Pore scale modeling of porous media flow including moving boundaries

Carina Bringedal
Iuliu Sorin Pop, Manuela Bastidas, Sohely Sharmin

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KNOWLEDGE IN ACTION

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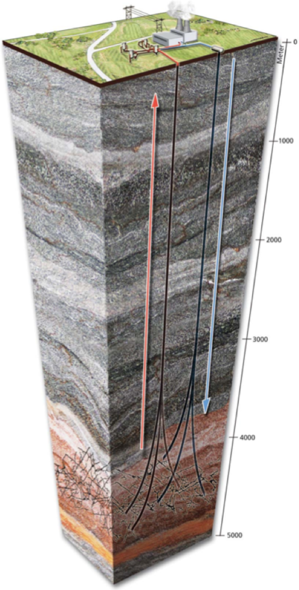
Motivation

- Geothermal energy extraction.
- Inject cold fluid into subsurface.
  Extract warm water.
- Model flow rates and heat transport.



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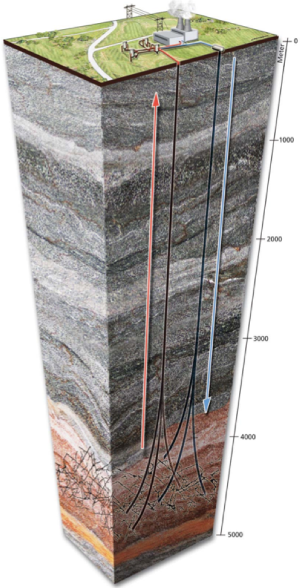

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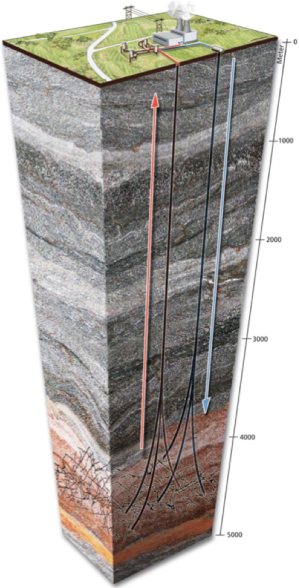
- Mineral precipitation and dissolution can affect the fluid flow and heat transport through the reservoir.



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- Inject cold fluid into subsurface. Extract warm water.
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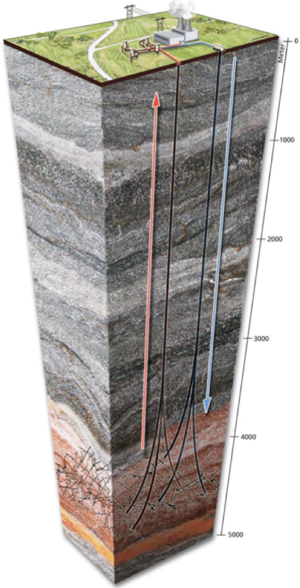
- Chemical reactions?
- Mineral precipitation and dissolution can affect the fluid flow and heat transport through the reservoir.
- If two fluids present: Their flow will affect each other.



Darcy scale description

- Flow at reservoir scale usually modeled using Darcy's law:

$$\mathbf{v} = -\frac{\mathbf{K}}{\mu}\nabla P$$

- Permeability $\mathbf{K}$ gives effective description of the average flow through an elementary representative volume.

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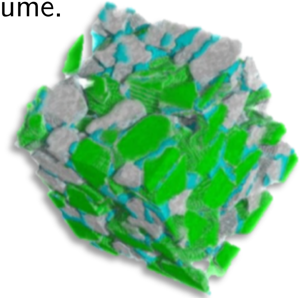
- Not necessarily interested in detailed behavior, but in large scale effect.



V. Cnudde, UGent

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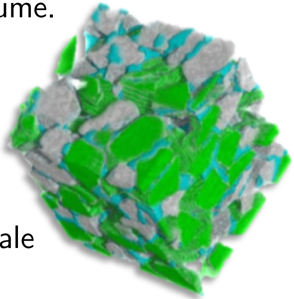
- Start with pore scale description $\rightarrow$ Upscale to Darcy scale.



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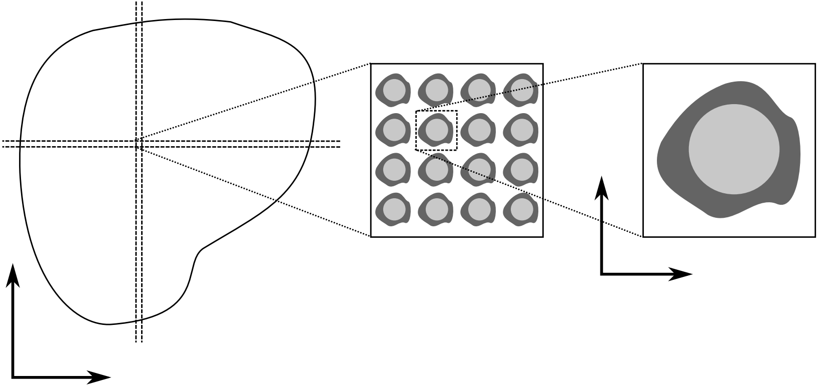
Darcy scale vs pore scale



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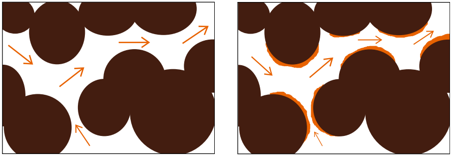
Overview

1. Pore scale formulation
   - Mineral precipitation and dissolution
   - Two-phase flow
   - Evolving domains
2. Upscaling using homogenization
   - Asymptotic expansions
   - Thin strip
   - Periodic porous medium
3. Effective behavior on Darcy scale

Pore scale formulation

- Pore scale processes can affect fluid flow through the reservoir.
- Mineral precipitation and dissolution.



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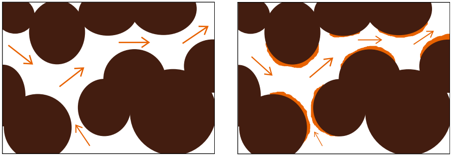
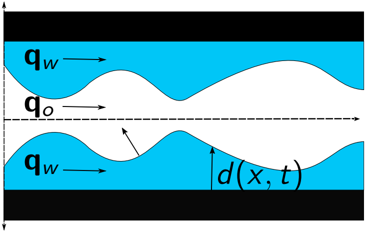


- Two-phase/Unsaturated flow.



Pore scale formulation

- Formulate pore scale description including a moving boundary.
  - → Separate explicitly between grain and void space.
  - → Conservation equations in evolving domains.

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Pore scale formulation

- Formulate pore scale description including a moving boundary.
  - $\rightarrow$ Separate explicitly between grain and void space.
  - $\rightarrow$ Conservation equations in evolving domains.
- How does the permeability and effective heat transfer change when a chemical reaction affects the pore geometry?

Pore scale formulation

- Formulate pore scale description including a moving boundary.
  - → Separate explicitly between grain and void space.
  - → Conservation equations in evolving domains.
- How does the permeability and effective heat transfer change when a chemical reaction affects the pore geometry?
- How does the (relative) permeability change when two fluids are present?

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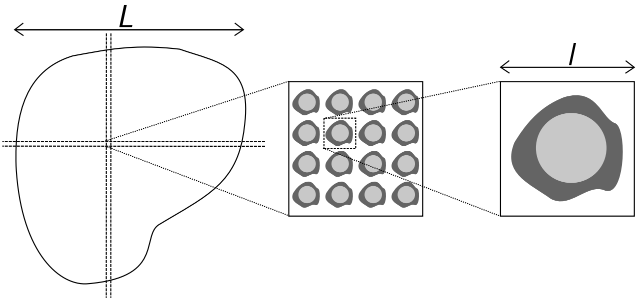
Pore scale formulation



- Consider domain with perforations.
- Scale separator: $\varepsilon = l/L$.
- Fluid domain: $\Omega^\varepsilon(t)$.
- Grain domain: $G^\varepsilon(t)$.
- Interface between them: $\Gamma^\varepsilon(t)$.

Pore scale formulation

- In fluid-filled void space, mass and momentum conservation:

$$\partial_t \rho_f + \nabla \cdot (\rho_f \mathbf{q}^\varepsilon) = 0 \qquad \text{in } \Omega^\varepsilon(t)$$

$$\varepsilon^2 \Big(\partial_t (\rho_f \mathbf{q}^\varepsilon) + \nabla \cdot (\rho_f \mathbf{q}^\varepsilon \otimes \mathbf{q}^\varepsilon) \Big) = -\nabla p^\varepsilon + \varepsilon^2 \Big(\nabla \cdot (\mu \mathcal{D}(\mathbf{q}^\varepsilon) - \frac{2}{3} \nabla (\mu \nabla \cdot \mathbf{q}^\varepsilon) \Big) \qquad \text{in } \Omega^\varepsilon(t)$$

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- If constant density and viscosity:

$$\nabla \cdot \mathbf{q}^\varepsilon = 0 \qquad \text{in } \Omega^\varepsilon(t)$$

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- Boundary conditions?
  - No-slip at grain wall.
  - Rankine-Hugoniot jump condition: $\mathbf{n}^\varepsilon \cdot [\mathbf{j}_a] = v_n[a]$ on $\Gamma^\varepsilon(t)$

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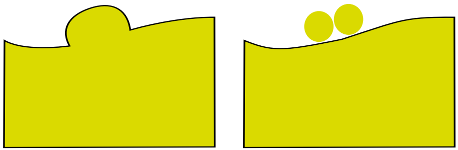
Mineral precipitation and dissolution

- Grain-fluid boundary moving due to chemical reactions:

$$\mathbf{n}^{\varepsilon} \cdot (-\rho_f \mathbf{q}^{\varepsilon}) = v_n(2\rho - \rho_f) \qquad \text{on } \Gamma^{\varepsilon}(t)$$



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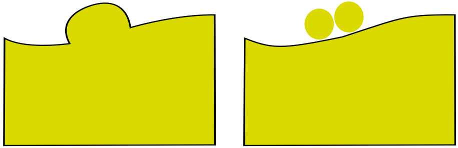
$$\mathbf{q}^\varepsilon = v_n \frac{\rho_f - 2\rho}{\rho_f} \mathbf{n}^\varepsilon \qquad \text{on } \Gamma^\varepsilon(t)$$



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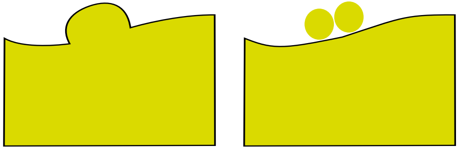
$$\mathbf{n}^\varepsilon \cdot (D \nabla u^\varepsilon - \mathbf{q}^\varepsilon u^\varepsilon) = v_n(\rho - u^\varepsilon) \qquad \text{on } \Gamma^\varepsilon(t)$$

$$\rho v_n = -\varepsilon f(u^\varepsilon, T_f^\varepsilon) \qquad \text{on } \Gamma^\varepsilon(t)$$



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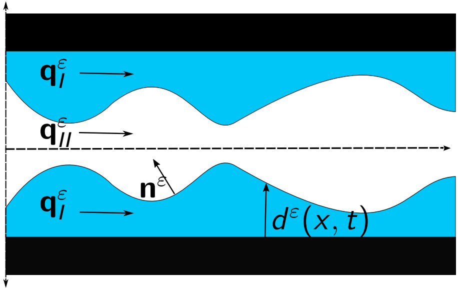
Two-phase flow

- Fluid-fluid boundary moving due to surface tension:

$$\mathbf{q}_I^\varepsilon \cdot \mathbf{n}^\varepsilon = \mathbf{q}_{II}^\varepsilon \cdot \mathbf{n}^\varepsilon = v_n$$

$$(\sigma_I^\varepsilon - \sigma_{II}^\varepsilon) \cdot \mathbf{n}^\varepsilon = \frac{\varepsilon^3}{Ca}\mathbf{n}^\varepsilon$$



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Evolving domains

- The domain formulation and evolution of them must also be accounted for.
- In the general case, use level set:

$$S^\varepsilon(t,\mathbf{x}) = \begin{cases} < 0 & \text{if } \mathbf{x} \in \Omega^\varepsilon(t) \\ 0 & \text{if } \mathbf{x} \in \Gamma^\varepsilon(t) \\ > 0 & \text{if } \mathbf{x} \in G^\varepsilon(t) \end{cases}$$

$$\mathbf{n}^\varepsilon = \frac{\nabla S^\varepsilon}{|\nabla S^\varepsilon|}$$

$$\partial_t S^\varepsilon + v_n |\nabla S^\varepsilon| = 0$$

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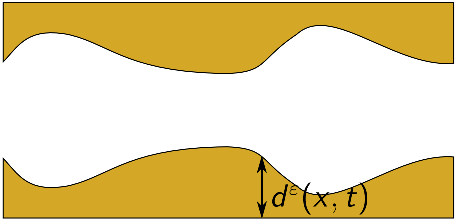
Evolving domains

- The domain formulation and evolution of them must also be accounted for.
- Can be simplified if considering a thin strip:



$$\mathbf{n}^\varepsilon = \frac{(\varepsilon \partial_x d^\varepsilon, -1)}{\sqrt{1 + (\varepsilon \partial_x d^\varepsilon)^2}}$$

$$\varepsilon \partial_t d^\varepsilon + v_n \sqrt{1 + (\varepsilon \partial_x d^\varepsilon)^2} = 0$$

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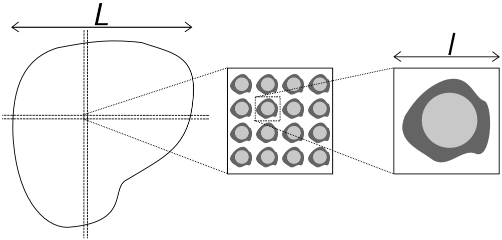
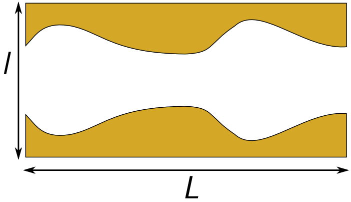
Upscaling using homogenization

- Goal: Find effective description at Darcy scale of the pore scale formulation.
- Scale separator $\varepsilon = l/L$.





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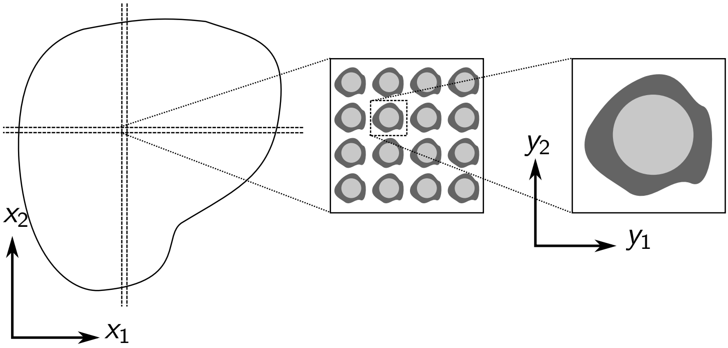
Asymptotic expansions

- Homogenization ansatz: $u(t, \mathbf{x}) = u_0(t, \mathbf{x}, \mathbf{y}) + \varepsilon u_1(t, \mathbf{x}, \mathbf{y}) + \dots$



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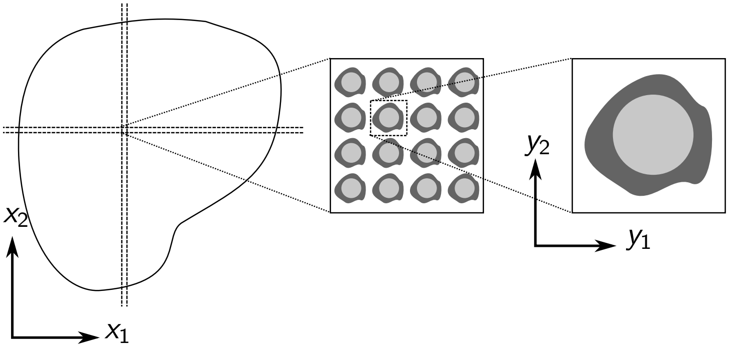

- Scaling of local variable: $\nabla = \nabla_{\mathbf{x}} + \frac{1}{\varepsilon}\nabla_{\mathbf{y}}$.

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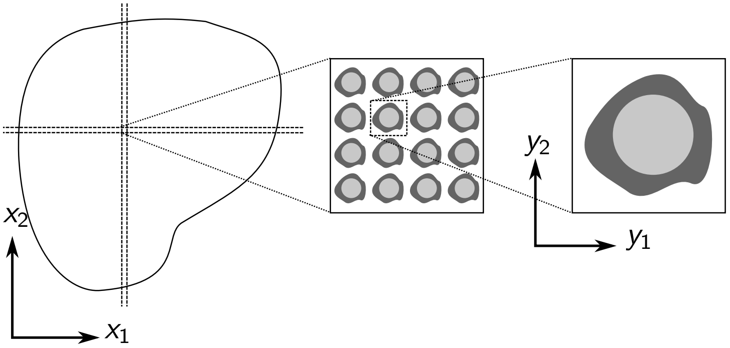

- Scaling of local variable: $\nabla = \nabla_{\mathbf{x}} + \frac{1}{\varepsilon}\nabla_{\mathbf{y}}$.
- Insert expansions, collect dominating terms, account for moving boundaries and find effective quantities.

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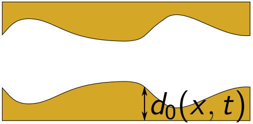
Thin strip

- Dependence on transversal variable eliminated: 2D $\rightarrow$ 1D.
- Coupled reactive and heat transport:



C. Bringedal, I. Berre, I.S. Pop, F.A. Radu, A model for non-isothermal flow and mineral precipitation and dissolution in a thin strip, Journal of Computational and Applied Mathematics, 2015.

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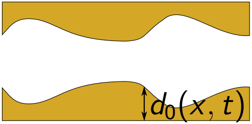

$$\partial_t\Big((1-2d_0)\rho_{f0} + 2d_0 2\rho\Big) + \partial_x(\rho_{f0}\bar{q}_0) = 0$$

$$\overline{q}_0(x,t) = -\frac{(1-2d_0(x,t))^3}{12\mu_{f0}}\partial_x p_0(x,t)$$

$$\partial_t\Big((1-2d_0)\rho_{f0}T_0 + 2d_0\varsigma\rho T_0\Big) + \partial_x(\rho_{f0}\overline{q}_0 T_0)$$

$$= \partial_x\Big((1-2d_0)\kappa_f\partial_x T_0 + 2d_0\kappa_g\partial_x T_0\Big)$$

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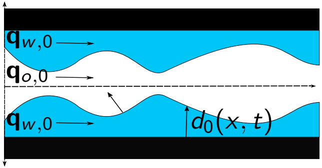
- Two-phase flow with surface tension:



Ongoing work by Sohely Sharmin.

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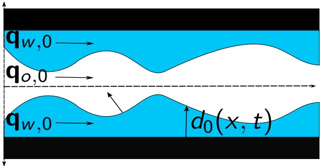

$$\partial_t d_0 = -\partial_x \overline{q}_{w,0} \qquad \partial_t d_0 = \partial_x \overline{q}_{o,0}$$

$$\overline{q}_{w,0} = -\frac{d_0^3}{3}\partial_x p_{w,0} - \frac{d_0^2(1-d_0)}{2M}\partial_x p_{o,0}$$

$$\overline{q}_{o,0} = -\frac{(1-d_0)^3}{3}\partial_x p_{o,0} - \frac{d_0(1-d_0)^2}{M}\partial_x p_{o,0} - \frac{d_0^2(1-d_0)}{2}\partial_x p_{w,0}$$

$$\overline{Ca}_o p_{o,0} - \overline{Ca}_w p_{w,0} = \partial_{xx} d_0$$

Ongoing work by Sohely Sharmin.

Periodic porous medium

- Two-scale model: Separation between macroscale variable $\mathbf{x}$ and microscale $\mathbf{y}$.
- Coupled reactive and heat transport.
- In macroscale $\mathbf{x}$, solve for $u_0(t,\mathbf{x})$, $p_0(t,\mathbf{x})$, $\overline{\mathbf{q}_0}(t,\mathbf{x})$, $T_0(t,\mathbf{x})$:

C. Bringedal, I. Berre, I.S. Pop, F.A. Radu, Upscaling of non-isothermal reactive porous media flow with changing porosity, Transport in Porous Media, 2016.

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$$\partial_t\Big(|Y_0(t,\mathbf{x})|\rho_f + |G_0(t,\mathbf{x})|2\rho\Big) + \nabla_{\mathbf{x}}\cdot\Big(\rho_f\overline{\mathbf{q}_0}\Big) = 0$$

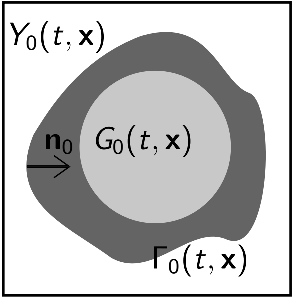
$$\overline{\mathbf{q}_0} = -\frac{1}{\mu_f}\mathcal{K}(t,\mathbf{x})\nabla_{\mathbf{x}}p_0$$

$$\partial_t\Big(|Y_0(t,\mathbf{x})|\rho_f T_0 + |G_0(t,\mathbf{x})|\varsigma\rho T_0\Big) + \nabla_{\mathbf{x}}\cdot\Big(\rho_f\overline{\mathbf{q}_0}T_0\Big)$$
$$= \nabla_x\cdot\Big(\kappa_f\mathcal{A}_f(t,\mathbf{x})\nabla_{\mathbf{x}}T_0 + \kappa_g\mathcal{A}_g(t,\mathbf{x})\nabla_{\mathbf{x}}T_0\Big)$$

C. Bringedal, I. Berre, I.S. Pop, F.A. Radu, Upscaling of non-isothermal reactive porous media flow with changing porosity, Transport in Porous Media, 2016.

Periodic porous medium

- For each macroscale point $\mathbf{x}$, update level set equation and solve cell problems in $\mathbf{y}$ for components of matrices $\mathcal{K}$, $\mathcal{A}_f$ and $\mathcal{A}_g$:



$$k_{ij}(t,\mathbf{x}) = \frac{1}{|Y_0|}\int_{Y_0} w_i^j(\mathbf{y})d\mathbf{y}$$

$$\text{where } \mathbf{e}_j + \nabla_{\mathbf{y}}\Pi^j + \nabla_{\mathbf{y}}^2\mathbf{w}^j = 0$$

$$\text{and } \nabla_{\mathbf{y}}\cdot\mathbf{w}^j = 0 \text{ in } Y_0,$$

$$\text{and } \mathbf{w}^j(y) = 0 \text{ on } \Gamma_0.$$

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$$a_{ij}^f = \int_{Y_0} (\delta_{ij} + \partial_{y_i}\Theta^j(y))dy, \qquad a_{ij}^g = \int_{G_0} (\delta_{ij} + \partial_{y_i}\Theta^j(y))dy.$$

where $\nabla_y^2 \Theta_f^j(y) = 0$ in $Y_0(x,t)$ and $\nabla_y^2 \Theta_g^j(y) = 0$ in $G_0(x,t)$

and $\kappa_f \mathbf{n}_0 \cdot (\mathbf{e}_j + \nabla_y \Theta_f^j(y)) = \kappa_g \mathbf{n}_0 \cdot (\mathbf{e}_j + \nabla_y \Theta_g^j(y))$

and $\Theta_f^j(y) = \Theta_g^j(y)$ on $\Gamma_0(x,t)$,

C. Bringedal, I. Berre, I.S. Pop, F.A. Radu, Upscaling of non-isothermal reactive porous media flow with changing porosity, Transport in Porous Media, 2016.

Effective behavior on Darcy scale

- Effective behavior for flow and heat transport with changing porosity.
- Solve cell problems for various grain shapes/sizes.
- Obtain effective quantities as function of a parameter:

C. Bringedal, K. Kumar, Effective behavior near clogging in upscaled equations for non-isothermal reactive porous media flow, Transport in Porous Media, 2017.

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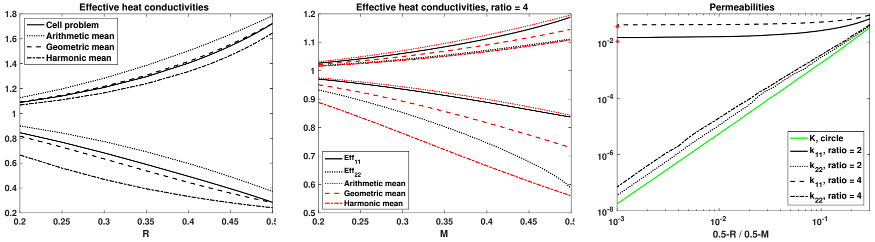
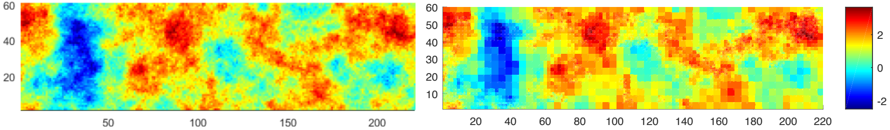
- Effective behavior for flow and heat transport with changing porosity.
- Solve cell problems for various grain shapes/sizes.
- Obtain effective quantities as function of a parameter:



C. Bringedal, K. Kumar, Effective behavior near clogging in upscaled equations for non-isothermal reactive porous media flow, Transport in Porous Media, 2017.

Effective behavior on Darcy scale

- Coupled pore scale - Darcy scale implementation.
- Use cell problem formulation to upscale permeabilities.



Ongoing work by Manuela Bastidas.

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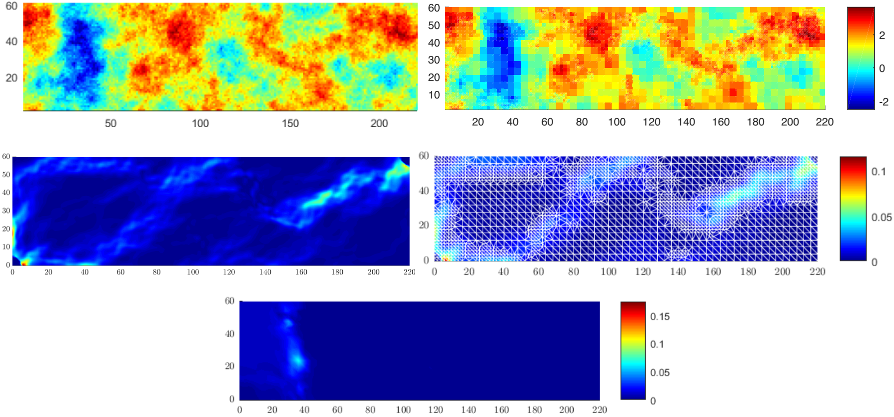

Ongoing work by Manuela Bastidas.

Summary

* Porous media flow: Coupled processes relevant for geothermal energy, CO_2 storage, groundwater contamination, etc.

Carina Bringedal (carina.bringedal@uhasselt.be)

Summary

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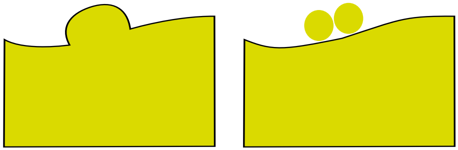
* Pore scale formulations including moving boundaries: Chemical reactions and two-phase flow.



Carina Bringedal (carina.bringedal@uhasselt.be)

Summary

- Porous media flow: Coupled processes relevant for geothermal energy, CO_2 storage, groundwater contamination, etc.
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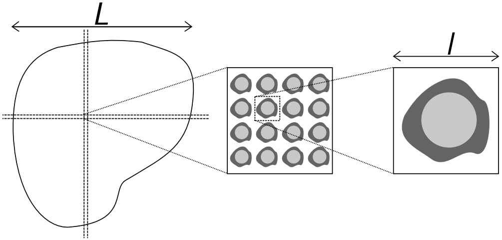
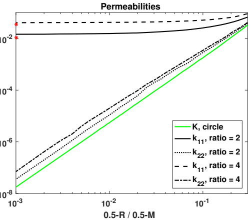
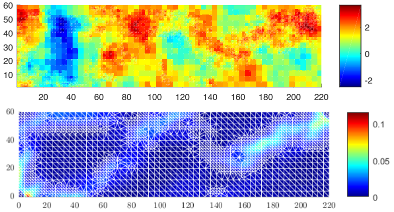


Carina Bringedal (carina.bringedal@uhasselt.be)

Summary

- Porous media flow: Coupled processes relevant for geothermal energy, CO_2 storage, groundwater contamination, etc.
- Pore scale formulations including moving boundaries: Chemical reactions and two-phase flow.
- Upscaling for effective descriptions incorporating effect of pore scale processes.
- Understanding effective behavior and numerical upscaling.



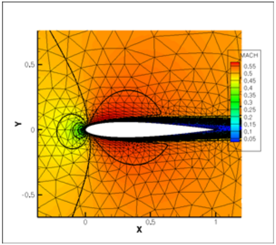


Thank you for your attention!

carina.bringedal@uhasselt.be
uhasselt.be/cmat

A bit of advertisement

Summer school on hyperbolic conservation laws



Hasselt, June 25-27.

Participation free of charge.

Registration deadline June 11.

See uhasselt.be/cmat for more info.

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