Automated non-destructive testing and evaluation

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Non-destructive testing and evaluation

1. Material properties
2. Defect existence
3. Defect properties
4. Object lifetime

Motivation
Inverse problems
Wave speed
Inverse scattering
Summary
Overview

Non-destructive testing and evaluation

Inverse problems

Automatic determination of wave speeds

Inverse scattering problems

Summary
Inverse problem formulation

\[
\min_{\bar{u}} \vec{J} \left( \vec{q} (\bar{u}), \bar{u} \right), \quad \text{(Objective)}
\]

where

\[
\vec{J} = \frac{1}{2} \left\| q_i^*(R, t) - q_i(R, t) \right\|^2 + \alpha R,
\]

such that

\[
\vec{c} = \vec{q} + \nabla \cdot \vec{F} (\vec{q}) - \vec{f} = 0. \quad \text{(Forward model)}
\]
Solving inverse problems
Using gradient information

\[ \vec{u}^* = \min_{\vec{u}} \tilde{J}(\vec{u}) \quad \Rightarrow \quad \nabla \tilde{J}(\vec{u}^*) = 0 \quad \text{(FONC)} \]

Newton (type) iterations:

\[ \nabla \tilde{J}(\vec{u}^*) = 0 \approx \nabla \tilde{J}(\vec{u}_k) + \frac{\partial}{\partial \vec{u}} \left( \nabla \tilde{J}(\vec{u}_k) \right) \vec{\rho}_k, \]

\[ \vec{\rho}_k = - \left( \nabla^2 \tilde{J}(\vec{u}_k) \right)^{-1} \nabla \tilde{J}(\vec{u}_k). \]
Solving inverse problems

Gradient computation

\[
\frac{dJ}{du} = - \frac{\partial J}{\partial q} \left( \frac{\partial c}{\partial q} \right)^{-1} \frac{\partial c}{\partial u} + \frac{\partial J}{\partial u}
\]

where \( J \) is the objective, \( c \) is the PDE constraint, \( u \) is the control and \( q \) are the PDE solutions.
Classical approach vs. AD

**Classical**

- Discrete forward equations $\xrightarrow{\text{manual implementation}}$ Forward code
- Discrete adjoint equations $\xrightarrow{\text{manual implementation}}$ Adjoint code
- human effort

**Automatic / algorithmic differentiation**

- Discrete forward equations $\xrightarrow{\text{manual implementation}}$ Forward code
- algorithmic differentiation
- Discrete adjoint equations $\xrightarrow{\text{algorithmic differentiation}}$ Adjoint code
Classical approach vs. Libadjoint

Classical

Discrete forward equations → Manual implementation → Forward code

Discrete adjoint equations → Manual implementation → Adjoint code

Human effort

Libadjoint

Discrete forward equations → FEniCS → Forward code

Discrete adjoint equations → FEniCS → Adjoint code

libadjoint
Automatic calibration of wave speeds

Problem set-up
Automatic calibration of wave speeds

Results

\[
\min_{c_l, c_t} \frac{1}{2} \left\| q_i^*(R, t) - q_i(R, t) \right\|^2
\]
Inverse scattering

Problem statement

- Defect/scatterer
- Receiver
- Source
Inverse scattering

Inverse scattering: pitfalls and outlook

- Bad stability of the problem
- Mesh independent optimisation
- Function space for the control
Summary and conclusion

- Goal: Automated non-destructive testing and evaluation
- Solve adjoint PDE’s to compute gradients
- Computationally demanding problem
- FEniCS and dolfin-adjoint make the job a lot easier
- Example: Automatic determination of wave speeds
- Outlook: Inverse scattering problems

Gradient based methods to solve inverse problems have great potential in non-destructive testing and evaluation (but require some hard math)
Automated non-destructive testing and evaluation

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Simulation of wave propagation problems for Automated Identification of Micro Defects with Discontinuous Properties.
Simulation of wave propagation problems for Automated Identification of Micro Defects with Discontinuous Properties

- Automated Identification
- Inverse Problem
- PDE Constrained Optimization
- Adjoint Based Techniques

- High Frequencies
- Dense Meshes
- Non-Linearity
- DG-FEM
- Time Domain
- Explicit Time Stepping
- Small Time Step Size
- Many Time Steps
- Large Memory Requirements
- Many FLOPS
- Computationally Demanding Problem
Simulation of wave propagation problems for Automated Identification of Micro Defects with Discontinuous Properties using Inverse Problem, PDE Constrained Optimization, Adjoint Based Techniques, High Frequencies, Dense Meshes, and Non-Linearity in DG-FEM Time Domain Explicit Stepping with Small Time Step Size Many Time Steps Large Memory Requirements Many FLOPS Computationally Demanding Problem.
Simulation of wave propagation problems for

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The diagram illustrates the process of simulating wave propagation problems involving micro defects with discontinuous properties, utilizing automated identification techniques, inverse problems, PDE constrained optimization, adjoint based techniques, high frequencies, dense meshes, and discontinuous properties through DG-FEM, time domain, and explicit time stepping methods.
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Computationally Demanding Problem