

Topological Data Analysis

SCS Spring Meeting 2022 Magnus Bakke Botnan





Topological data analysis (TDA) can broadly be described as a collection of data analysis methods that find structure in data. These methods include clustering, manifold estimation, nonlinear dimension reduction, mode estimation, ridge estimation and persistent homology.



Larry Wassermann, Topological Data Analysis Annual Review of Statistics and Its Application Vol. 5:501-532 (Volume publication date March 2018)





























1-st level connectivity: 5'->1

"TWO LOOPS ARE EQUIVALENT IF ONE CAN BE DEFORMED INTO THE OTHER



- D Topology delects obstructions





JORUS or Wedge?

¥ p. = 1 Bo = 1 p, = 2 p2 = 1 p, = 2 p2 = 1

But Cottonalogy Does THE JOB!

Detecting the Circle!









PERSISTENT HOMOLOGY

• Introduced by Edelsbrunner et al to study voids and cauties in proteins.



• Uses the fact that homology is functionial, i.e. if TST', then there is a linear map $Hp(T) \longrightarrow Hp(T')$.




























































Data - Homology

A simplicial complex is a topological space built from simple pieces: vertices, edges, triangles, tetrahedra,





Not a simplicial complex

A SIMPLICAL COMPLEX FROM DATA: THE Cech complex at scale E:



THE NERVE THEOREM



FROM ROD GHRIST'S SURVEY!

$$H'_{i}\left(\bigcup_{\varepsilon} B_{\varepsilon}(x_{i})\right) \cong H'_{i}\left(\bigcup_{\varepsilon}\right)$$

 $\hat{\mathcal{L}}$ can be computed
with linear algebra!























In 1971, John O'Keefe discovered the first component of this positioning system. He found a type of nerve cell in an area of the brain called the hippocampus that was always activated when a rat was at a certain place in a room. Other nerve cells were activated when the rat was at other places. O'Keefe concluded that these "place cells" formed a map of the room.

- The Nobel Committee, October 2014



PLACE FIELDS



Spreemann, Dunn, B., Baas 2018 (Phys. Rev E)

- Animal enters "room" (loosely defined environment)
- 2 Place cells establish a cover by *place fields*
- Animal moves around, place cells fire

























(Embedding is of course not known)







direction cells.

head

> Neural Comput. 2019 Jan;31(1):68-93. doi: 10.1162/neco_a_01150. Epub 2018 Nov 21.

Decoding of Neural Data Using Cohomological Feature Extraction

Erik Rybakken¹, Nils Baas², Benjamin Dunn³







Tracked tuning curves
 Decoded tuning curves



Toroidal topology of population activity in grid cells

Richard J. Gardner 🖂, Erik Hermansen, Marius Pachitariu, Yoram Burak, Nils A. Baas 🖂, Benjamin A.





18 0

Radius

16

Radius

18 0


Setup

30s guide to giotto-tda



The End