Solid/Fluid Thermal Coupling
Using the Immersed Boundary Method

Gianluca Iaccarino
Mechanical Engineering
Stanford University

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Non-Body-Fitted Approaches

- True Boundary
- IB with Forcing
- Stairstep/Mask
- Cut-Cell (Body-Fitted)
- Projection (Body-Fitted) OpenFoam SnappyHex
- IB reconstruction
4. IB Reconstruction

Solid/Fluid Thermal Coupling
Using the Immersed Boundary Method
Geometric IB Reconstruction

Linear Reconstruction

\[ \phi = a_1 x + a_2 y + a_3 \]

Quadratic Reconstruction

\[ \phi = a_1 n^2 + a_2 n + a_3 t + a_4 nt + a_5 \]

F: fluid points; B: boundary point; G: ghost point
Geometric IB Reconstruction

The quadratic reconstruction scheme requires 4 fluid nodes + BC

Remarks:

1) The fluid points F1-F4 are selected according to the angle between the IB and the grid
2) The image point is used to construct the interpolation and then the boundary value is reflected

F: fluid points; B: boundary point; G: ghost point; I: image of the ghost point
Physics-based IB Reconstruction

The quadratic reconstruction scheme requires 4 fluid nodes + BC

We can build a **better reconstruction** by using the conservation of momentum in a “ghost” control volume
(4 fluid nodes + BC + conservation of mass)

\[
\phi = a_1 n^2 + a_2 n + a_3 t + a_4 nt + a_5
\]

\[
U(P2-F2) - U(P3-F1) + V(F1-F2) = 0
\]

F: fluid points; B: boundary point;
G: ghost point; I: image of the ghost point
P: Additional surface points
Verification Test

Decaying vortex problem (exact solution of the Navier Stokes equations)

\[
p(x, y, t) = -\frac{1}{4} (\cos 2\pi x + \cos 2\pi y) e^{-4\pi^2 t / Re}
\]

\[
u(x, y, t) = -\cos \pi x \sin \pi y \ e^{-2\pi^2 t / Re}
\]

\[
v(x, y, t) = \sin \pi x \cos \pi y \ e^{-2\pi^2 t / Re}
\]

Achieved 2\textsuperscript{nd} order accuracy at the Immersed Boundary
Reconstruction Schemes

• **Flow in an impeller stirred tank using an immersed boundary method.**
  1D linear, low-Re flow

• **RANS solver with adaptive structured boundary non-conforming grids**
  S. Majumdar, G. Iaccarino and P. Durbin, CTR Annual Briefs, 2001
  2D linear & quadratic, image point reflection, laminar flow

• **Turbulence modeling in an immersed-boundary RANS method**
  G. Kalitzin and G. Iaccarino, CTR Annual Briefs, 2002
  3D linear & inverse distance, turbulent flow, RANS

• **Wall modeling for large-eddy simulation using an immersed boundary method**
  3D linear/logarithmic, turbulent flow, LES

• **Accurate and efficient immersed-boundary interpolations for viscous flows**
  S. Kang, G. Iaccarino and P. Moin, CTR Annual Briefs, 2004
  3D linear & quadratic, mass conservation constraint, turbulent flow, LES

• **Immersed boundary for compressible flow simulations on semi-structured meshes**
  M. de Tullio and G. Iaccarino, CTR Annual Briefs, 2005
  3D linear & quadratic, compressible, turbulent flow, RANS

• **Automatic mesh generation for LES in complex geometries**
  G. Iaccarino and F. Ham, CTR Annual Briefs, 2005
  3D linear, local and global mass conserving reconstructions, turbulent flow, LES
Mass Conservation

Forcing the IB “inside” the fluid domain eliminates the reflection!

However, The Reconstructed velocity at the virtual boundary still does not exactly conserve mass

\[ (\vec{V} \cdot \hat{n})_{IB} \neq 0 \]

both locally and globally

We can define a correction of the reconstruction based either on the global mass imbalance

\[ \alpha \sum (\vec{V} \cdot \hat{n})_{IB} A_{IB} \]
equally distributed at the IB nodes

or the local mass flux \( (\vec{V} \cdot \hat{n})_{IB} \)
Laminar Channel

Use Cartesian grids generated in a rotated axis
- Periodic domain (no boundary conditions)
- Uniform grids (no effects of the numerical accuracy)

Test accuracy, symmetry, convergence

Used $\alpha = 20, 10, -10$ and three grid resolutions

$\alpha = 10$
StairStep Solution

Remark: ibwalls are always inside the computational domain, hence we always obtain lower mass through (fixed pressure gradient!)
Mass Conservation

Effect of the mass correction applied to the ibwall interpolation

(Quadratic reconstruction)

Remark: the asymmetry is related to the grid not being top/bottom symmetric. The corrections eliminate it almost completely!
The solution correctly converges independently of the mass correction used.
Local mass correction provides a symmetric profile independently of the angle.
Verification Test

$L^\infty$ error in velocity

\[ \alpha = 15^\circ \]
**Numerical Method - Details**

Navier-Stokes Equations – Incompressible Fluid

\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j}
\]

\[
\frac{\partial u_i}{\partial x_i} = 0
\]

Fractional Step Method – Crank-Nicholson in time, RK for Convective Terms

\[
\left[ \frac{1}{\Delta t} - \frac{(\alpha_k + \beta_k)}{2Re} \frac{\partial^2}{\partial x_j \partial x_j} \right] \hat{u}_i^k = \frac{u_i^{k-1}}{\Delta t} - (\alpha_k + \beta_k) \frac{\partial p^{k-1}}{\partial x_i}
\]

\[
- \alpha_k \left( \frac{\partial u_i u_j}{\partial x_j} \right)^{k-1} - \beta_k \left( \frac{\partial u_i u_j}{\partial x_j} \right)^{k-2} + (\alpha_k + \beta_k) \frac{(\partial^2 u_i^{k-1})}{(\partial x_j \partial x_j)}
\]

\[
\frac{\partial^2 \phi}{\partial x_j \partial x_j} = \frac{1}{(\alpha_k + \beta_k) \Delta t} \frac{\partial \hat{u}_i^k}{\partial x_i}
\]

\[
u_i^k = \hat{u}_i^k - (\alpha_k + \beta_k) \Delta t \frac{\partial \phi}{\partial x_i}
\]

\[
p^k = p^{k-1} + \phi
\]

Momentum

Continuity

Momentum Predictor

Diverge-free Constraint

Momentum Corrector

Pressure update
## IB Reconstruction

### Linear

\[
\begin{align*}
    u_{i,c}^k &= \omega_{i,1} u_{i,1}^k + \omega_{i,2} u_{i,2}^k + \omega_{i,IB} u_{i,IB}^k \\
    \Delta u_{i,c} &= \omega_{i,1} \Delta u_{i,1} + \omega_{i,2} \Delta u_{i,2} + \omega_{i,IB} \Delta u_{i,IB}
\end{align*}
\]

### Revised Linear

\[
\begin{align*}
    u_{i,c}^k &= \omega_{i,1} u_{i,1}^k + \omega_{i,2} u_{i,2}^k + \omega_{i,IB} u_{i,IB}^k \\
    \Delta u_{i,c} &= \omega_{i,1} \Delta u_{i,1} + \omega_{i,2} \Delta u_{i,2} + \omega_{i,IB} \Delta u_{i,IB} \\
    \Delta u_{i,c} &= \hat{u}_{i,c}^k - \hat{u}_{i,c}^{k-1}
\end{align*}
\]

### Quadratic

\[
\begin{align*}
    \hat{u}_{i,c}^k(x_1, x_2) &= a_{i,1,1}^k x_1^2 + b_{i,1,1}^k x_1 + a_{i,2,2}^k x_2^2 + b_{i,2,2}^k x_2 + \hat{u}_{i,c}^k
\end{align*}
\]

### Quadratic + Momentum

\[
\begin{align*}
    \frac{\hat{u}_{i,c}^k}{\Delta t} - \frac{(\alpha_k + \beta_k)}{Re} (a_{i,1,1}^k + a_{i,2,2}^k) &= \frac{u_{i,c}^{k-1}}{\Delta t} \\
    - (\alpha_k + \beta_k) \frac{\partial p^{k-1}}{\partial x_i} - \alpha_k \left(\frac{\partial u_i u_j}{\partial x_j}\right)^{k-1} \\
    - \beta_k \left(\frac{\partial u_i u_j}{\partial x_j}\right)^{k-2} + \frac{(\alpha_k + \beta_k)}{2Re} \frac{\partial^2 u_i^{k-1}}{\partial x_j \partial x_j}
\end{align*}
\]
IB Reconstruction

Decaying Taylor Vortices

Exact Solution of NS Eqs.
LES of airfoil flow

Re ~ 150,000 - Grid size ~7M

Local mesh refinement technique reduces the total number of mesh points by 70% compared to a (single-block) Cartesian mesh with similar mesh resolution near the wall.
LES of airfoil flow

Re ~ 150,000 - Grid size ~7M

Small separation bubble near LE prompts transition to turbulence

Instantaneous $x$-velocity and $x$-vorticity
LES of airfoil flow

Re ~ 150,000 - Grid size ~7M

Streamwise velocity
Mean Flow Predictions

Over the airfoil

Downstream
Acoustic Predictions

Comparisons of velocity profiles in the wake and pressure distribution on the surface is very favorable.

Much more challenging is the prediction of the wall pressure fluctuations.

Wall pressure spectrum at the trailing edge ($x/c=0.98$)
Summary

IB faces (ibwalls) are **always inside** the computational domain: effectively the **reconstruction is always an interpolation**

Use of Locally Refined Grid creates additional complexity in the reconstruction step

Enforcing mass conservation is NOT trivial but important and lead to more accurate results!
5. Solid/Fluid Thermal Coupling

Solid/Fluid Thermal Coupling
Using the Immersed Boundary Method
Multi-Domain Coupling

Because of discontinuities in physical properties (conductivity, etc) it is preferred to apply boundary conditions rather than differentiate across the interface!
Multi-Domain Coupling

\[
\begin{align*}
T_{\text{fluid}} &= T_{\text{solid}} \\
\left. k_{\text{solid}} \frac{\partial T}{\partial n} \right|_{\text{solid}} &= \left. k_{\text{fluid}} \frac{\partial T}{\partial n} \right|_{\text{fluid}}
\end{align*}
\]

Interface Conditions
Multi-Domain Coupling

Interface Conditions

\[
\begin{align*}
T_{\text{fluid}} &= T_{\text{solid}} \\
k_{\text{solid}} \frac{\partial T}{\partial n}_{\text{solid}} &= k_{\text{fluid}} \frac{\partial T}{\partial n}_{\text{fluid}} + \eta \times (T_{\text{solid}} - T_{\text{fluid}})
\end{align*}
\]
Multi-Domain Coupling

\[ T_{\text{fluid}}^n = T_{\text{solid}}^{n-1} \]

\[ k_{\text{solid}} \left. \frac{\partial T}{\partial n} \right|_{\text{solid}} ^ n = k_{\text{fluid}} \left. \frac{\partial T}{\partial n} \right|_{\text{fluid}} ^ n + \eta \times \left( T_{\text{solid}}^n - T_{\text{fluid}}^n \right) \]

\[ T_{\text{fluid}}^{n+1} = T_{\text{solid}}^n \]

\[ k_{\text{solid}} \left. \frac{\partial T}{\partial n} \right|_{\text{solid}} ^ {n+1} = k_{\text{fluid}} \left. \frac{\partial T}{\partial n} \right|_{\text{fluid}} ^ {n+1} + \eta \times \left( T_{\text{solid}}^{n+1} - T_{\text{fluid}}^{n+1} \right) \]

Staggered coupling procedure
The energy equation is NOT solved across the subdomains, but boundary conditions are formally applied at the interface.

We apply the IB reconstruction method on both sides of the interface.
IB Multi-Domain Coupling

What is actually coupled?

Continuity of temperature

\[ T_{\text{fluid}} = \overline{T_{\text{solid}}} \]

Continuity of heat flux

\[ k_{\text{solid}} \frac{\partial T}{\partial n}_{\text{solid}} = k_{\text{fluid}} \frac{\partial T}{\partial n}_{\text{fluid}} \]

Overbar indicates surface interpolation operators (on the “true” surface)

Asymmetric boundary condition enforcement
Interface Interpolation

We need the actual interface geometry....
Interface Grid Reconstruction

- Original STL
- Stairstep Tommie grid
- Projected Tommie grid
Interface Interpolation

\[ \Gamma_{\text{fluid}} \quad \text{fluid-solid} \quad \Gamma_{\text{solid}} \]

Projection  Interpolation  Projection
Validation Test, I

Flow generated by a heated sphere
Gr = 10^4
Constant temperature on the sphere
Natural convection, Boussinesq approximation

Mesh: ~2.2M elements
Validation Test, II

Validations

Test, II

Based on the channel half width = 500
Experiments by Laskowski et al. 2007
Computational Mesh

Grid size: 5-12M elements
Heating starts at $x=0$ on the bottom plate. The BL is thermally unstable...
The Effect of Solid Conduction

Highly unsteady turbulent flow field in the wake result in motion of the separation points on the cylinder

Thermal plumes from the upstream BL hit the cylinder at the stagnation point

As a result the wall temperature is NOT stationary
The Effect of Solid Conduction

![Graph showing the effect of solid conduction on heat transfer.](image-url)
Instantaneous Temperature Field

(a) At the outer cylinder ($r=0.79\,cm$)

(b) At $r=0.68\,cm$

(c) At $r=0.57\,cm$

(d) At $r=0.47\,cm$
6. Applications

Solid/Fluid Thermal Coupling
Using the Immersed Boundary Method
Electronic Component Units
Electronic Components

A simple test case: Conduction + Natural Convection

2 Hot Spots (CPUs)
1 Plastic Mold
in a air-filled cavity

Body-Fitted
420K Cells

IB
380K Cells
Electronic Components

A simple test case: Conduction + Natural Convection

2 Hot Spots (CPUs)
1 Plastic Mold
in a air-filled cavity

Body-Fitted
420K Cells

IB
380K Cells
Temperature Field

Body Fitted  Immersed Boundary
Velocity Field

Body Fitted

Immersed Boundary
Electronic Component Unit

A complex test case: Conduction + Forced Convection

- Cartesian mesh
  - ~ 5.2 million cells
- Mesh contains ~10 solid zones for calculating conjugate heat transfer

STL Geometry
Electronic Component Unit

**CASE 1:**
$T_{\text{max}} = 322\ K$

**CASE 2:**
$T_{\text{max}} = 325\ K$
Electric Motors
The “world” is NOT Cartesian

Many of the industrial applications have moving/rotating parts and Cartesian grids are not ideal

Example: Valeo Electric Motor
Beyond Cartesian Grids

The grids are not generated in the Physical Space but in a notional, integer space

- Addition of cells, and geometrical info are fast
- Tri-segment intersections are “exact”
Beyond Cartesian Grids

A coordinate transformation can be applied to the entire process

• Cylindrical-to-Cartesian
• Any generic invertible transformation

Physical Space (Real X Real)  
Transformed Space (Real X Real)  
Notional Space (Integer X Integer)
From Cartesian to Cylindrical

Example: Valeo Electric Motor
Alternator

- Cylindrical mesh
  - ~16.2 million cells
- Mesh contains ~40 solid zones for calculating conjugate heat transfer
Alternator

- Cylindrical mesh
  - ~16.2 million cells
- Mesh contains ~40 solid zones for calculating conjugate heat transfer
Alternator

<table>
<thead>
<tr>
<th>Flow Rate (CFM)</th>
<th>Pressure Drop (in. water)</th>
<th>Experiment</th>
<th>Simulation</th>
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<tbody>
<tr>
<td>2.0</td>
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<td>0.087</td>
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</tr>
<tr>
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<tr>
<td>5.0</td>
<td>0.195-0.225</td>
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Simulations vs. Experiments

<table>
<thead>
<tr>
<th></th>
<th>Simulation</th>
<th>Test results</th>
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<tr>
<td>Stator copper</td>
<td>453</td>
<td>471</td>
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<tr>
<td>Stator iron</td>
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<tr>
<td>Rear bearing</td>
<td>450</td>
<td>389</td>
</tr>
<tr>
<td>diodes</td>
<td>440</td>
<td>418</td>
</tr>
</tbody>
</table>

Temperature
7. Closing

Solid/Fluid Thermal Coupling
Using the Immersed Boundary Method
Beyond CHT

Nutrient Transport in Coral Reefs

Hazard Dispersion In Urban Environments
References


**IB for Incompressible LES:** Verzicco, Fatica, Iaccarino, Orlandi, AIAA J. Vol. 40, pp. 177-191, 2002


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