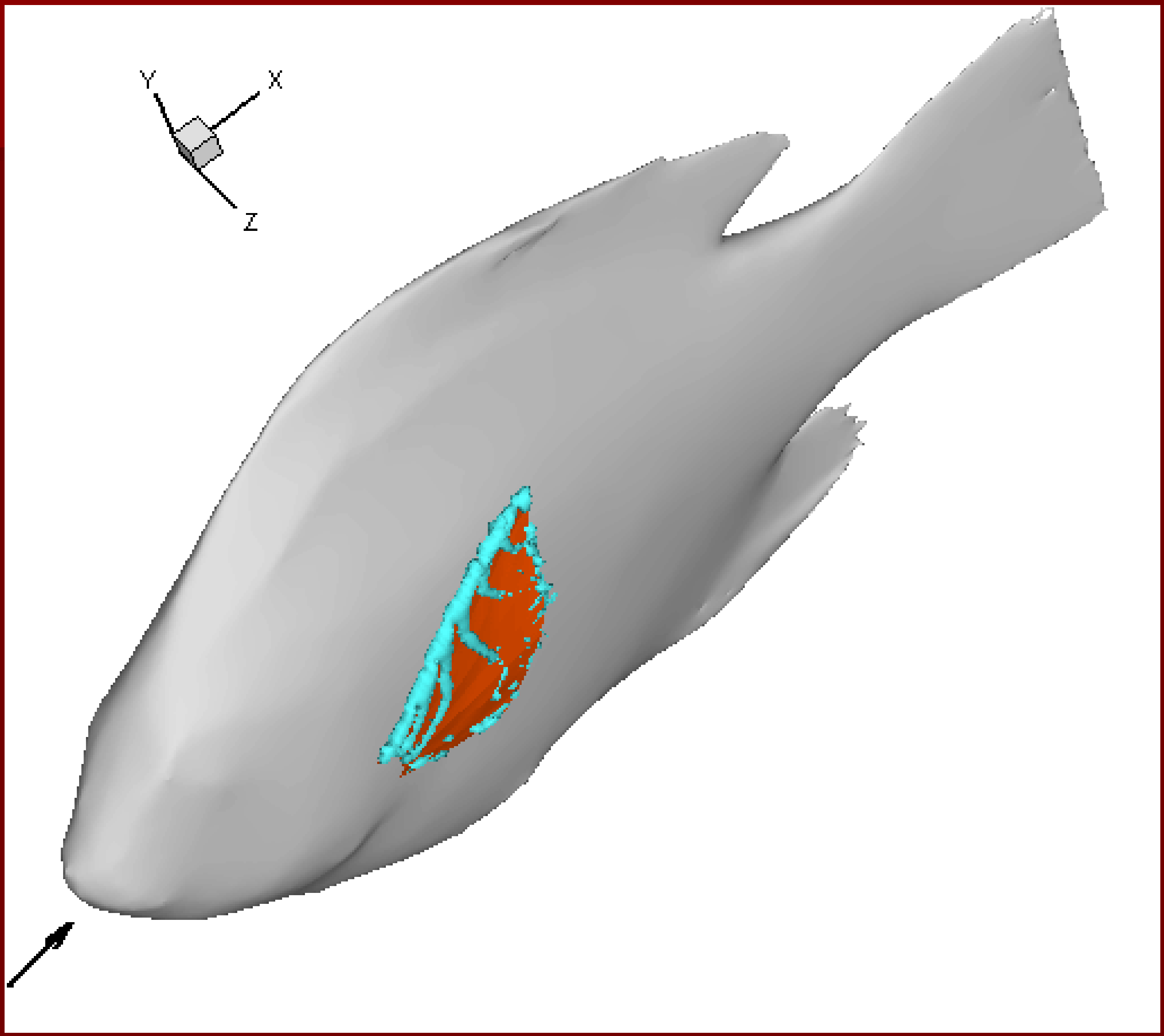


- "Propulsive Efficiency of the Underwater Dolphin Kick in Humans", Journal of Biomechanical Engineering, Vol. 131, May 2009

- "A computational method for analysis of underwater dolphin kick hydrodynamics in human swimming", Sports Biomechanics, 8(1), pp. 60-77, March 2009.

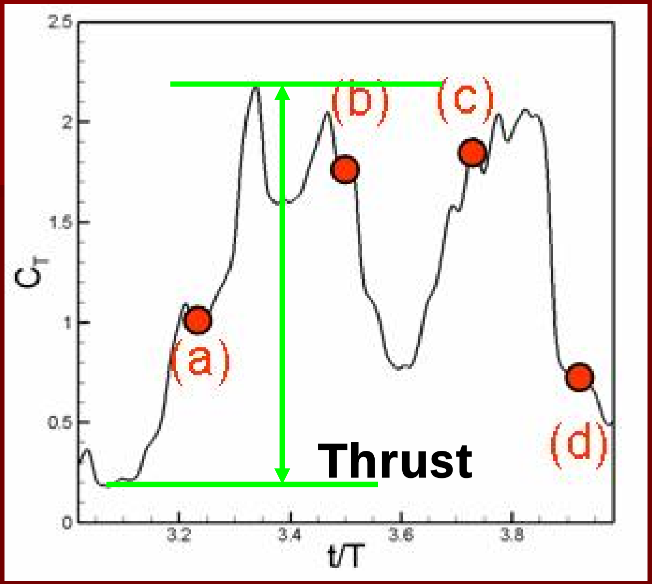
- "A comparison of the kinematics of the dolphin kick in humans and cetaceans", Human Movement Science, Vol.28, pp.99-112, 2009

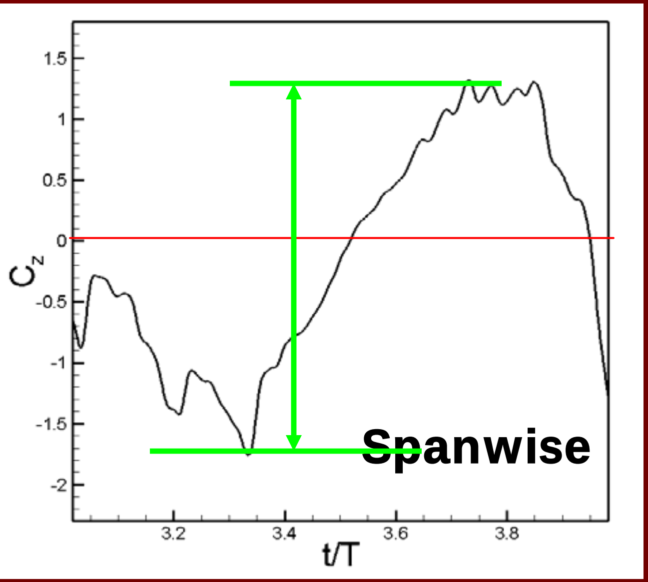
Labriform Propulsion in Fish

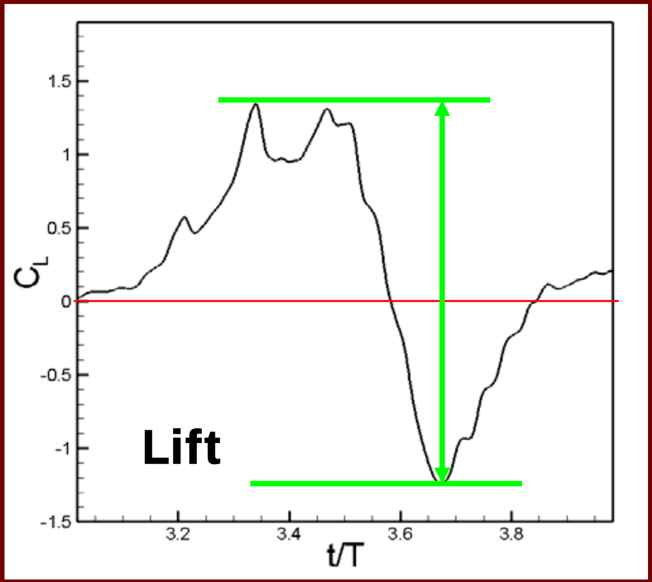


- "Wake Topology and Hydrodynamic Performance of Low-Aspect-Ratio Flapping airfoil", J. Fluid Mechanics (2006) Vol 566 pp 309-343 .
- Low-dimensional models and performance scaling of a highly deformable fish pectoral fin; J. Fluid Mech. (2009), vol. 631, pp. 311–342.
- "Computational modelling and analysis of the hydrodynamics of a highly deformable fish pectoral fin." (2010) , J. Fluid Mech. , doi:10.1017/S0022112009992941.

Validation against Experiments







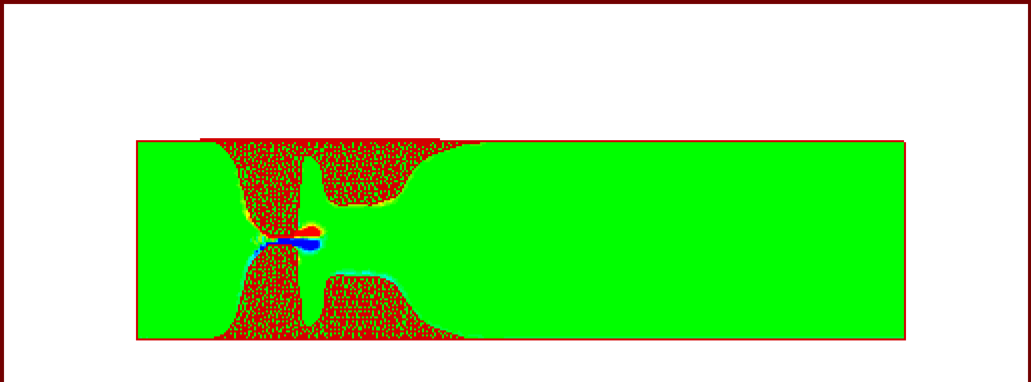
Peak to Peak

	CFD	PIV
Thrust	2.43	2.4
Lift	2.51	2.7
Span	3.61	3.7

Mean thrust Coeff.	CFD	PIV
	1.18	1.09

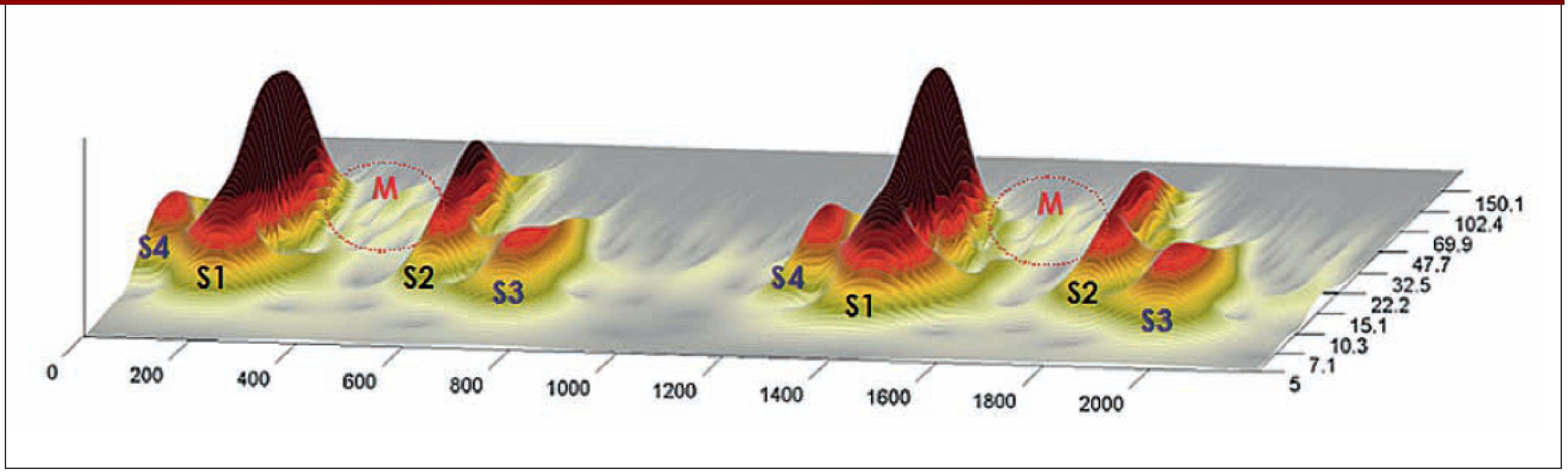
Flow-Induced Vibration (FEM)

- Simulation Details
  - 2D Simulation
  - Geometry based notionally on CT scan of human larynx
  - ViCar3D for air-flow
  - Finite-Element for VF
  - VF not fully adducted
- Observations
  - Kelvin-Helmholtz vortices
  - Bistable Jet
  - Sustained vibrations of vocal folds.



(400K points)

Flow-Induced Sound in Biomedicine



b

Figure 6

Acoustic cardiography report and time-frequency analysis of heart sounds for a subject with a systolic murmur. The upper panel (a) shows an acoustic cardiography report for a subject with a systolic murmur and the panels below (b) illustrate the 2D and 3D scalogram views for a few beats from the acoustic cardiography rhythm strip shown above. Since murmurs have higher frequency components than diastolic heart sounds, they do not have to be high in intensity to be detected by the human ear. Thus in this case, it will be easier for the human ear to detect the murmur than the low frequency third and fourth heart sound.

- Non-invasive Diagnosis


Review article

SWISS MED WKLY 2008;138(31-32):439-452 · www.smw.ch

Peer reviewed article

Beyond auscultation – acoustic cardiography in the diagnosis and assessment of cardiac disease

Paul Erne

Division of Cardiology, Kantonsspital Luzern, Lucerne, Switzerland

Flow-Induced Noise in Engineering Applications







- Noise is un-desirable
- Seeking source mechanism to reduce or control the noise

Current Approach

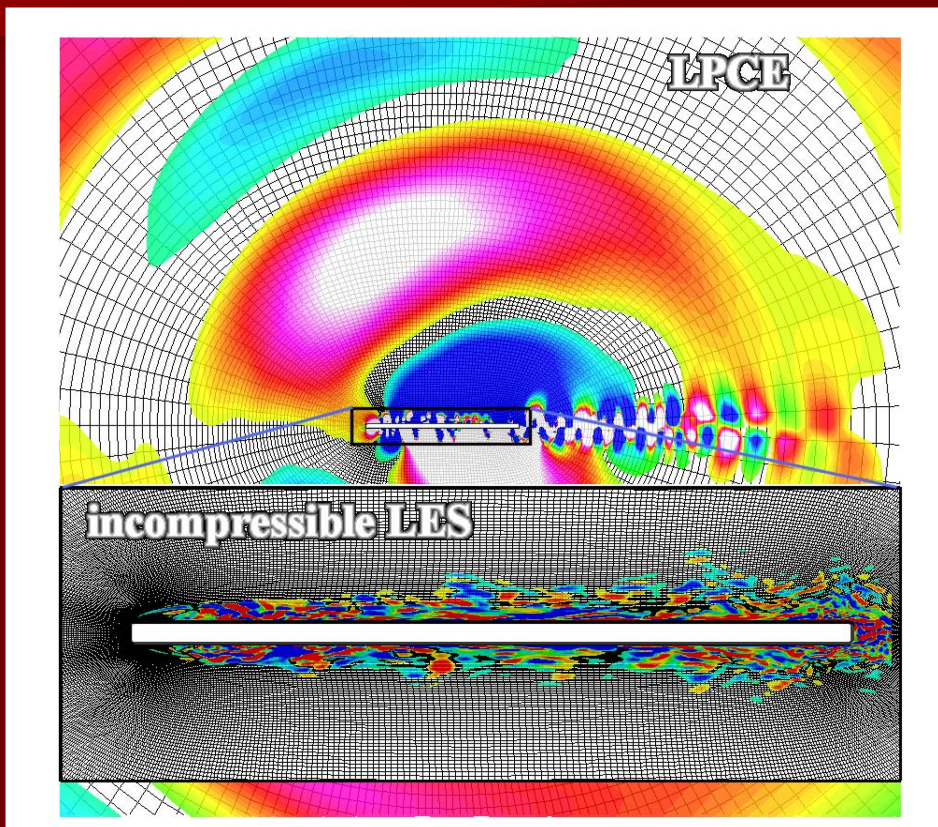
- Low Mach number regime (M < 0.3)
  large scale disparity between flow and acoustics

  Two-step hybrid method for flow and acoustic field computation

- Complex geometry
  : hard to generate good computational grid

  • Immersed Boundary Method on non-body conformal Cartesian grid.
  • Challenge: how to achieve higher order accuracy?
  • 6^{th}-order Pade Scheme

Hybrid method for Low Mach Number Aeroacoustics



Two-step approach, one-way coupled

$$u(x,t) = U(x,t) + u'(x,t)$$

Total,
Compressible

incompressible

perturbed

Incompressible N-S

LPCE*

*** Linearized Perturbed Compressible Equations**

Hydrodynamic/Acoustic Splitting method:

Efficient method to solve low Mach number aeroacoustic problem

Linearized Perturbed Compressible Equations

LPCE (Seo & Moon, JCP, 2006)

$$\rho(\vec{x},t) = \rho_0 + \rho'(\vec{x},t)$$
$$\vec{u}(\vec{x},t) = \vec{U}(\vec{x},t) + \vec{u}'(\vec{x},t)$$
$$p(\vec{x},t) = P(\vec{x},t) + p'(\vec{x},t)$$

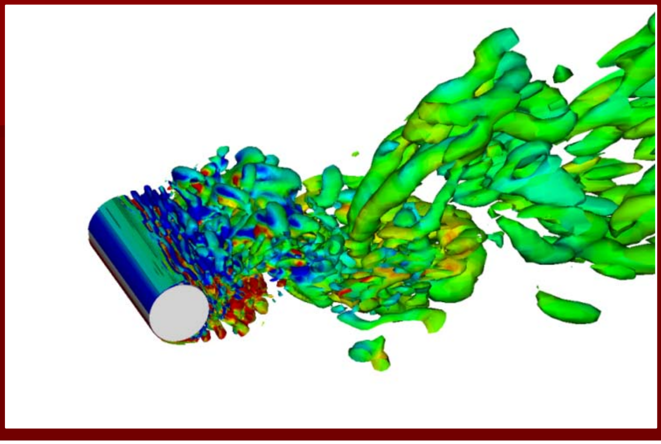
$$\frac{\partial \rho'}{\partial t} + (\vec{U}\cdot\nabla)\rho' + \rho_0(\nabla\cdot\vec{u}') = 0$$

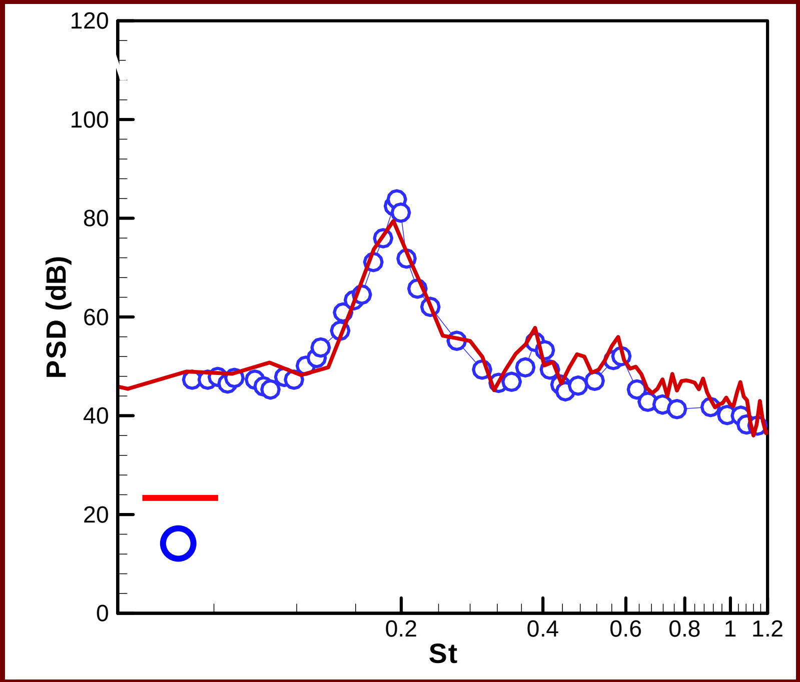
$$\frac{\partial \vec{u}'}{\partial t} + \nabla(\vec{u}'\cdot\vec{U}) + \frac{1}{\rho_0}\nabla p' = 0$$

$$\frac{\partial p'}{\partial t} + (\vec{U}\cdot\nabla)p' + \gamma P(\nabla\cdot\vec{u}') + (\vec{u}'\cdot\nabla)P = -\frac{DP}{Dt}$$

- **Subtracting INS from CNS**
- **Linearization**
- **Suppressing the generation and evolution of vortical component on the acoustic field.**

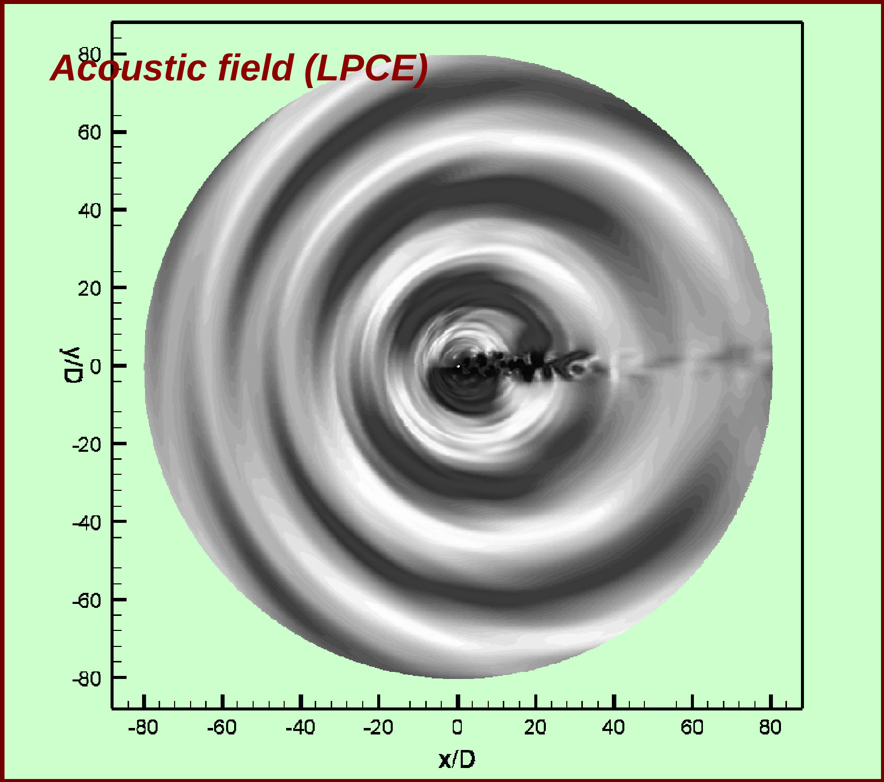
Validation of INS/LPCE Hybrid Method





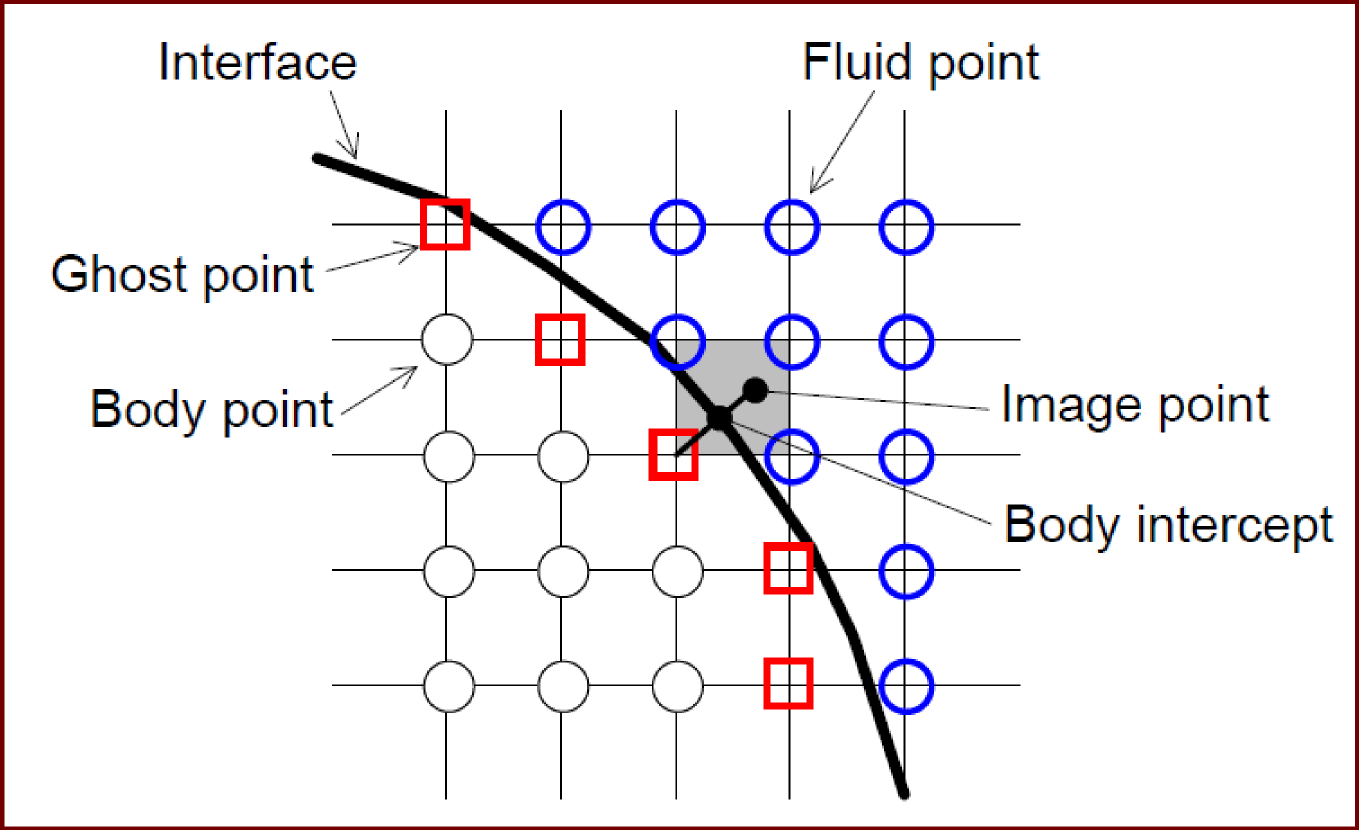
Noise generated by turbulent flow over a circular cylinder at Re_D = 46000 , M = 0.21

(Seo and Moon, JSV, 2007)



(Other cases : Moon et al., CF, 2010)

Original Immersed boundary method ?? (Mittal et al., JCP, 2008)



Sharp interface IBM based on the ghost-cell method

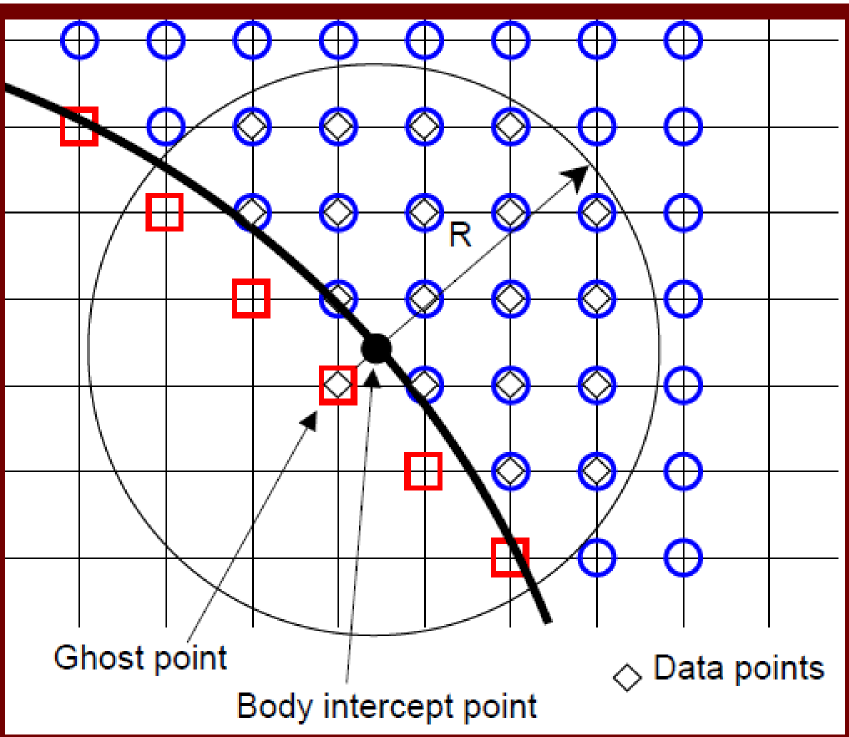
IBM for Acoustic solver (LPCE)

Approximating polynomial method (Luo et al., JCP, 2008)

$$\phi(x',y',z') \simeq \Phi(x',y',z') = \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=0}^{N} c_{ijk}(x')^i(y')^j(z')^k, \qquad i+j+k \le N$$

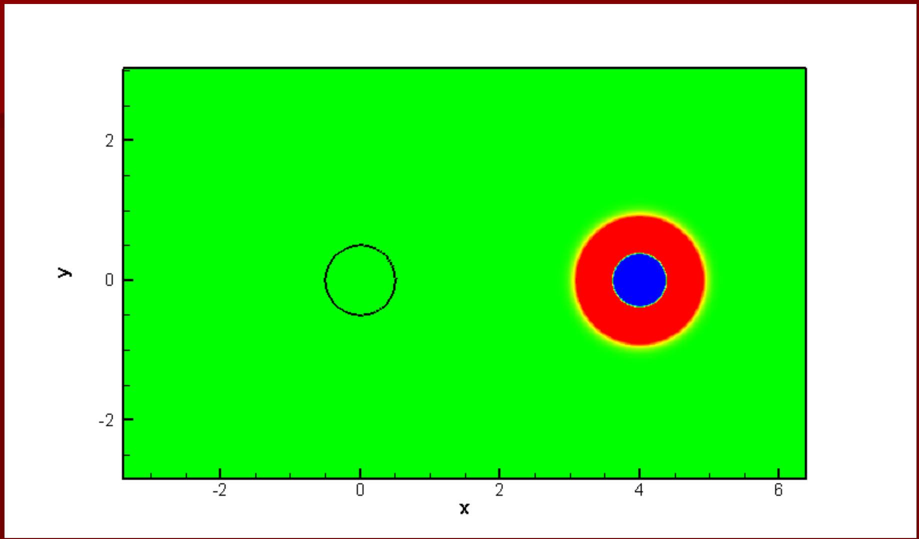
$$\varepsilon = \sum_{m=1}^{M} w_m{}^2 \left[\Phi(x'_m, y'_m, z'_m) - \phi(x'_m, y'_m, z'_m)\right]^2$$

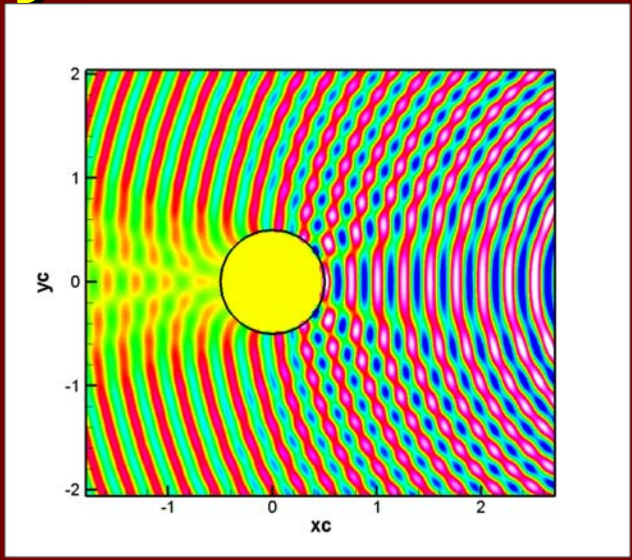
$$\vec{x}' = \vec{x} - \vec{x}_{BI}$$

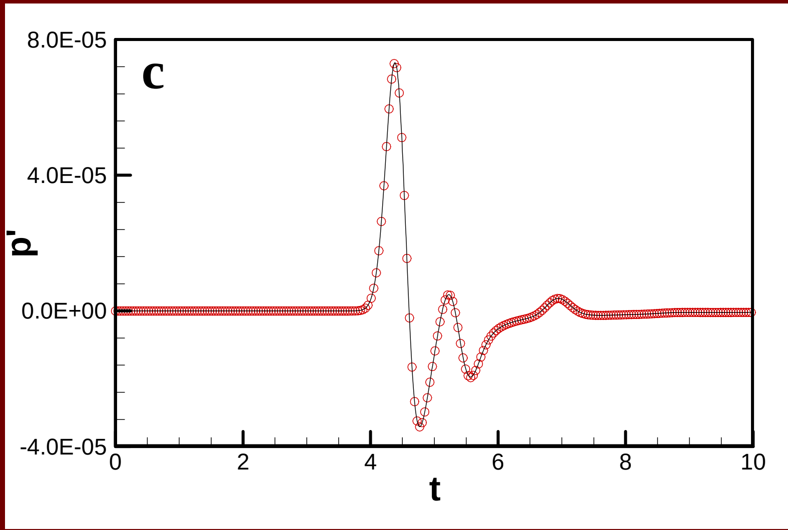


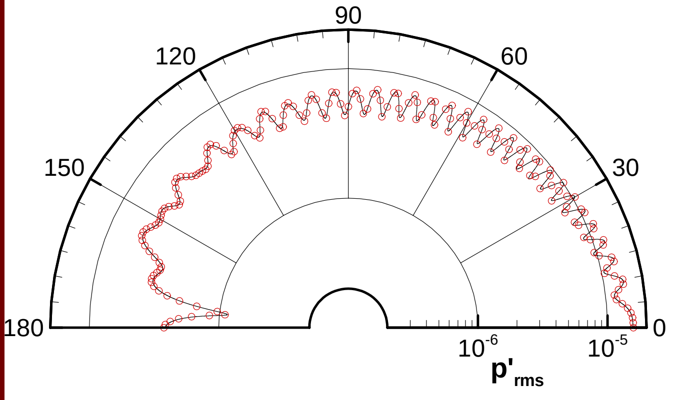
N	Number of coefficients	
	2D	3D
1	3	4
2	6	10
3	10	20
4	15	35

Benchmark: Sound Scattering by a Circular Cylinder





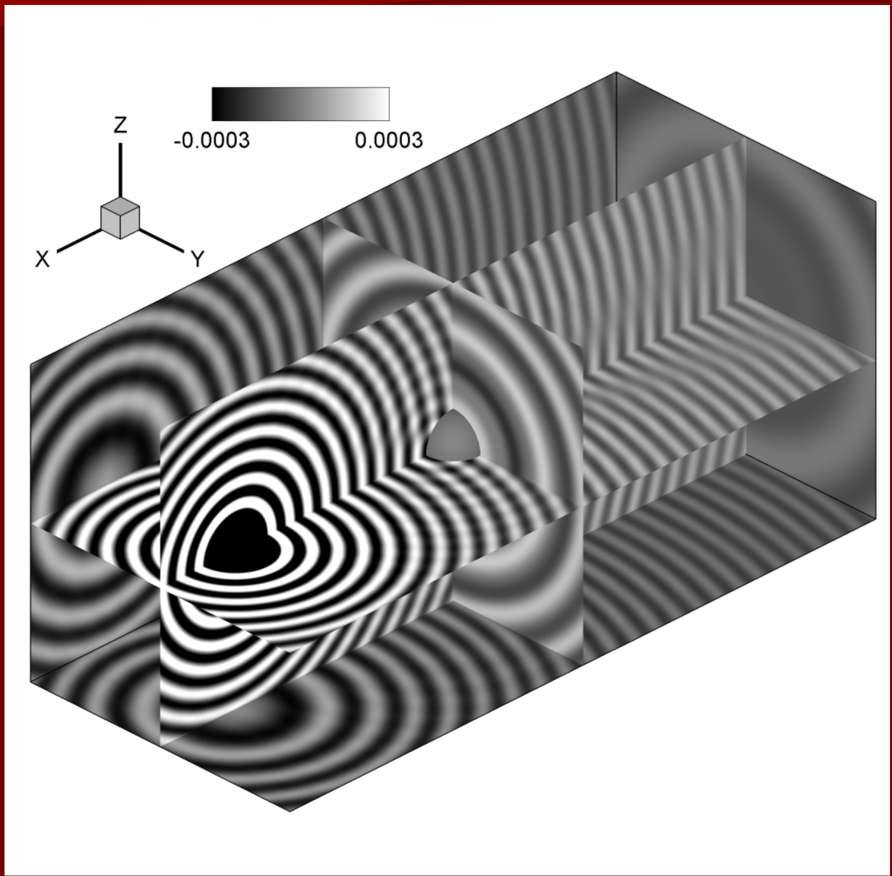


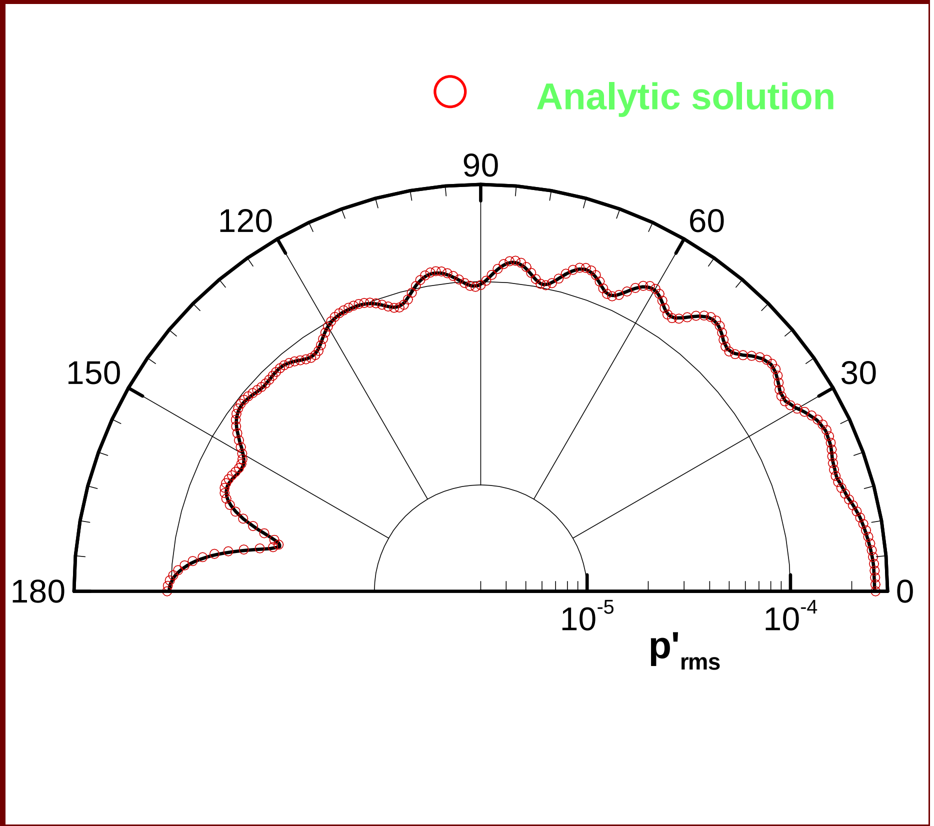


Directivity at r=5

Sound Scattering by a Sphere

3D, time-harmonic wave scattering

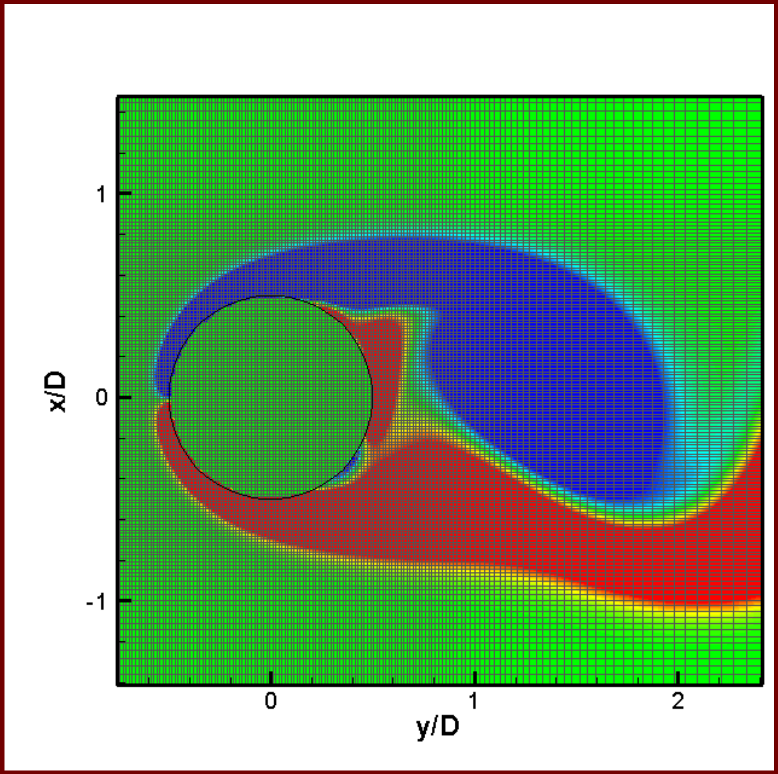


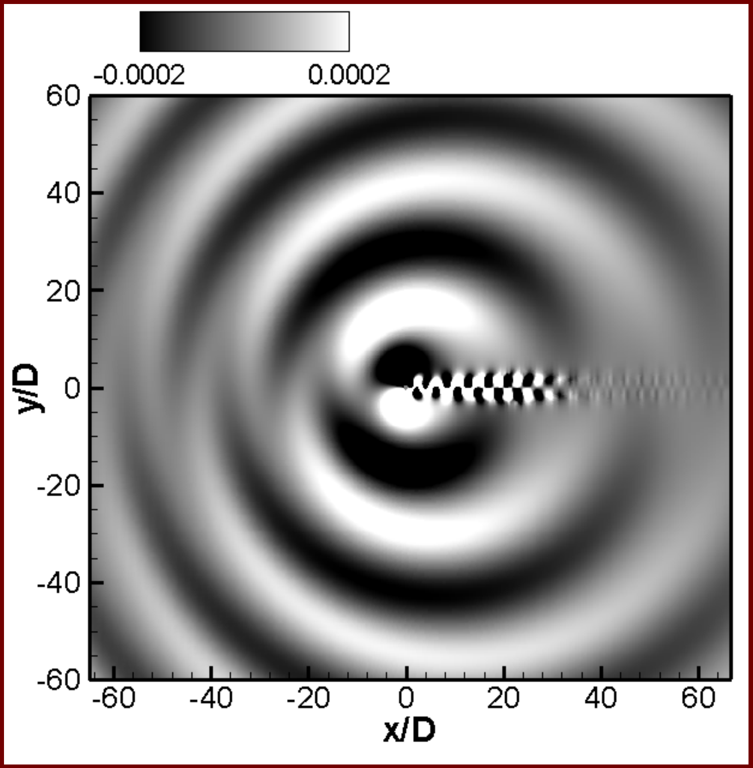


Flow induced Noise

IBM, incompressible flow solver (*Vicar3D*) + IBM LPCE solver (*carLPCE*)

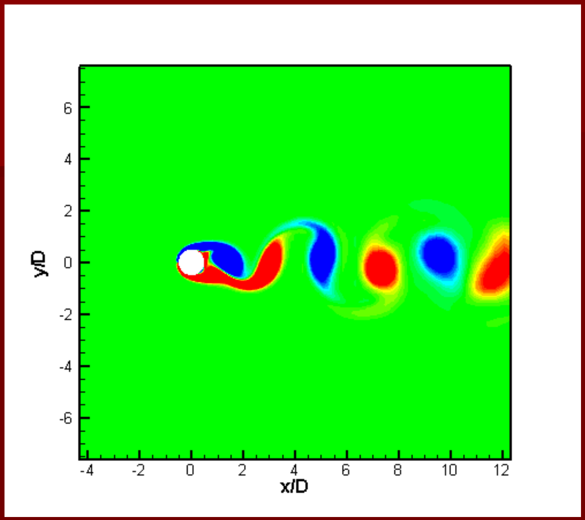
Tonal noise from a circular cylinder at $M_\infty = 0.2$, $Re_D = 200$

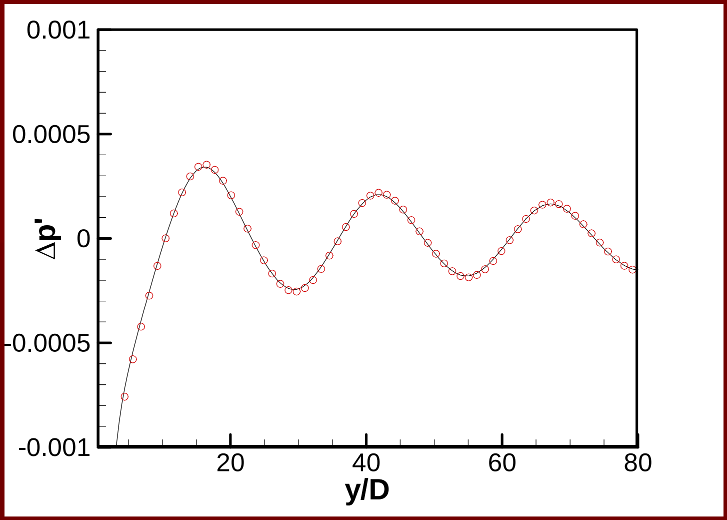


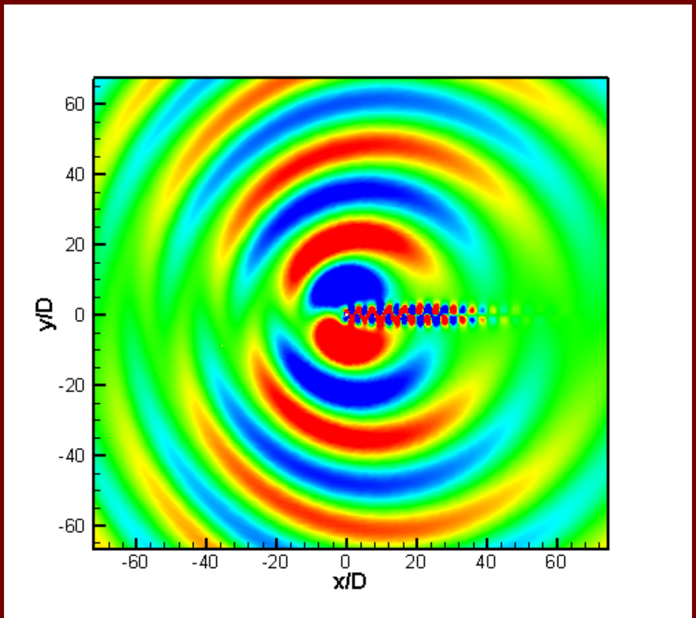


Comparison with DNS

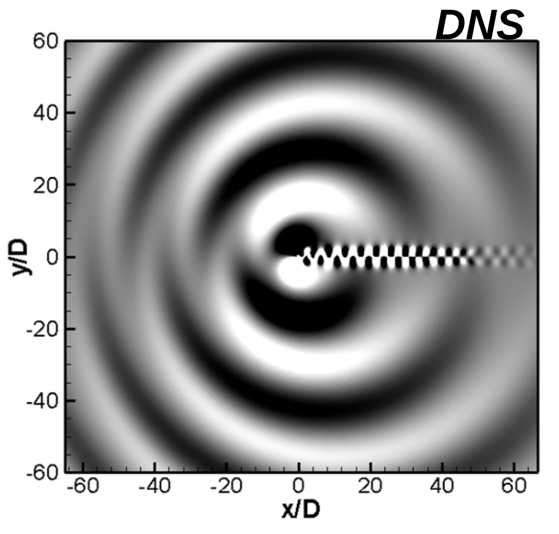
ViCar3D







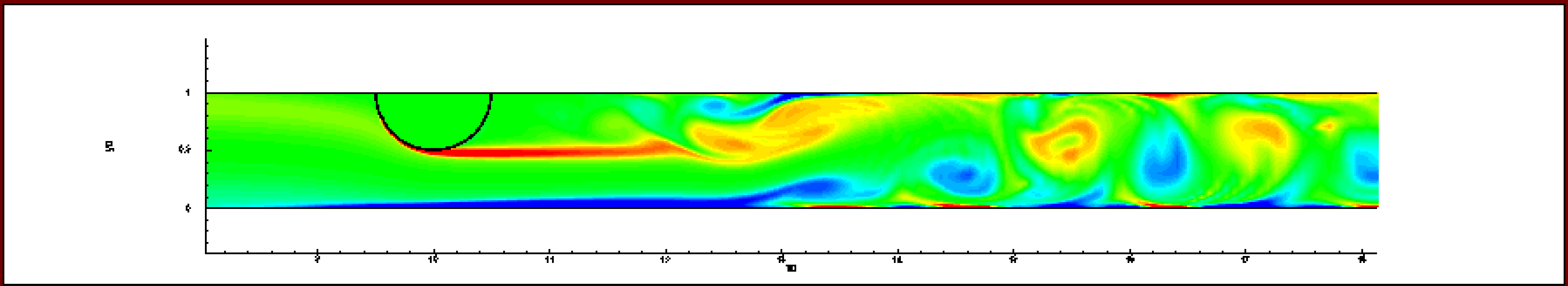
carLPCE



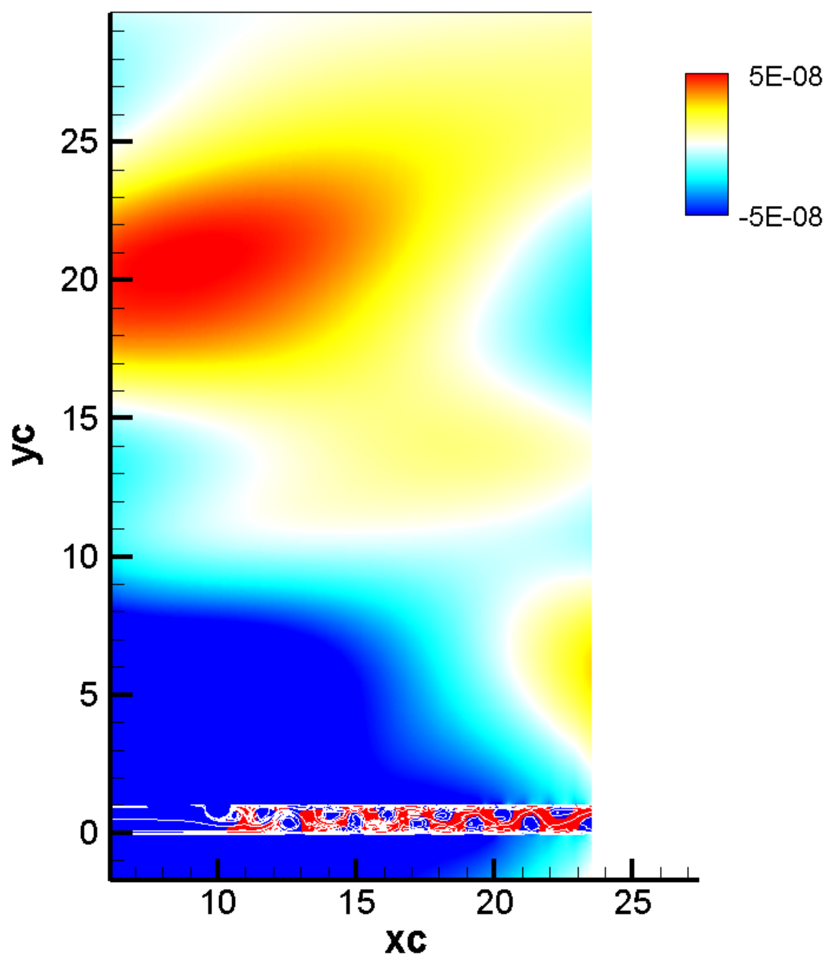
DNS : full compressible N-S Eqs. on a body-fitted O-grid

Modeling of Arterial Murmurs (Bruits)

Re=2000
50% Constriction



Modeled Sound



Closing

- Immersed boundary methods are well suited for
  - Complex geometries
  - Moving boundaries
  - Multi-physics
- Challenges
  - High Reynolds number flows? → local refinement?
  - Strict conservation? → Cut-Cell?

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Questions?