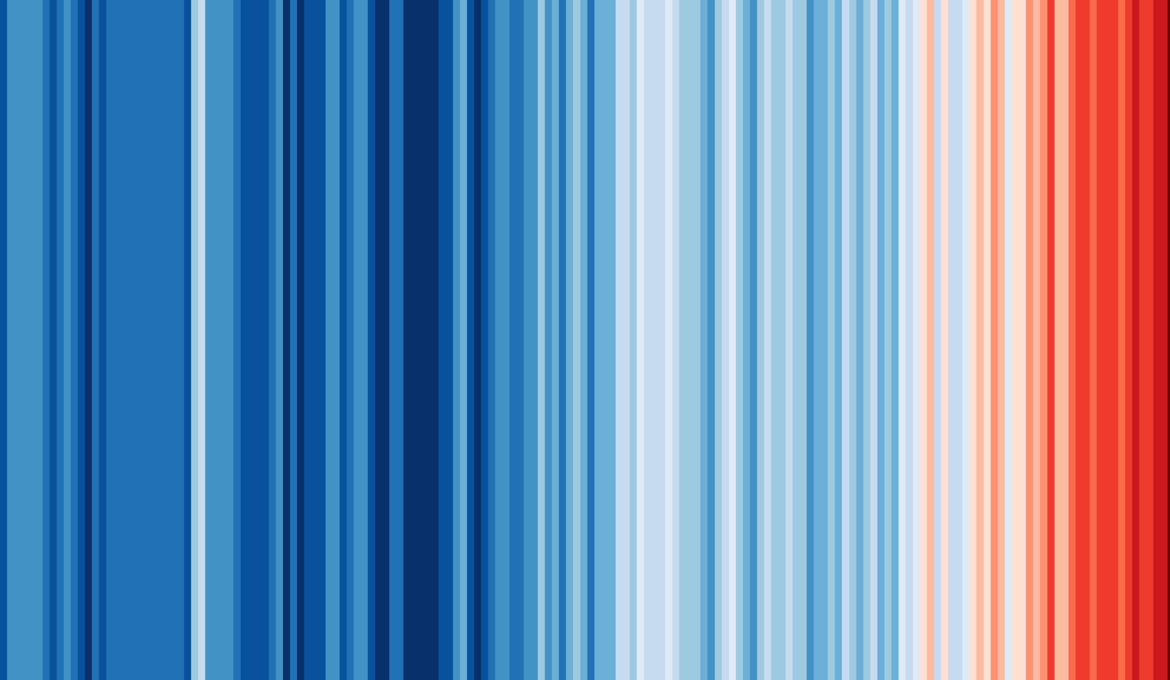
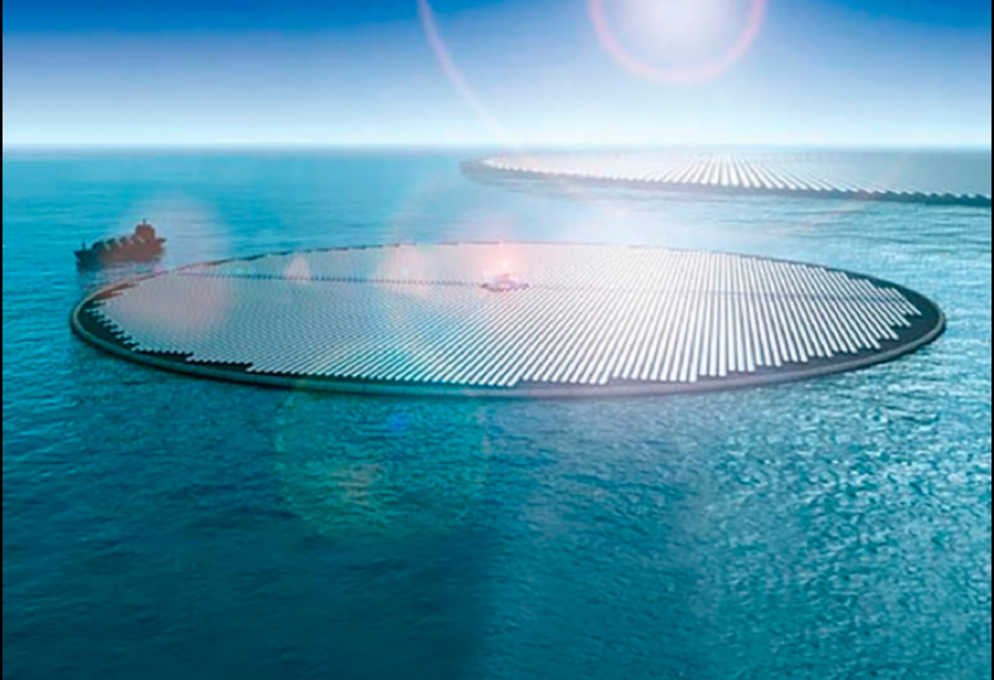


Warming Stripes, Ed Hawkings, Climate Lab Book



Patterson et al. 2019

Image by OceanSun

Image by OceanSun

On the modeling of wrinkling instabilities using isogeometric shell analysis

Hugo Verhelst [1,2,@] Matthias Möller [1] Henk den Besten [2]

[1] Numerical Analysis, Faculty of Electrical Engineering, Mathematics and Computer Science (EEMCS), Delft University of Technology
[2] Maritime and Transport Technology, Faculty of Mechanical, Maritime and Materials Engineering (3mE), Delft University of Technology

[@] h.m.verhelst@tudelft.nl

06-05-2022

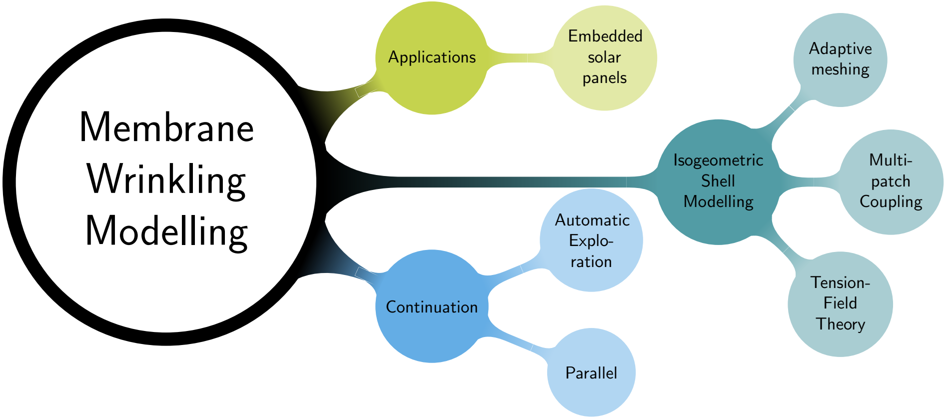
Outline

Outline

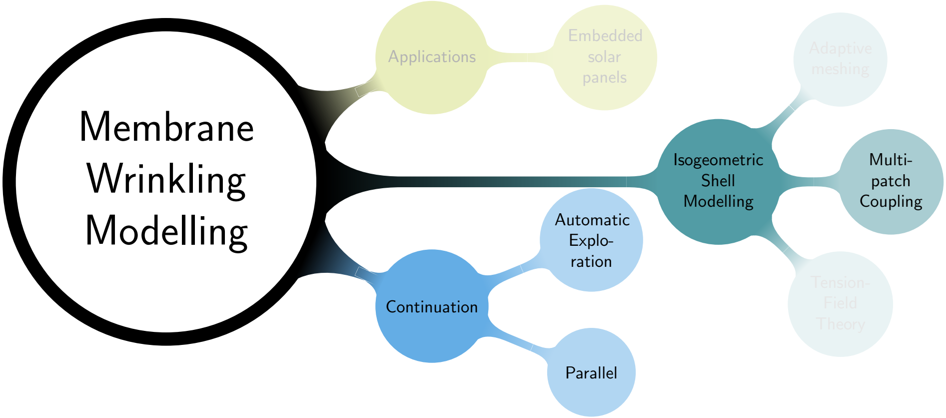
Introduction - Aim

Develop a robust and efficient strategy for the modelling of wrinkles using isogeometric analysis

Introduction - Aim



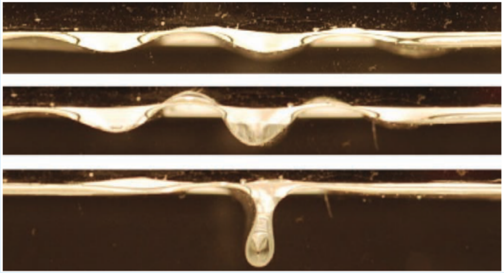
Introduction - Aim



Outline

Wrinkling Basics - Energy Balance

Foundation vs Bending



Folding in a floating sheet [1]

Tension vs Bending

Wrinkling in a tensioned sheet [2]

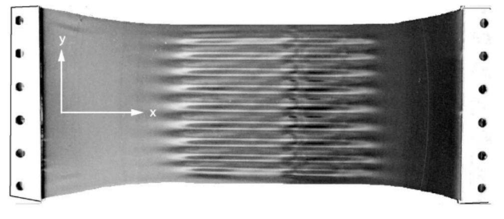
[1] L. Pocivavsek et al. (2008). "Stress and fold localization in thin elastic membranes". In: *Science*. ISSN: 00368075. DOI: 10.1126/science.1154069

Wrinkling Basics - Energy Balance

Foundation vs Bending

Folding in a floating sheet [1]

Tension vs Bending



Wrinkling in a tensioned sheet [2]

[1] L. Pocivavsek et al. (2008). "Stress and fold localization in thin elastic membranes". In: *Science*. ISSN: 00368075. DOI: 10.1126/science.1154069

[2] E. Cerda et al. (2002). "Wrinkling of an elastic sheet under tension". In: *Nature* 419.6907, pp. 579–580. ISSN: 0028-0836. DOI: 10.1038/419579b

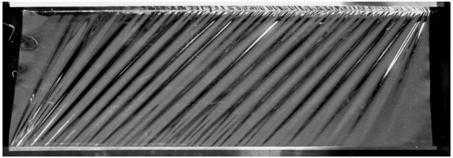
Wrinkling Basics - Buckling Instabilities

[3] W. Wong et al. (2006). "Wrinkled membranes I: experiments". In: *Journal of Mechanics of Materials and Structures* 1.1, pp. 3–25. ISSN: 1559-3959. DOI: 10.2140/jomms.2006.1.3

Wrinkling Basics - Buckling Instabilities

©U.S. Climate Resilience Toolkit

$\equiv$



Buckling/Wrinkling of a sheared sheet[3]

[3] W. Wong et al. (2006). "Wrinkled membranes I: experiments". In: *Journal of Mechanics of Materials and Structures* 1.1, pp. 3–25. ISSN: 1559-3959. DOI: 10.2140/jomms.2006.1.3

Outline

Isogeometric Shell Analysis - Isogeometric Analysis

Isogeometric Analysis[4] bridges Computer Aided Design (CAD) and Finite Element Analysis (FEA) by employing the same ('*iso*') functions for the representation of the geometry and solutions.

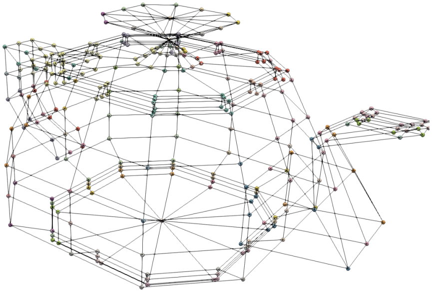
$$\mathcal{S}(\xi,\eta) = \sum_{i,j} \mathrm{C}_{ij}\varphi_{ij}(\xi,\eta)$$

"Let no man ignorant of geometry enter here." [5]

[4] T. Hughes et al. (2005). "Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement". In: *Comput. Methods Appl. Mech. Eng.* 194.39-41, pp. 4135–4195

[5] J. A. Cottrell et al. (2009). *Isogeometric Analysis: Toward Integration of CAD and FEA*. Wiley, pp. 1–335. ISBN: 9780470748732. DOI: 10.1002/9780470749081

Isogeometric Shell Analysis - Isogeometric Analysis



Multi-Patch surface $\mathcal{S} = \bigcup_i \mathcal{S}_i$

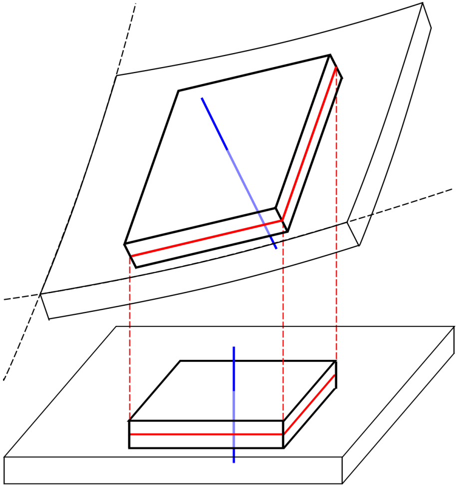
Control Net, $C_k = \bigcup_{ij} C_{ij}$ of patches $\mathcal{S}_k$, $k = 0, 1, \ldots$

Isogeometric Shell Analysis - Isogeometric Kirchhoff-Love Shell

Definition (Kirchhoff Hypothesis)

In the Kirchhoff hypothesis, the following is assumed

1. Straight lines perpendicular to the mid-plane before deformation remain perpendicular to the mid-surface after deformation;
2. The normals rotate such that they remain perpendicular to the mid-surface after deformation;
3. The normals do not experience elongation (i.e. they are inextensible).



Isogeometric Shell Analysis - Variational Formulation

Variational formulation[6]

Find $\mathrm{u} \in \mathcal{V}$ s.t.

$$\mathcal{N}(\mathrm{u},\phi) = \int_\Omega \boldsymbol{n}(\mathrm{u}) : \boldsymbol{\varepsilon}'(\mathrm{u},\phi) + \boldsymbol{m}(\mathrm{u}) : \boldsymbol{\kappa}'(\mathrm{u},\phi)\,\mathrm{d}\Omega - \int_\Omega \mathrm{f}(\mathrm{u})\cdot\phi\,\mathrm{d}\Omega = 0 \quad \forall\phi\in\mathcal{V}$$

With $\boldsymbol{n} = n^{\alpha\beta}\,\mathring{\mathrm{g}}_\alpha \otimes \mathring{\mathrm{g}}_\beta$ and $\boldsymbol{m} = m^{\alpha\beta}\,\mathring{\mathrm{g}}_\alpha \otimes \mathring{\mathrm{g}}_\beta$ the membrane force and bending moment tensors,

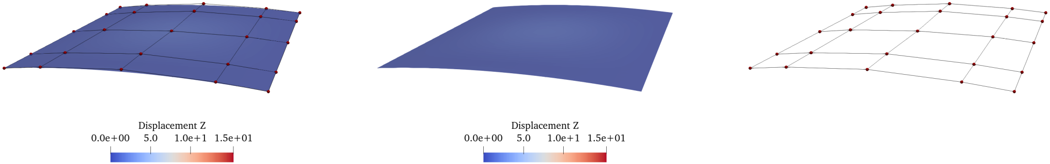
$$n^{\alpha\beta} = \int_{[-t/2,t/2]} S^{\alpha\beta}\,\mathrm{d}\theta_3 \qquad\qquad m^{\alpha\beta} = \int_{[-t/2,t/2]} \theta_3 S^{\alpha\beta}\,\mathrm{d}\theta_3$$

With $S^{\alpha\beta}$ the stress tensor depending on the (un)deformed geometry $\mathring{\mathcal{S}}$ ($\mathcal{S}$), containing linear, hyperelastic [7] or other constitutive models.

[6] J. Kiendl et al. (2009). "Isogeometric shell analysis with Kirchhoff–Love elements". In: *Computer Methods in Applied Mechanics and Engineering* 198.49-52, pp. 3902–3914. ISSN: 0045-7825. DOI: 10.1016/J.CMA.2009.08.013

[7] J. Kiendl et al. (2015). "Isogeometric collocation methods for the Reissner–Mindlin plate problem". In: *Computer Methods in Applied Mechanics and Engineering* 284, pp. 489–507. ISSN: 0045-7825. DOI: 10.1016/J.CMA.2014.09.011; **Verhelst, H.M.** et al. (2021). "Stretch-Based Hyperelastic Material Formulations for Isogeometric Kirchhoff–Love Shells with Application to Wrinkling". In: *Computer-Aided Design* 139, p. 103075. ISSN: 00104485. DOI: 10.1016/j.cad.2021.103075

Isogeometric Shell Analysis - Isogeometric Kirchhoff-Love Shell



Collapse of a shallow roof. Left: solution + control points; middle: solution; right: control points

Isogeometric Shell Analysis - Trade-off

- \+ Geometric exactness
- \+ k-refinement (i.e. arbitrary smoothness across elements)
- +/- No rotational degrees of freedom; but requires C^1 smooth (i.e. quadratic) basis
- \- Higher smoothness $\implies$ bigger band-with in (stiffness) matrix

Isogeometric Shell Analysis - Trade-off

- \+ Geometric exactness
- \+ *k*-refinement (i.e. arbitrary smoothness across elements)
- +/- No rotational degrees of freedom; but requires C^1 smooth (i.e. quadratic) basis
- \- Higher smoothness $\Longrightarrow$ bigger band-with in (stiffness) matrix

Isogeometric Shell Analysis - Trade-off

- \+ Geometric exactness
- \+ *k*-refinement (i.e. arbitrary smoothness across elements)
- +/- No rotational degrees of freedom; but requires C^1 smooth (i.e. quadratic) basis
- \- Higher smoothness $\Longrightarrow$ bigger band-with in (stiffness) matrix

Isogeometric Shell Analysis - Trade-off

+ Geometric exactness
+ k-refinement (i.e. arbitrary smoothness across elements)

+/- No rotational degrees of freedom; but requires C^1 smooth (i.e. quadratic) basis

- Higher smoothness $\implies$ bigger band-with in (stiffness) matrix

Isogeometric Shell Analysis - Smooth Multi-Patch Analysis

Requirements

The curvature term in the variational formulation requires C_1-smoothness everywhere; also on patch boundaries.

- Mortar-based methods [8]
- Weak coupling methods (e.g. penalty, Nitsche) [9]
- **Strong coupling methods**, e.g. D-Patch, Almost-C_1, Approximate-C_1, Exact-C_1 [10]

---

[8] Horger et al. 2019; Dornisch et al. 2015

[9] Guo et al. 2019; Coox et al. 2017; Liu et al. 2019; Herrema et al. 2019; "Coupling of non-conforming trimmed isogeometric Kirchhoff-Love shells via a projected super-penalty approach"; Leonetti et al. 2020; Zhao et al. 2022

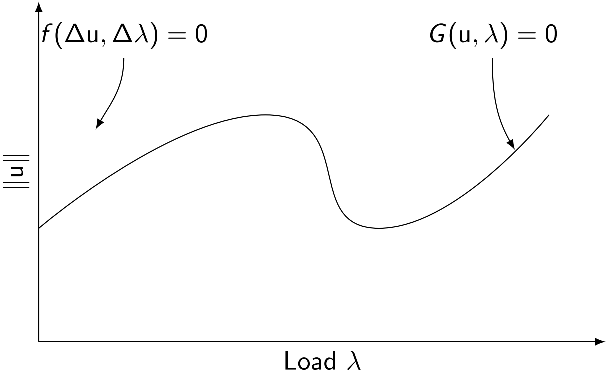
[10] **Verhelst, H.M.** et al. In preparation(b); Toshniwal et al. 2017; Takacs et al. 2022; Weinmüller et al. 2021; Farahat et al. In preparation

Outline

Path-Following Methods - Basics

Arc-Length Methods

Find $G(\mathrm{u}, \lambda) = 0$ by steps constrained by equation $f(\Delta \mathrm{u}, \Delta\lambda)$

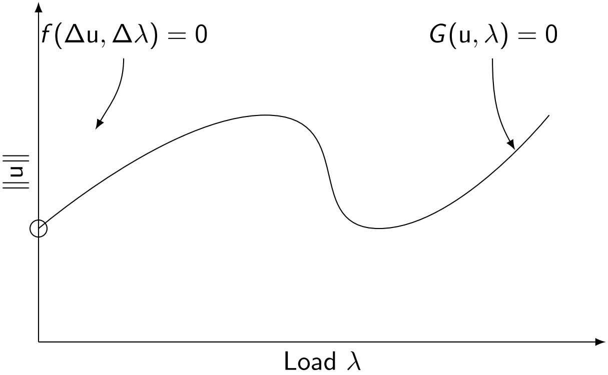


Path-Following Methods - Basics

Arc-Length Methods

Find $G(\mathrm{u},\lambda) = 0$ by steps constrained by equation $f(\Delta\mathrm{u},\Delta\lambda)$

1. Start at (u_i,λ_i), $G(\mathrm{u}_i,\lambda_i) = 0$
2. Find $\Delta(\mathrm{u}_i,\Delta\lambda_i)$ s.t. $G(\mathrm{u}_i,\lambda_i) = 0$ and $f(\Delta\mathrm{u},\Delta\lambda) = 0$
3. $(\mathrm{u}_{i+1},\lambda_{i+1}) = (\mathrm{u}_i,\lambda_i) + \Delta(\mathrm{u}_i,\Delta\lambda_i)$
4. Set $i = i + 1$ and go to 1

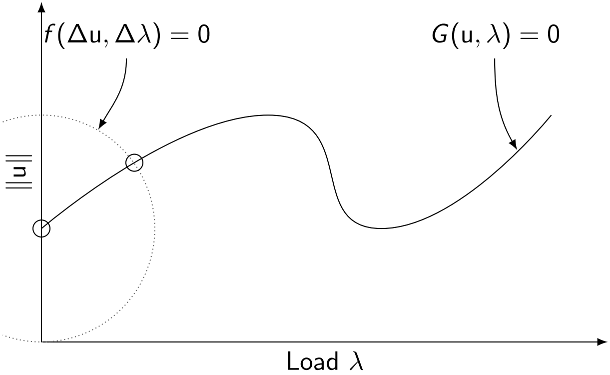


Path-Following Methods - Basics

Arc-Length Methods

Find $G(\mathrm{u}, \lambda) = 0$ by steps constrained by equation $f(\Delta\mathrm{u}, \Delta\lambda)$

1. Start at $(\mathrm{u}_i, \lambda_i)$, $G(\mathrm{u}_i, \lambda_i) = 0$
2. Find $\Delta(\mathrm{u}_i, \Delta\lambda_i)$ s.t. $G(\mathrm{u}_i, \lambda_i) = 0$ and $f(\Delta\mathrm{u}, \Delta\lambda) = 0$
3. $(\mathrm{u}_{i+1}, \lambda_{i+1}) = (\mathrm{u}_i, \lambda_i) + \Delta(\mathrm{u}_i, \Delta\lambda_i)$
4. Set $i = i + 1$ and go to 1

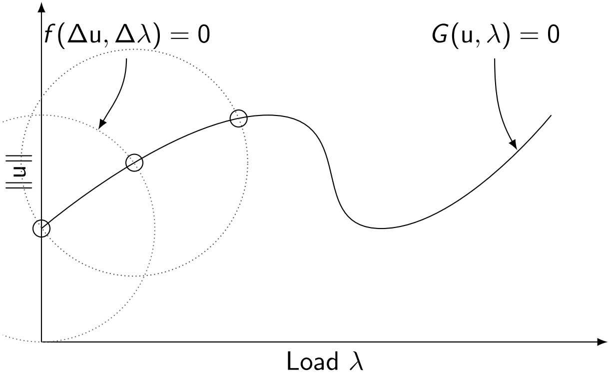


Path-Following Methods - Basics

Arc-Length Methods

Find $G(\mathrm{u}, \lambda) = 0$ by steps constrained by equation $f(\Delta\mathrm{u}, \Delta\lambda)$

1. Start at $(\mathrm{u}_i, \lambda_i)$, $G(\mathrm{u}_i, \lambda_i) = 0$
2. Find $\Delta(\mathrm{u}_i, \Delta\lambda_i)$ s.t. $G(\mathrm{u}_i, \lambda_i) = 0$ and $f(\Delta\mathrm{u}, \Delta\lambda) = 0$
3. $(\mathrm{u}_{i+1}, \lambda_{i+1}) = (\mathrm{u}_i, \lambda_i) + \Delta(\mathrm{u}_i, \Delta\lambda_i)$
4. Set $i = i + 1$ and go to 1

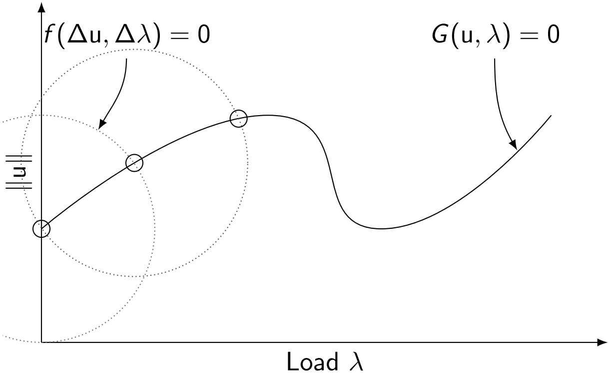


Path-Following Methods - Basics

Arc-Length Methods

Find $G(\mathrm{u},\lambda)=0$ by steps constrained by equation $f(\Delta\mathrm{u},\Delta\lambda)$

1. Start at (u_i,λ_i), $G(\mathrm{u}_i,\lambda_i)=0$
2. Find $\Delta(\mathrm{u}_i,\Delta\lambda_i)$ s.t. $G(\mathrm{u}_i,\lambda_i)=0$ and $f(\Delta\mathrm{u},\Delta\lambda)=0$
3. $(\mathrm{u}_{i+1},\lambda_{i+1}) = (\mathrm{u}_i,\lambda_i)+\Delta(\mathrm{u}_i,\Delta\lambda_i)$
4. Set $i=i+1$ and go to 1

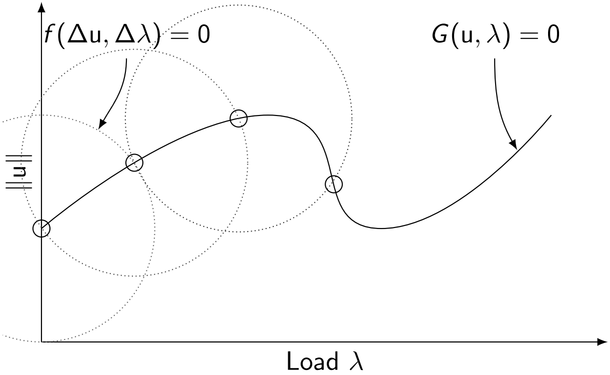


Path-Following Methods - Basics

Arc-Length Methods

Find $G(\mathrm{u}, \lambda) = 0$ by steps constrained by equation $f(\Delta\mathrm{u}, \Delta\lambda)$

1. Start at $(\mathrm{u}_i, \lambda_i)$, $G(\mathrm{u}_i, \lambda_i) = 0$
2. Find $\Delta(\mathrm{u}_i, \Delta\lambda_i)$ s.t. $G(\mathrm{u}_i, \lambda_i) = 0$ and $f(\Delta\mathrm{u}, \Delta\lambda) = 0$
3. $(\mathrm{u}_{i+1}, \lambda_{i+1}) = (\mathrm{u}_i, \lambda_i) + \Delta(\mathrm{u}_i, \Delta\lambda_i)$
4. Set $i = i + 1$ and go to 1

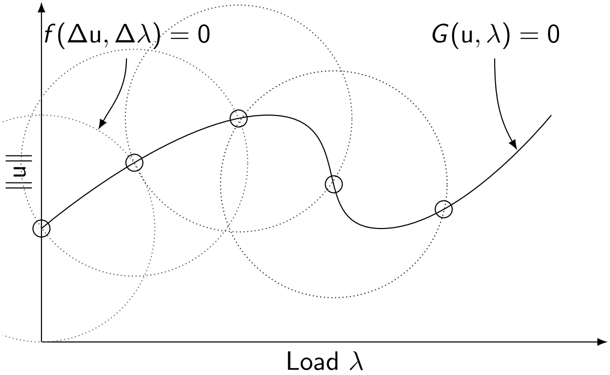


Path-Following Methods - Basics

Arc-Length Methods

Find $G(\mathrm{u},\lambda)=0$ by steps constrained by equation $f(\Delta\mathrm{u},\Delta\lambda)$

1. Start at (u_i,λ_i), $G(\mathrm{u}_i,\lambda_i)=0$
2. Find $\Delta(\mathrm{u}_i,\Delta\lambda_i)$ s.t. $G(\mathrm{u}_i,\lambda_i)=0$ and $f(\Delta\mathrm{u},\Delta\lambda)=0$
3. $(\mathrm{u}_{i+1},\lambda_{i+1}) = (\mathrm{u}_i,\lambda_i)+\Delta(\mathrm{u}_i,\Delta\lambda_i)$
4. Set $i=i+1$ and go to 1

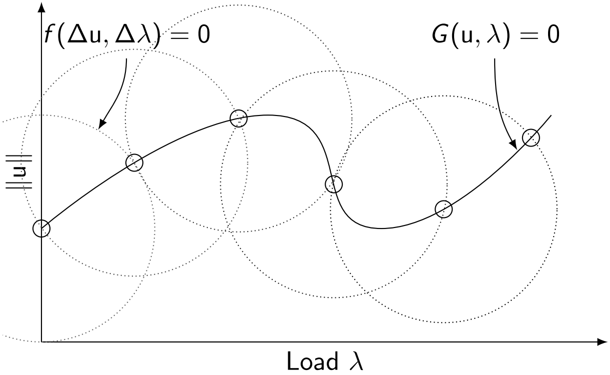


Path-Following Methods - Basics

Arc-Length Methods

Find $G(\mathrm{u}, \lambda) = 0$ by steps constrained by equation $f(\Delta \mathrm{u}, \Delta\lambda)$

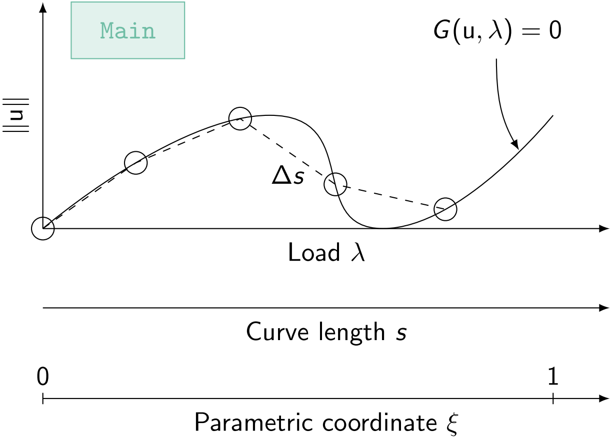
1. Start at $(\mathrm{u}_i, \lambda_i)$, $G(\mathrm{u}_i, \lambda_i) = 0$
2. Find $\Delta(\mathrm{u}_i, \Delta\lambda_i)$ s.t. $G(\mathrm{u}_i, \lambda_i) = 0$ and $f(\Delta \mathrm{u}, \Delta\lambda) = 0$
3. $(\mathrm{u}_{i+1}, \lambda_{i+1}) = (\mathrm{u}_i, \lambda_i) + \Delta(\mathrm{u}_i, \Delta\lambda_i)$
4. Set $i = i + 1$ and go to 1



Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM [11]

1. Compute coarse approximation of the path



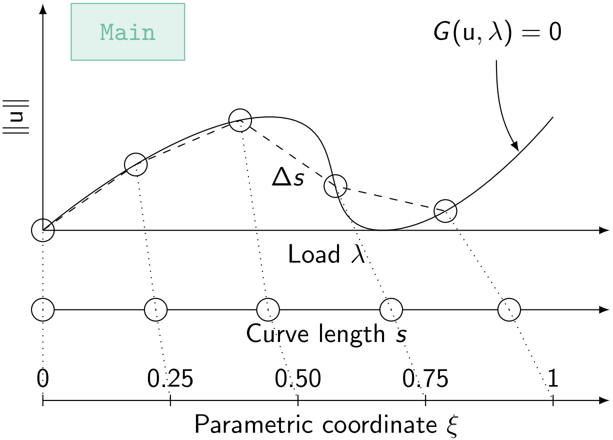
● = Main; ● = Worker;

[11] Verhelst, H.M. et al. (In preparation[a]). "An Adaptive Parallel Arc-Length Method".

Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM [11]

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain $\xi \in [0, 1]$



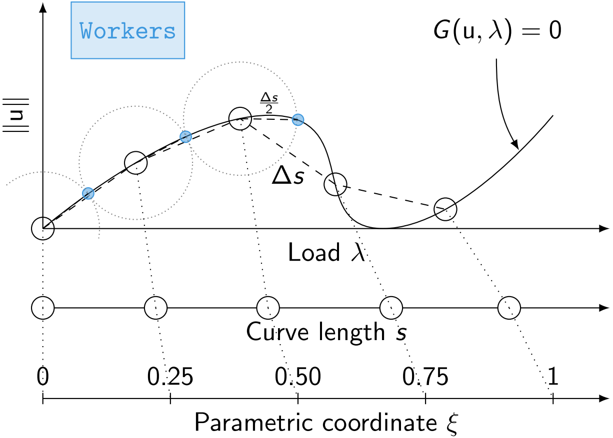
● = Main; ● = Worker;

[11] Verhelst, H.M. et al. (In preparation[a]). "An Adaptive Parallel Arc-Length Method".

Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM [11]

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain $\xi \in [0, 1]$
3. Perform fine steps in each coarse subdomain



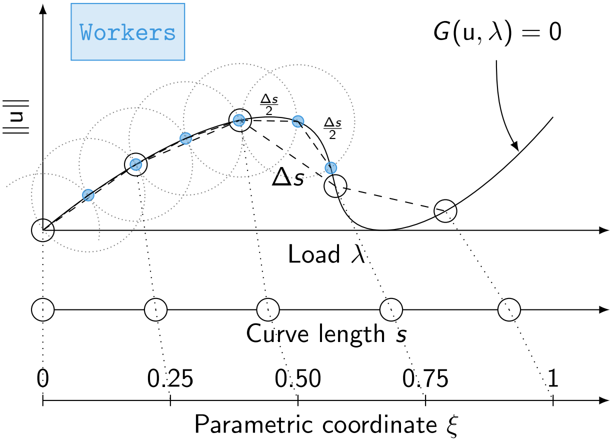
● = Main; ● = Worker;

[11] Verhelst, H.M. et al. (In preparation[a]). "An Adaptive Parallel Arc-Length Method".

Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM [11]

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain $\xi \in [0, 1]$
3. Perform fine steps in each coarse subdomain



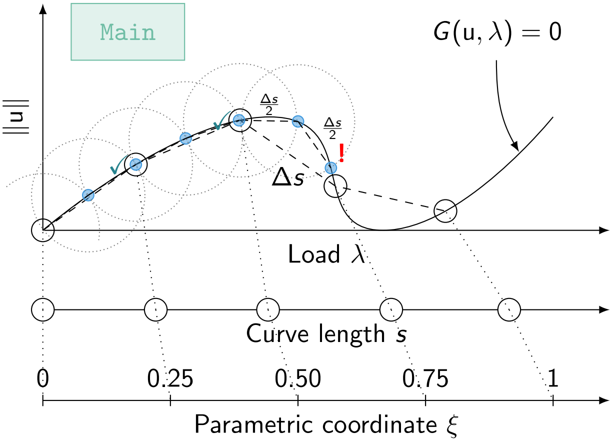
● = Main; ● = Worker;

[11] Verhelst, H.M. et al. (In preparation[a]). "An Adaptive Parallel Arc-Length Method".

Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM [11]

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain $\xi \in [0, 1]$
3. Perform fine steps in each coarse subdomain
4. Check the distance between the fine and coarse approximation (using $f(\Delta \mathrm{u}, \Delta\lambda)$)



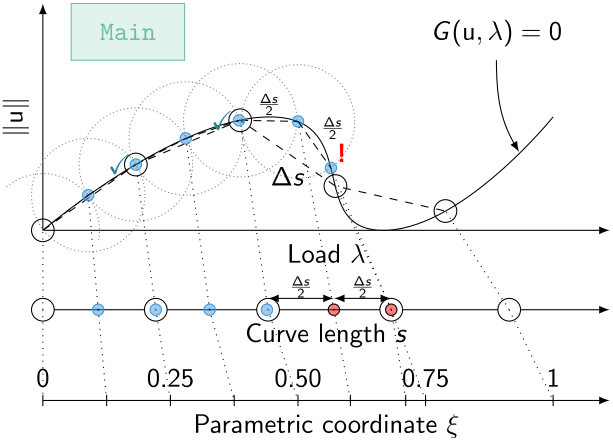
● = Main; ● = Worker;

[11] Verhelst, H.M. et al. (In preparation[a]). "An Adaptive Parallel Arc-Length Method".

Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM [11]

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain $\xi \in [0,1]$
3. Perform fine steps in each coarse subdomain
4. Check the distance between the fine and coarse approximation (using $f(\Delta \mathrm{u}, \Delta\lambda)$)
5. **if** $\varepsilon <$TOL: Rescale and mark



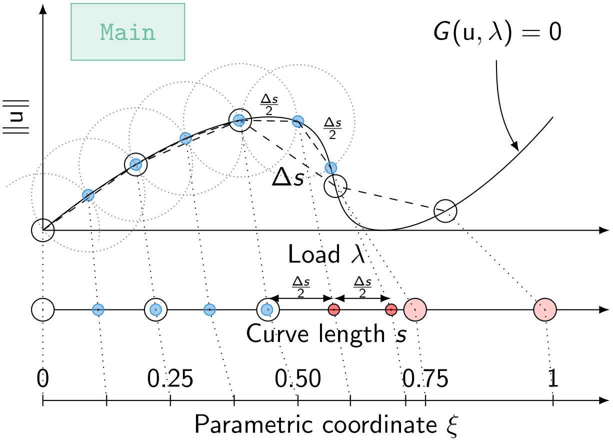
● = Main; ● = Worker;

[11] Verhelst, H.M. et al. (In preparation[a]). "An Adaptive Parallel Arc-Length Method".

Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM [11]

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain $\xi \in [0, 1]$
3. Perform fine steps in each coarse subdomain
4. Check the distance between the fine and coarse approximation (using $f(\Delta u, \Delta\lambda)$)
5. **if** $\varepsilon <$TOL: Rescale and mark
6. Go to 2 for marked segments



● = Main; ● = Worker;

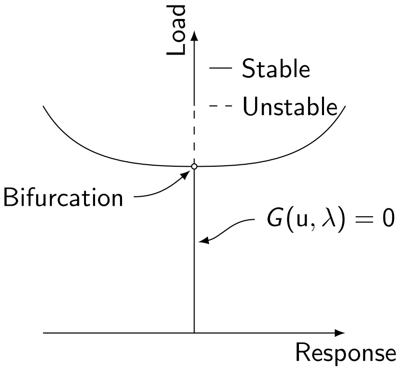
[11] Verhelst, H.M. et al. (In preparation[a]). "An Adaptive Parallel Arc-Length Method".

Path-Following Methods - Bifurcations

Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

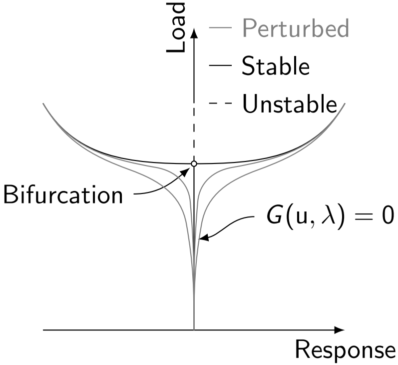
Path-Following Methods - Bifurcations



Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

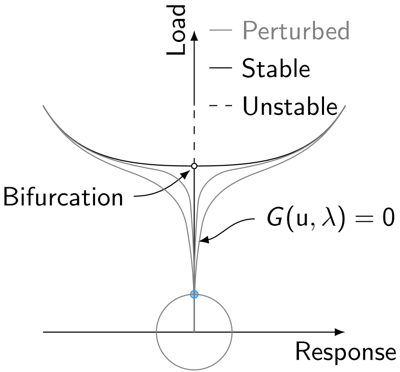
Path-Following Methods - Bifurcations



Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

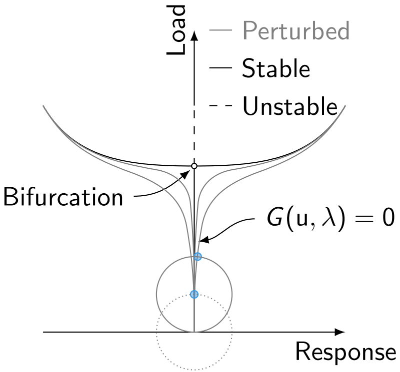
Path-Following Methods - Bifurcations



Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

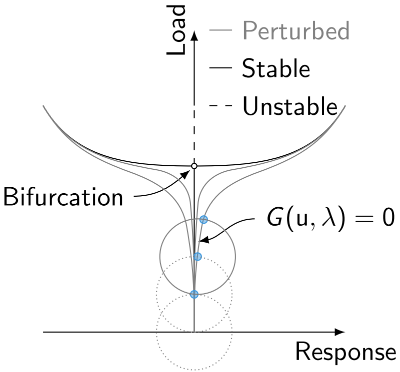
Path-Following Methods - Bifurcations



Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

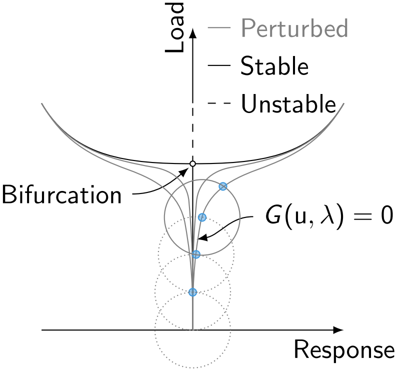
Path-Following Methods - Bifurcations



Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

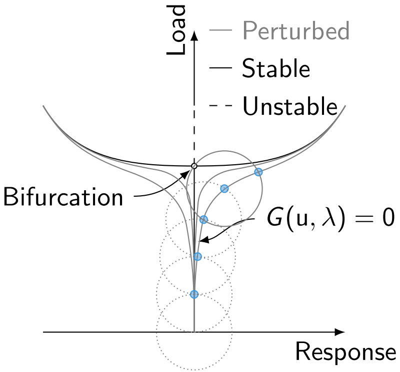
Path-Following Methods - Bifurcations



Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

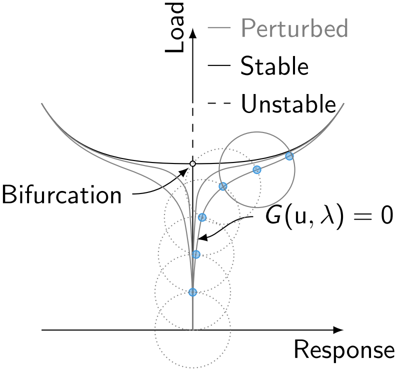
Path-Following Methods - Bifurcations



Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

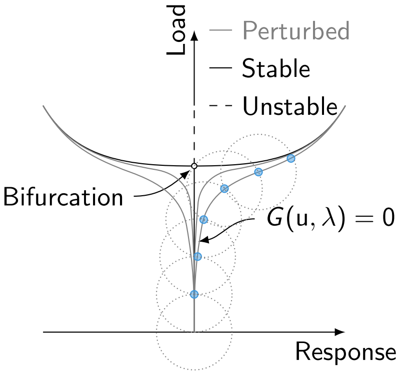
Path-Following Methods - Bifurcations



Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

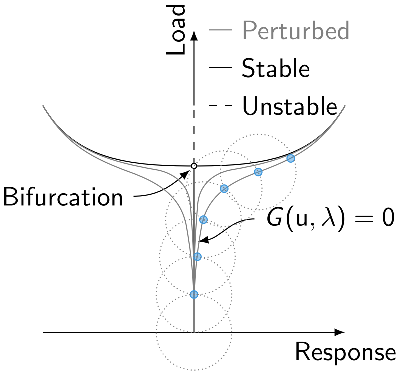
Path-Following Methods - Bifurcations



Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

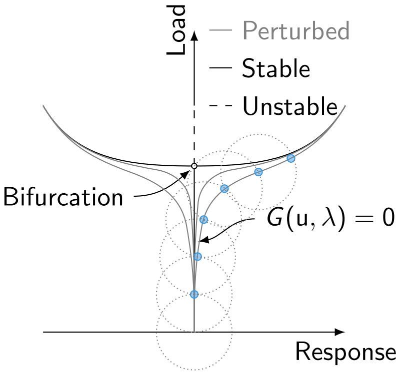
Path-Following Methods - Bifurcations



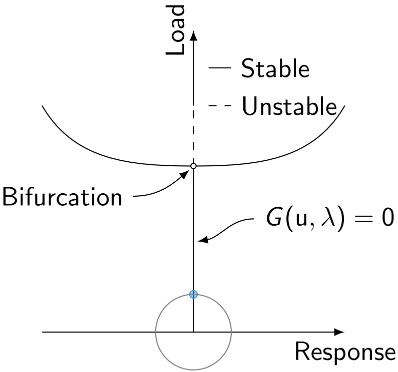
Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations

Path-Following Methods - Bifurcations

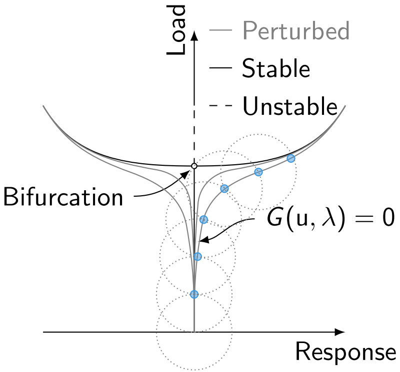


Effect of initial perturbations to avoid passing bifurcation points.

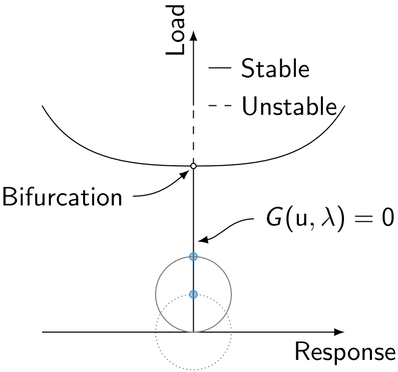


Arc-Length Method without perturbations

Path-Following Methods - Bifurcations

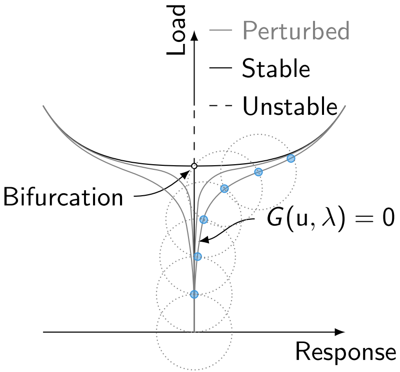


Effect of initial perturbations to avoid passing bifurcation points.

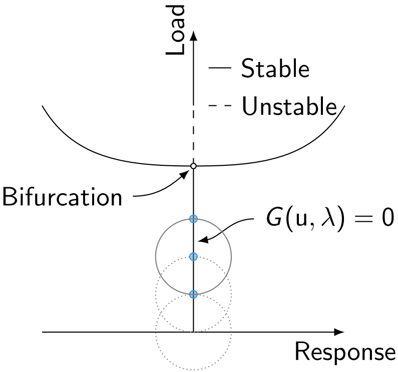


Arc-Length Method without perturbations

Path-Following Methods - Bifurcations

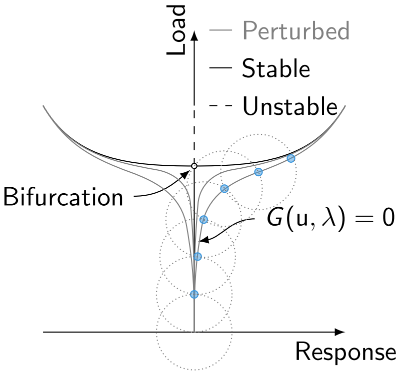


Effect of initial perturbations to avoid passing bifurcation points.

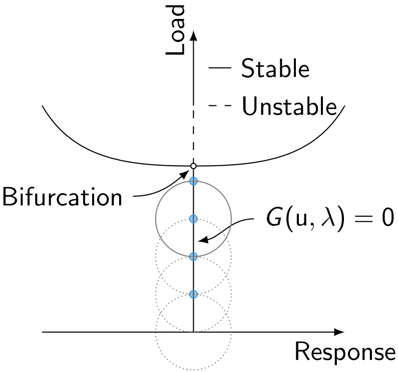


Arc-Length Method without perturbations

Path-Following Methods - Bifurcations

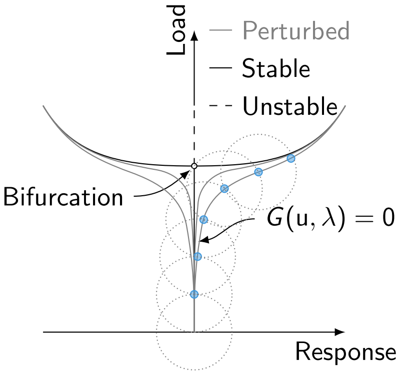


Effect of initial perturbations to avoid passing bifurcation points.

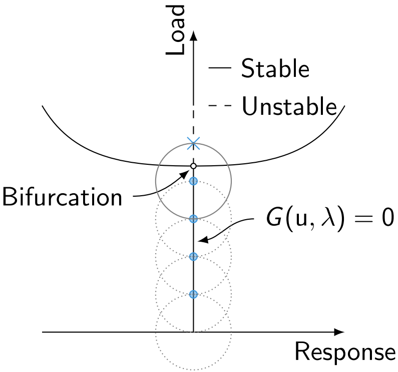


Arc-Length Method without perturbations

Path-Following Methods - Bifurcations

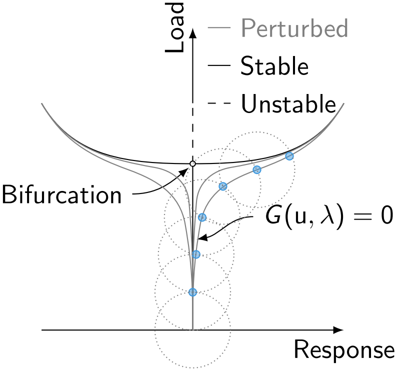


Effect of initial perturbations to avoid passing bifurcation points.

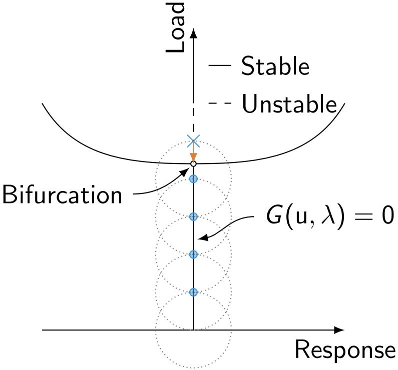


Arc-Length Method without perturbations

Path-Following Methods - Bifurcations

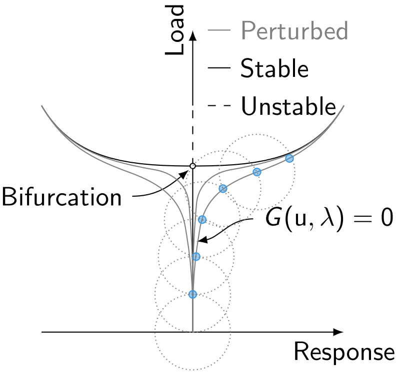


Effect of initial perturbations to avoid passing bifurcation points.

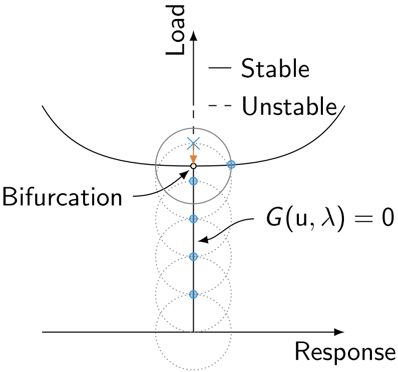


Arc-Length Method without perturbations

Path-Following Methods - Bifurcations

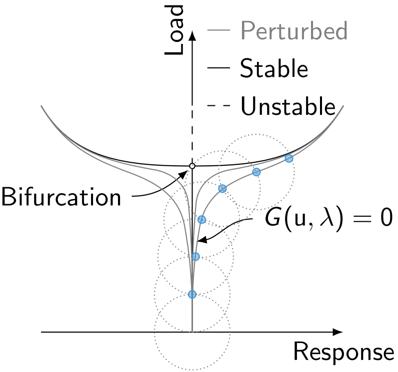


Effect of initial perturbations to avoid passing bifurcation points.

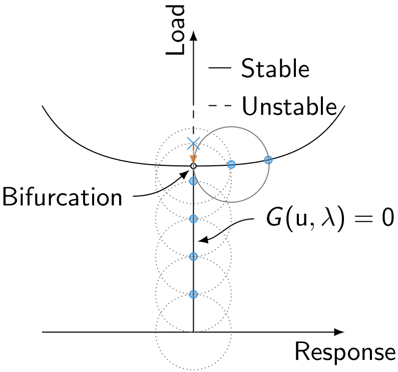


Arc-Length Method without perturbations

Path-Following Methods - Bifurcations

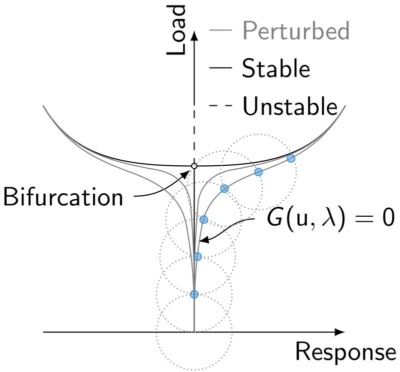


Effect of initial perturbations to avoid passing bifurcation points.

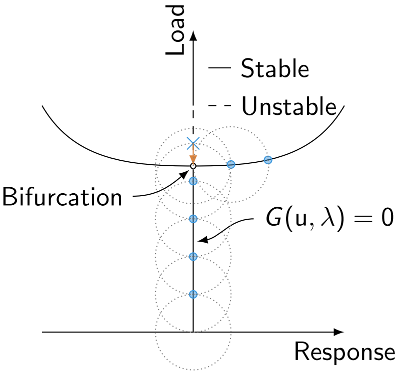


Arc-Length Method without perturbations

Path-Following Methods - Bifurcations



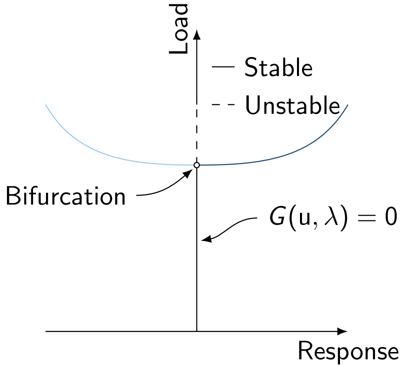
Effect of initial perturbations to avoid passing bifurcation points.



Arc-Length Method without perturbations

Path-Following Methods - Across-Branch Parallelism (Exploration)

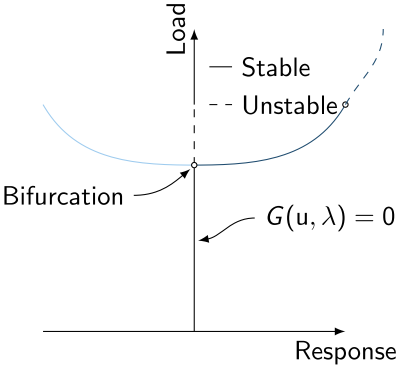
Given Bifurcation detection, in addition to bifurcation tangents, multiple branches can be explored in parallel[12]



[12] J. Thies et al. (2021). "Towards Scalable Automatic Exploration of Bifurcation Diagrams for Large-Scale Applications". In: *Lecture Notes in Computational Science and Engineering* 139, pp. 981–989. ISSN: 21977100. DOI: 10.1007/978-3-030-55874-1_97/FIGURES/5

Path-Following Methods - Across-Branch Parallelism (Exploration)

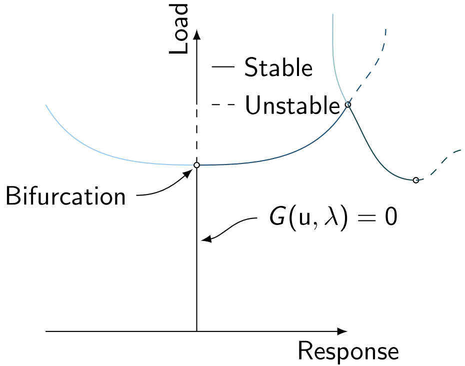
Given Bifurcation detection, in addition to bifurcation tangents, multiple branches can be explored in parallel[12]



[12] J. Thies et al. (2021). "Towards Scalable Automatic Exploration of Bifurcation Diagrams for Large-Scale Applications". In: *Lecture Notes in Computational Science and Engineering* 139, pp. 981–989. ISSN: 21977100. DOI: 10.1007/978-3-030-55874-1_97/FIGURES/5

Path-Following Methods - Across-Branch Parallelism (Exploration)

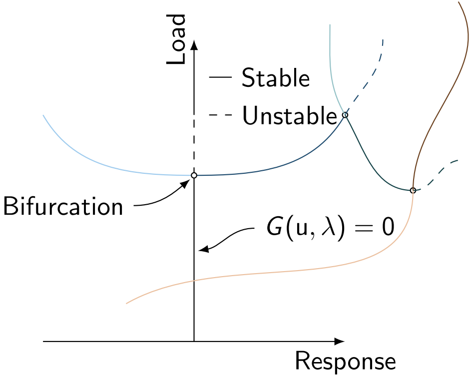
Given Bifurcation detection, in addition to bifurcation tangents, multiple branches can be explored in parallel[12]



[12] J. Thies et al. (2021). "Towards Scalable Automatic Exploration of Bifurcation Diagrams for Large-Scale Applications". In: *Lecture Notes in Computational Science and Engineering* 139, pp. 981–989. ISSN: 21977100. DOI: 10.1007/978-3-030-55874-1_97/FIGURES/5

Path-Following Methods - Across-Branch Parallelism (Exploration)

Given Bifurcation detection, in addition to bifurcation tangents, multiple branches can be explored in parallel[12]

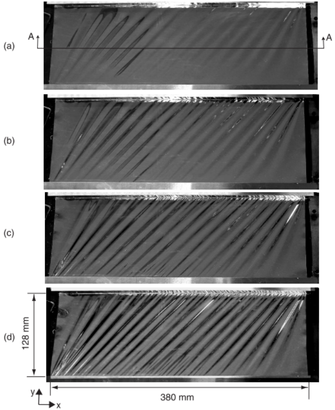


[12] J. Thies et al. (2021). “Towards Scalable Automatic Exploration of Bifurcation Diagrams for Large-Scale Applications”. In: *Lecture Notes in Computational Science and Engineering* 139, pp. 981–989. ISSN: 21977100. DOI: 10.1007/978-3-030-55874-1_97/FIGURES/5

Outline

Results - Shear Wrinkling

Numerical results



Experimental results[13].

[13] W. Wong et al. (2006). "Wrinkled membranes I: experiments". In: *Journal of Mechanics of Materials and Structures* 1.1, pp. 3–25. ISSN: 1559-3959. DOI: 10.2140/jomms.2006.1.3

Outline

Conclusions and Future Work - Conclusions

- Wrinkling occurs when in membrane-like[14] offshore solar platforms; hence (isogeometric) shell modelling is used
  - Isogeometric Kirchhoff-Love shells require C^1-smoothness across patch interfaces, but...
  - Provide geometric exactness, arbitrary smoothness in a rotation-free setting.
- Wrinkling is a buckling instability, hence continuation methods are important
  - Continuation without a priori perturbations provides robustness, and...
  - Enables automatic exploration for subsequent bifurcations
  - Paralellization in continuation methods can be performed across and within branches

[14] With finite bending stiffness because of the energy balance!

Conclusions and Future Work - Conclusions

- Wrinkling occurs when in membrane-like[14] offshore solar platforms; hence (isogeometric) shell modelling is used
  - Isogeometric Kirchhoff-Love shells require C^1-smoothness across patch interfaces, but...
  - Provide geometric exactness, arbitrary smoothness in a rotation-free setting.
- Wrinkling is a buckling instability, hence continuation methods are important
  - Continuation without a priori perturbations provides robustness, and...
  - Enables automatic exploration for subsequent bifurcations
  - Paralellization in continuation methods can be performed across and within branches

[14] With finite bending stiffness because of the energy balance!

Conclusions and Future Work - Conclusions

- Wrinkling occurs when in membrane-like[14] offshore solar platforms; hence (isogeometric) shell modelling is used
  - Isogeometric Kirchhoff-Love shells require C^1-smoothness across patch interfaces, but...
  - Provide geometric exactness, arbitrary smoothness in a rotation-free setting.
- Wrinkling is a buckling instability, hence continuation methods are important
  - Continuation without a priori perturbations provides robustness, and...
  - Enables automatic exploration for subsequent bifurcations
  - Parallellization in continuation methods can be performed across and within branches

[14] With finite bending stiffness because of the energy balance!

Conclusions and Future Work - Conclusions

- Wrinkling occurs when in membrane-like[14] offshore solar platforms; hence (isogeometric) shell modelling is used
  - Isogeometric Kirchhoff-Love shells require C^1-smoothness across patch interfaces, but...
  - Provide geometric exactness, arbitrary smoothness in a rotation-free setting.
- Wrinkling is a buckling instability, hence continuation methods are important
  - Continuation without a priori perturbations provides robustness, and...
  - Enables automatic exploration for subsequent bifurcations
  - Paralellization in continuation methods can be performed across and within branches

[14] With finite bending stiffness because of the energy balance!

Conclusions and Future Work - Conclusions

- Wrinkling occurs when in membrane-like[14] offshore solar platforms; hence (isogeometric) shell modelling is used
  - Isogeometric Kirchhoff-Love shells require C^1-smoothness across patch interfaces, but...
  - Provide geometric exactness, arbitrary smoothness in a rotation-free setting.
- Wrinkling is a buckling instability, hence continuation methods are important
  - Continuation without a priori perturbations provides robustness, and...
  - Enables automatic exploration for subsequent bifurcations
  - Paralellization in continuation methods can be performed across and within branches

[14] With finite bending stiffness because of the energy balance!

Conclusions and Future Work - Conclusions

- Wrinkling occurs when in membrane-like[14] offshore solar platforms; hence (isogeometric) shell modelling is used
  - Isogeometric Kirchhoff-Love shells require C^1-smoothness across patch interfaces, but...
  - Provide geometric exactness, arbitrary smoothness in a rotation-free setting.
- Wrinkling is a buckling instability, hence continuation methods are important
  - Continuation without a priori perturbations provides robustness, and...
  - Enables automatic exploration for subsequent bifurcations
  - Paralellization in continuation methods can be performed across and within branches

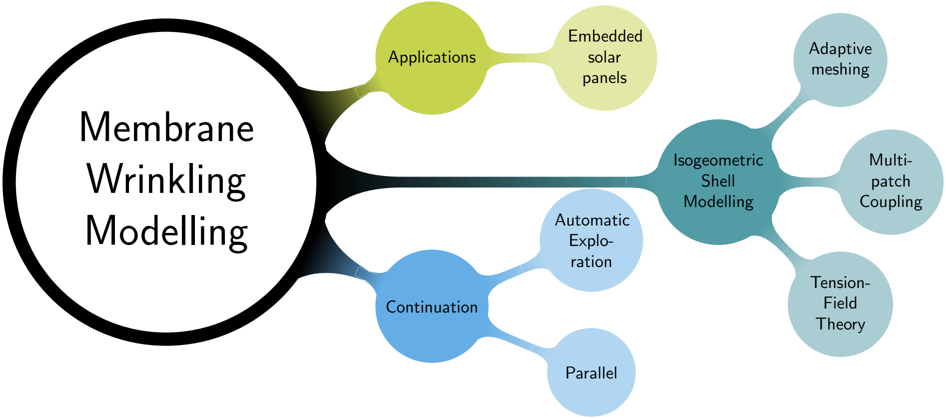
[14] With finite bending stiffness because of the energy balance!

Conclusions and Future Work - Conclusions

- Wrinkling occurs when in membrane-like[14] offshore solar platforms; hence (isogeometric) shell modelling is used
  - Isogeometric Kirchhoff-Love shells require C^1-smoothness across patch interfaces, but...
  - Provide geometric exactness, arbitrary smoothness in a rotation-free setting.
- Wrinkling is a buckling instability, hence continuation methods are important
  - Continuation without a priori perturbations provides robustness, and...
  - Enables automatic exploration for subsequent bifurcations
  - Paralellization in continuation methods can be performed across and within branches

[14] With finite bending stiffness because of the energy balance!

Conclusions and Future Work - Future Work



Conclusions and Future Work - Is this open-source?

Geometry + Simulation Modules
(G+SMo, pronounced gismo)

YES!

- Isogeometric analysis core `github.com/gismo/gismo`
- Thin shell analysis `github.com/gismo/gsKLShell`
- Arc-length methods `github.com/gismo/gsStructuralAnalysis`

Almost

- Smooth patch coupling `github.com/gismo/gsUnstructuredSplines`, expected Summer 2022
- Automatic exploration `PyNCT`, expected fall 2022

Conclusions and Future Work - Is this open-source?

Geometry + Simulation Modules
(G+SMo, pronounced gismo)

YES!

Isogeometric analysis core `github.com/gismo/gismo`

Thin shell analysis `github.com/gismo/gsKLShell`

Arc-length methods `github.com/gismo/gsStructuralAnalysis`

Almost

Smooth patch coupling `github.com/gismo/gsUnstructuredSplines`, expected Summer 2022

Automatic exploration PyNCT, expected fall 2022

Conclusions and Future Work - Is this open-source?

Geometry + Simulation Modules
(G+SMo, pronounced gismo)

YES!

Isogeometric analysis core `github.com/gismo/gismo`

Thin shell analysis `github.com/gismo/gsKLShell`

Arc-length methods `github.com/gismo/gsStructuralAnalysis`

Almost

Smooth patch coupling `github.com/gismo/gsUnstructuredSplines`, expected Summer 2022

Automatic exploration `PyNCT`, expected fall 2022

Conclusions and Future Work - Is this open-source?

Geometry + Simulation Modules
(G+SMo, pronounced gismo)

YES!

Isogeometric analysis core `github.com/gismo/gismo`

Thin shell analysis `github.com/gismo/gsKLShell`

Arc-length methods `github.com/gismo/gsStructuralAnalysis`

Almost

Smooth patch coupling `github.com/gismo/gsUnstructuredSplines`, expected Summer 2022

Automatic exploration `PyNCT`, expected fall 2022

Conclusions and Future Work - Is this open-source?

Geometry + Simulation Modules
(G+SMo, pronounced gismo)

YES!

Isogeometric analysis core `github.com/gismo/gismo`

Thin shell analysis `github.com/gismo/gsKLShell`

Arc-length methods `github.com/gismo/gsStructuralAnalysis`

Almost

Smooth patch coupling `github.com/gismo/gsUnstructuredSplines`, expected Summer 2022

Automatic exploration `PyNCT`, expected fall 2022

Conclusions and Future Work - Is this open-source?

Geometry + Simulation Modules
(G+SMo, pronounced gismo)

YES!

Isogeometric analysis core `github.com/gismo/gismo`

Thin shell analysis `github.com/gismo/gsKLShell`

Arc-length methods `github.com/gismo/gsStructuralAnalysis`

Almost

Smooth patch coupling
`github.com/gismo/gsUnstructuredSplines`,
expected Summer 2022

Automatic exploration `PyNCT`, expected fall 2022

Conclusions and Future Work - Is this open-source?

Geometry + Simulation Modules
(G+SMo, pronounced gismo)

YES!

Isogeometric analysis core `github.com/gismo/gismo`

Thin shell analysis `github.com/gismo/gsKLShell`

Arc-length methods `github.com/gismo/gsStructuralAnalysis`

Almost

Smooth patch coupling
`github.com/gismo/gsUnstructuredSplines`,
expected Summer 2022

Automatic exploration `PyNCT`, expected fall 2022

On the modeling of wrinkling instabilities using isogeometric shell analysis

Hugo Verhelst [1,2,@] Matthias Möller [1] Henk den Besten [2]

[1] Numerical Analysis, Faculty of Electrical Engineering, Mathematics and Computer Science (EEMCS), Delft University of Technology
[2] Maritime and Transport Technology, Faculty of Mechanical, Maritime and Materials Engineering (3mE), Delft University of Technology

[@] h.m.verhelst@tudelft.nl

06-05-2022

References I

Cerda, E., K. Ravi-Chandar, and L. Mahadevan (2002). "Wrinkling of an elastic sheet under tension". In: *Nature* 419.6907, pp. 579–580. ISSN: 0028-0836. DOI: 10.1038/419579b.

Coox, L., F. Maurin, F. Greco, E. Deckers, D. Vandepitte, and W. Desmet (2017). "A flexible approach for coupling NURBS patches in rotationless isogeometric analysis of Kirchhoff–Love shells". In: *Computer Methods in Applied Mechanics and Engineering* 325, pp. 505–531. ISSN: 0045-7825. DOI: 10.1016/J.CMA.2017.07.022.

Coradello, L., J. Kiendl, and A. Buffa. "Coupling of non-conforming trimmed isogeometric Kirchhoff-Love shells via a projected super-penalty approach". In: (). arXiv: 2104.13804v1.

Cottrell, J. A., T. J. Hughes, and Y. Bazilevs (2009). *Isogeometric Analysis: Toward Integration of CAD and FEA*. Wiley, pp. 1–335. ISBN: 9780470748732. DOI: 10.1002/9780470749081.

Dornisch, W., G. Vitucci, and S. Klinkel (2015). "The weak substitution method - an application of the mortar method for patch coupling in NURBS-based isogeometric analysis". In: *International Journal for Numerical Methods in Engineering* 103.3, pp. 205–234. ISSN: 00295981. DOI: 10.1002/nme.4918.

Farahat, A., **Verhelst, H.M.**, M. Kapl, and J. Kiendl (In preparation). "An Exact Analysis-Suitble C^1 Construction for Isogeometric Shell Analysis".

References II

Guo, Y., H. Do, and M. Ruess (2019). "Isogeometric stability analysis of thin shells: From simple geometries to engineering models". In: *International Journal for Numerical Methods in Engineering*. ISSN: 00295981. DOI: 10.1002/nme.6020.

Herrema, A. J., E. L. Johnson, D. Proserpio, M. C. Wu, J. Kiendl, and M.-C. Hsu (2019). "Penalty coupling of non-matching isogeometric Kirchhoff–Love shell patches with application to composite wind turbine blades". In: *Computer Methods in Applied Mechanics and Engineering* 346, pp. 810–840. ISSN: 0045-7825. DOI: 10.1016/J.CMA.2018.08.038.

Horger, T., A. Reali, B. Wohlmuth, and L. Wunderlich (2019). "A hybrid isogeometric approach on multi-patches with applications to Kirchhoff plates and eigenvalue problems". In: *Computer Methods in Applied Mechanics and Engineering* 348, pp. 396–408. ISSN: 00457825. DOI: 10.1016/j.cma.2018.12.038.

Hughes, T., J. Cottrell, and Y. Bazilevs (2005). "Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement". In: *Comput. Methods Appl. Mech. Eng.* 194.39-41, pp. 4135–4195.

Kiendl, J., F. Auricchio, L. Beirão da Veiga, C. Lovadina, and A. Reali (2015). "Isogeometric collocation methods for the Reissner–Mindlin plate problem". In: *Computer Methods in Applied Mechanics and Engineering* 284, pp. 489–507. ISSN: 0045-7825. DOI: 10.1016/J.CMA.2014.09.011.

References III

Kiendl, J., K.-U. Bletzinger, J. Linhard, and R. Wüchner (2009). "Isogeometric shell analysis with Kirchhoff–Love elements". In: *Computer Methods in Applied Mechanics and Engineering* 198.49-52, pp. 3902–3914. ISSN: 0045-7825. DOI: 10.1016/J.CMA.2009.08.013.

Leonetti, L., F. S. Liguori, D. Magisano, J. Kiendl, A. Reali, and G. Garcea (2020). "A robust penalty coupling of non-matching isogeometric Kirchhoff–Love shell patches in large deformations". In: *Computer Methods in Applied Mechanics and Engineering* 371, p. 113289. ISSN: 0045-7825. DOI: https://doi.org/10.1016/j.cma.2020.113289.

Liu, Z., J. Cheng, M. Yang, P. Yuan, C. Qiu, W. Gao, and J. Tan (2019). "Isogeometric analysis of large thin shell structures based on weak coupling of substructures with unstructured T-splines patches". In: *Advances in Engineering Software* 135, p. 102692. ISSN: 18735339. DOI: 10.1016/j.advengsoft.2019.102692.

Patterson, B. D., F. Mo, A. Borgschulte, M. Hillestad, F. Joos, T. Kristiansen, S. Sunde, and J. A. van Bokhoven (2019). "Renewable CO2 recycling and synthetic fuel production in a marine environment". In: *Proceedings of the National Academy of Sciences of the United States of America* 116.25, pp. 12212–12219. ISSN: 10916490. DOI: 10.1073/pnas.1902335116.

Pocivavsek, L., R. Dellsy, A. Kern, S. Johnson, B. Lin, K. Y. C. Lee, and E. Cerda (2008). "Stress and fold localization in thin elastic membranes". In: *Science*. ISSN: 00368075. DOI: 10.1126/science.1154069.

References IV

Takacs, T. and D. Toshniwal (2022). "Almost-C^1 splines: Biquadratic splines on unstructured quadrilateral meshes and their application to fourth order problems". In: DOI: 10.48550/arxiv.2201.11491. arXiv: 2201.11491.

Verhelst, H.M., M. Möller, J. Den Besten, A. Mantzaflaris, and M. Kaminski (2021). "Stretch-Based Hyperelastic Material Formulations for Isogeometric Kirchhoff–Love Shells with Application to Wrinkling". In: *Computer-Aided Design* 139, p. 103075. ISSN: 00104485. DOI: 10.1016/j.cad.2021.103075.

Verhelst, H.M., M. Möller, and J. Den Besten (In preparation[a]). "An Adaptive Parallel Arc-Length Method".

Verhelst, H.M., P. Weinmüller, T. Takacs, and D. Toshniwal (In preparation[b]). "A Qualitative Comparison of C^1 Smooth Multipatch Constructions for Isogeometric Analysis".

Thies, J., M. Wouters, R. S. Hennig, and W. Vanroose (2021). "Towards Scalable Automatic Exploration of Bifurcation Diagrams for Large-Scale Applications". In: *Lecture Notes in Computational Science and Engineering* 139, pp. 981–989. ISSN: 21977100. DOI: 10.1007/978-3-030-55874-1_97/FIGURES/5.

Toshniwal, D., H. Speleers, and T. J. Hughes (2017). "Smooth cubic spline spaces on unstructured quadrilateral meshes with particular emphasis on extraordinary points: Geometric design and isogeometric analysis considerations". In: *Computer Methods in Applied Mechanics and Engineering* 327, pp. 411–458. ISSN: 00457825. DOI: 10.1016/j.cma.2017.06.008.

References V

Weinmüller, P. and T. Takacs (2021). "Construction of approximate C1 bases for isogeometric analysis on two-patch domains". In: *Computer Methods in Applied Mechanics and Engineering* 385, p. 114017. ISSN: 0045-7825. DOI: 10.1016/J.CMA.2021.114017. arXiv: 2103.02980.

Wong, W. and S. Pellegrino (2006). "Wrinkled membranes I: experiments". In: *Journal of Mechanics of Materials and Structures* 1.1, pp. 3–25. ISSN: 1559-3959. DOI: 10.2140/jomms.2006.1.3.

Zhao, H., X. Liu, A. H. Fletcher, R. Xiang, J. T. Hwang, and D. Kamensky (2022). "An open-source framework for coupling non-matching isogeometric shells with application to aerospace structures". In: *Computers & Mathematics with Applications* 111, pp. 109–123. ISSN: 0898-1221. DOI: 10.1016/J.CAMWA.2022.02.007.