On the modeling of wrinkling instabilities using isogeometric shell analysis

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Outline

1 Introduction
2 Wrinkling Basics
3 Isogeometric Shell Analysis
4 Path-Following Methods
5 Results
6 Conclusions and Future Work
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Introduction - Aim

Develop a robust and efficient strategy for the modelling of wrinkles using isogeometric analysis
Introduction - Aim

Membrane Wrinkling Modelling

- Continuation
- Parallel
- Isogeometric Shell Modelling
  - Automatic Exploration
  - Adaptive meshing
- Multi-patch Coupling
- Tension-Field Theory
- Applications
  - Embedded solar panels
- Verhelst et al. Wrinkling instabilities with IGA shells 06-05-2022
Introduction - Aim

Membrane Wrinkling Modelling

- Applications
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- Automatic exploration
- Parallel
- Continuation

Isogeometric Shell Modelling

- Adaptive meshing
- Multi-patch Coupling
- Tension-Field Theory

Applications

- Embeddedsolar panels

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Wrinkling instabilities with IGA shells
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5 Results

6 Conclusions and Future Work
Wrinkling Basics - Energy Balance

Foundation vs Bending

Folding in a floating sheet

Wrinkling in a tensioned sheet

1L. Pocivavsek et al. (2008). “Stress and fold localization in thin elastic membranes”. In: Science. ISSN: 00368075. DOI: 10.1126/science.1154069

2E. Cerda et al. (2002). “Wrinkling of an elastic sheet under tension”. In: Nature. 419.6907, pp. 579–580. ISSN: 0028-0836. DOI: 10.1038/419579b
Wrinkling Basics - Energy Balance

**Foundation vs Bending**

Folding in a floating sheet

**Tension vs Bending**

Wrinkling in a tensioned sheet

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Wrinkling Basics - Buckling Instabilities

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Wrinkling Basics - Buckling Instabilities

Buckling/Wrinkling of a sheared sheet

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Isogeometric Analysis bridges Computer Aided Design (CAD) and Finite Element Analysis (FEA) by employing the same ('iso') functions for the representation of the geometry and solutions.

\[ S(\xi, \eta) = \sum_{i,j} C_{ij} \varphi_{ij}(\xi, \eta) \]

"Let no man ignorant of geometry enter here."
Multi-Patch surface $S = \bigcup_i S_i$

Control Net, $C_k = \bigcup_{ij} C_{ij}$ of patches $S_k$, $k = 0, 1, \ldots$
Isogeometric Shell Analysis - Isogeometric Kirchhoff-Love Shell

**Definition (Kirchhoff Hypothesis)**

In the **Kirchhoff hypothesis**, the following is assumed:

1. Straight lines perpendicular to the mid-plane before deformation remain perpendicular to the mid-surface after deformation;
2. The normals rotate such that they remain perpendicular to the mid-surface after deformation;
3. The normals do not experience elongation (i.e. they are inextensible).
Isogeometric Shell Analysis - Variational Formulation

Variational formulation\(^6\)

Find \(u \in \mathcal{V}\) s.t.

\[
\mathcal{N}(u, \phi) = \int_{\Omega} n(u) : \varepsilon'(u, \phi) + m(u) : \kappa'(u, \phi) \, d\Omega - \int_{\Omega} f(u) \cdot \phi \, d\Omega = 0 \quad \forall \phi \in \mathcal{V}
\]

With \(n = n^\alpha{}^\beta \mathbf{g}_\alpha \otimes \mathbf{g}_\beta\) and \(m = m^\alpha{}^\beta \mathbf{g}_\alpha \otimes \mathbf{g}_\beta\) the membrane force and bending moment tensors,

\[
n^\alpha{}^\beta = \int_{[-t/2, t/2]} S^\alpha{}^\beta \, d\theta_3 \quad \quad m^\alpha{}^\beta = \int_{[-t/2, t/2]} \theta_3 S^\alpha{}^\beta \, d\theta_3
\]

With \(S^\alpha{}^\beta\) the stress tensor depending on the (un)deformed geometry \(\mathbf{g}' (\mathbf{S})\), containing linear, hyperelastic\(^7\) or other constitutive models.

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Isogeometric Shell Analysis - Isogeometric Kirchhoff-Love Shell

Collapse of a shallow roof. Left: solution + control points; middle: solution; right: control points
Isogeometric Shell Analysis - Trade-off

+ Geometric exactness
  + $k$-refinement (i.e. arbitrary smoothness across elements)
  +/- No rotational degrees of freedom; but requires $C^1$ smooth (i.e. quadratic) basis
  - Higher smoothness $\implies$ bigger band-with in (stiffness) matrix
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Isogeometric Shell Analysis - Smooth Multi-Patch Analysis

Requirements

The curvature term in the variational formulation requires $C_1$-smoothness everywhere; also on patch boundaries.

- Mortar-based methods \(^8\)
- Weak coupling methods (e.g. penalty, Nitsche) \(^9\)
- **Strong coupling methods**, e.g. D-Patch, Almost-$C_1$, Approximate-$C_1$, Exact-$C_1$ \(^{10}\)

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\(^8\) Horger et al. 2019; Dornisch et al. 2015


\(^{10}\) Verhelst, H.M. et al. In preparation(b); Toshniwal et al. 2017; Takacs et al. 2022; Weinmüller et al. 2021; Farahat et al. In preparation
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**Arc-Length Methods**

Find $G(u, \lambda) = 0$ by steps constrained by equation $f(\Delta u, \Delta \lambda)$

1. Start at $(u_i, \lambda_i)$, $G(u_i, \lambda_i) = 0$
2. Find $\Delta(u_i, \Delta \lambda_i)$ s.t. $G(u_i, \lambda_i) = 0$ and $f(\Delta u, \Delta \lambda) = 0$
3. $(u_{i+1}, \lambda_{i+1}) = (u_i, \lambda_i) + \Delta(u_i, \Delta \lambda_i)$
4. Set $i = i + 1$ and go to 1
Path-Following Methods - Basics

Arc-Length Methods

Find \( G(u, \lambda) = 0 \) by steps constrained by equation \( f(\Delta u, \Delta \lambda) \)

1. Start at \((u_i, \lambda_i), G(u_i, \lambda_i) = 0\)
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\[ f(\Delta u, \Delta \lambda) = 0 \]
\[ G(u, \lambda) = 0 \]

Load \( \lambda \)
**Path-Following Methods - Basics**

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![Diagram](image.png)

$f(\Delta u, \Delta \lambda) = 0$

$G(u, \lambda) = 0$

Load $\lambda$
Path-Following Methods - Basics

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![Diagram of path-following methods with steps 1 to 4 explained visually]
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**Path-Following Methods - Basics**

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![Graph showing the path-following methods process](image)
Arc-Length Methods

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Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM

1. Compute coarse approximation of the path

- = Main;  = Worker;

\[ G(u, \lambda) = 0 \]

\[ \|u\| \]

\[ \Delta s \]

\[ s \]

\[ \lambda \]

\[ \xi \in [0, 1] \]

\[ G(u, \lambda) = 0 \]

\[ \Delta s \]

\[ 0 \]

\[ 1 \]

\[ \xi \]

\[ \|u\| \]


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Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM \textsuperscript{11}

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain $\xi \in [0, 1]$

\textsuperscript{11}Verhelst, H.M. et al. (In preparation\textsuperscript{[a]}). "An Adaptive Parallel Arc-Length Method".

---

\[ G(u, \lambda) = 0 \]

\[ \Delta s \]

\[ \text{Main} \]

\[ \text{Load } \lambda \]

\[ \text{Curve length } s \]

0 0.25 0.50 0.75 1

Parametric coordinate $\xi$

\[ \|n\| \]

\[ \text{Main} \]

\[ \text{Workers} \]

\[ \bullet \text{ = Main; } \circ \text{ = Worker; } \]

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Parallel Adaptive ALM\(^{11}\)

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain \(\xi \in [0, 1]\)
3. Perform fine steps in each coarse subdomain

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\(G(u, \lambda) = 0\)

\(\|u\|\)

\(\xi\)

\(\Delta s\)

\(\Delta \xi\)

\(\lambda\)

\(\xi\)

Main; \(\bullet\) = Worker;
Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM ¹¹

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Path-Following Methods - Within-Branch Parallelization

Parallel Adaptive ALM

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain $\xi \in [0, 1]$
3. Perform fine steps in each coarse subdomain
4. Check the distance between the fine and coarse approximation (using $f(\Delta u, \Delta \lambda)$)

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Verhelst, H.M. et al. (In preparation[a]). "An Adaptive Parallel Arc-Length Method".
**Path-Following Methods - Within-Branch Parallelization**

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4. Check the distance between the fine and coarse approximation (using $f(\Delta u, \Delta \lambda)$)
5. **if** $\varepsilon < \text{TOL}$: Rescale and mark

$L = \text{Main}; \bullet = \text{Worker};$

---

Parallel Adaptive ALM

1. Compute coarse approximation of the path
2. Map the coarse approximation on a parametric domain $\xi \in [0, 1]$
3. Perform fine steps in each coarse subdomain
4. Check the distance between the fine and coarse approximation (using $f(\Delta u, \Delta \lambda)$)
5. If $\varepsilon < \text{TOL}$: Rescale and mark
6. Go to 2 for marked segments

$\bullet = \text{Main}; \bullet = \text{Worker};$

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Path-Following Methods - Bifurcations

\[ G(u, \lambda) = 0 \]

Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations
Path-Following Methods - Bifurcations

Effect of initial perturbations to avoid passing bifurcation points.
Path-Following Methods - Bifurcations

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Path-Following Methods - Bifurcations

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Path-Following Methods - Bifurcations

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Arc-Length Method without perturbations

$G(u, \lambda) = 0$
Path-Following Methods - Bifurcations

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Arc-Length Method without perturbations
Path-Following Methods - Bifurcations

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Path-Following Methods - Bifurcations

Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations
Path-Following Methods - Bifurcations

Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations
**Path-Following Methods - Bifurcations**

Effect of initial perturbations to avoid passing bifurcation points. Arc-Length Method without perturbations.
Path-Following Methods - Bifurcations

Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations
Path-Following Methods - Bifurcations

Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations
Path-Following Methods - Bifurcations

$G(u, \lambda) = 0$

Effect of initial perturbations to avoid passing bifurcation points.

Arc-Length Method without perturbations
Given Bifurcation detection, in addition to bifurcation tangents, multiple branches can be explored in parallel\textsuperscript{12}

\[ G(u, \lambda) = 0 \]
Given **Bifurcation detection**, in addition to bifurcation tangents, multiple branches can be explored in parallel\(^\text{12}\).

\[^\text{12}\text{ J. Thies et al. (2021). “Towards Scalable Automatic Exploration of Bifurcation Diagrams for Large-Scale Applications”. In: Lecture Notes in Computational Science and Engineering 139, pp. 981–989. ISSN: 21977100. DOI: 10.1007/978-3-030-55874-1_97/FIGURES/5}\]
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Path-Following Methods - Across-Branch Parallelism (Exploration)

Given Bifurcation detection, in addition to bifurcation tangents, multiple branches can be explored in parallel\textsuperscript{12}
Outline

1. Introduction
2. Wrinkling Basics
3. Isogeometric Shell Analysis
4. Path-Following Methods
5. Results
6. Conclusions and Future Work
Results - Shear Wrinkling

Numerical results

Experimental results\textsuperscript{13}.

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Conclusions and Future Work - Conclusions

- **Wrinkling** occurs when in membrane-like\(^{14}\) offshore solar platforms; hence (isogeometric) shell modelling is used
  - Isogeometric Kirchhoff-Love shells require \(C^1\)-smoothness across patch interfaces, but...
  - Provide geometric exactness, arbitrary smoothness in a rotation-free setting.
- **Wrinkling** is a buckling instability, hence continuation methods are important
  - Continuation without a priori perturbations provides robustness, and...
  - Enables automatic exploration for subsequent bifurcations
  - Parallelization in continuation methods can be performed across and within branches

\(^{14}\)With finite bending stiffness because of the energy balance!
Conclusions and Future Work - Conclusions

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Conclusions and Future Work - Future Work

Membrane
Wrinkling
Modelling

Applications
Embedded
colar
panels
Isogeometric
Shell
Modelling
Automatic
Exploration
Isogeometric
Shell
Modelling
Multi-patch
Coupling
Tension-
Field
Theory
Adaptive
meshing

Continuation
Parallel

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Conclusions and Future Work - Is this open-source?

Geometry + Simulation Modules (G+SMo, pronounced gismo)

YES!

- Isogeometric analysis core: github.com/gismo/gismo
- Thin shell analysis: github.com/gismo/gsKLShell
- Arc-length methods: github.com/gismo/gsStructuralAnalysis
- Almost smooth patch coupling: github.com/gismo/gsUnstructuredSplines, expected Summer 2022
- Automatic exploration: PyNCT, expected Fall 2022
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**Almost**

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Geometry + Simulation Modules (G+SMo, pronounced gismo)

YES!

- Isogeometric analysis core github.com/gismo/gismo
- Thin shell analysis github.com/gismo/gsKLShell
- Arc-length methods github.com/gismo/gsStructuralAnalysis
- Almost Smooth patch coupling github.com/gismo/gsUnstructuredSplines, expected Summer 2022
- Automatic exploration PyNCT, expected fall 2022
On the modeling of wrinkling instabilities using isogeometric shell analysis

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References II


References IV


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Verhelst, H.M., P. Weinmüller, T. Takacs, and D. Toshniwal (In preparation[b]). “A Qualitative Comparison of $C^1$ Smooth Multipatch Constructions for Isogeometric Analysis”.


