KU LEUVEN kulak

Automated non-destructive testing and evaluation

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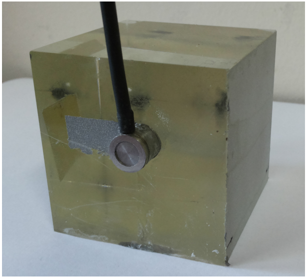
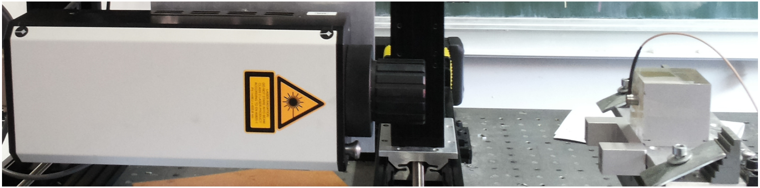
KU Leuven Kulak

WSC Spring Meeting, May 13th 2016, Utrecht

Non-destructive testing and evaluation



1. Material properties
2. Defect existence
3. Defect properties
4. Object lifetime



Overview

Non-destructive testing and evaluation

Inverse problems

Automatic determination of wave speeds

Inverse scattering problems

Summary

Inverse problem formulation

$$\min_{\vec{u}} \vec{J}\left(\vec{q}\left(\vec{u}\right), \vec{u}\right), \qquad \text{(Objective)}$$

where

$$\vec{J} = \frac{1}{2}\left\|q_i^*(R,t) - q_i(R,t)\right\|^2 + \alpha\mathcal{R},$$

such that

$$\vec{c} = \dot{\vec{q}} + \nabla \cdot \vec{F}\left(\vec{q}\right) - \vec{f} = 0. \qquad \text{(Forward model)}$$

Solving inverse problems

Using gradient information

$$\vec{u}^* = \min_{\vec{u}} \vec{J}(\vec{u}) \quad \Rightarrow \quad \nabla \vec{J}(\vec{u}^*) = 0 \quad \text{(FONC)}$$

Newton (type) iterations:

$$\nabla \vec{J}\left(\vec{u}^*\right) = 0 \approx \nabla \vec{J}\left(\vec{u}_k\right) + \frac{\partial}{\partial u}\left(\nabla \vec{J}\left(\vec{u}_k\right)\right) \vec{p}_k,$$

$$\vec{p}_k = -\left(\nabla^2 \vec{J}\left(\vec{u}_k\right)\right)^{-1} \nabla \vec{J}\left(\vec{u}_k\right).$$

Solving inverse problems

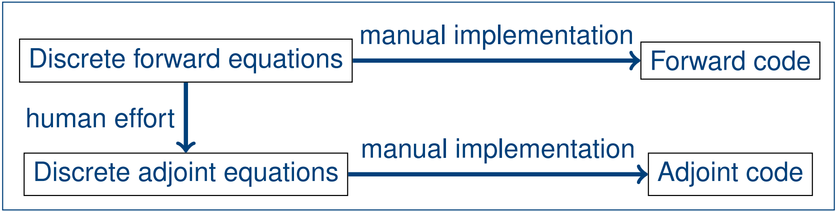
Gradient computation

$$\frac{\mathrm{d}J}{\mathrm{d}u} = \overbrace{-\frac{\partial J}{\partial q}\left(\frac{\partial c}{\partial q}\right)}^{\text{Adjoint PDE}}\underbrace{\vphantom{\frac{\partial J}{\partial q}}^{-1}\frac{\partial c}{\partial u}}_{\text{Tangent linear PDE}} + \frac{\partial J}{\partial u}$$

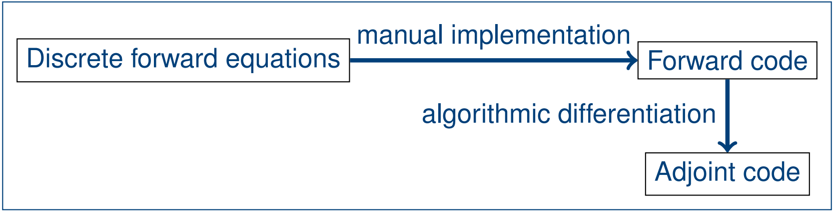
where J is the objective, c is the PDE constraint, u is the control and q are the PDE solutions

Classical approach vs. AD

Classical



Automatic / algorithmic differentiation



Classical approach vs. Libadjoint

Classical

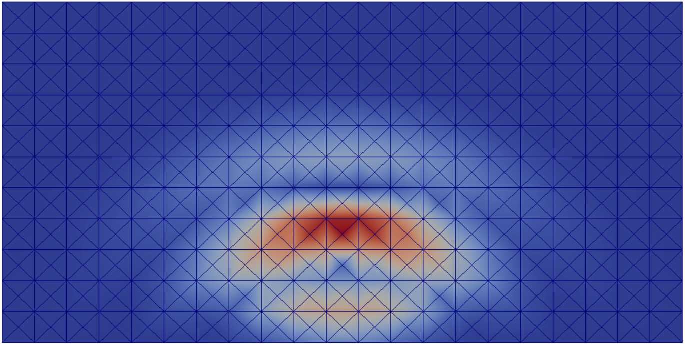
$$\begin{tikzcd}[column sep=huge, row sep=large]
\boxed{\text{Discrete forward equations}} \arrow[r, "\text{Manual implementation}"] \arrow[d, "\text{Human effort}"'] & \boxed{\text{Forward code}} \\
\boxed{\text{Discrete adjoint equations}} \arrow[r, "\text{Manual implementation}"] & \boxed{\text{Adjoint code}}
\end{tikzcd}$$

Libadjoint

$$\begin{tikzcd}[column sep=huge, row sep=large]
\boxed{\text{Discrete forward equations}} \arrow[r, "\text{FEniCS}"] \arrow[d, "\text{libadjoint}"'] & \boxed{\text{Forward code}} \\
\boxed{\text{Discrete adjoint equations}} \arrow[r, "\text{FEniCS}"] & \boxed{\text{Adjoint code}}
\end{tikzcd}$$

Automatic calibration of wave speeds

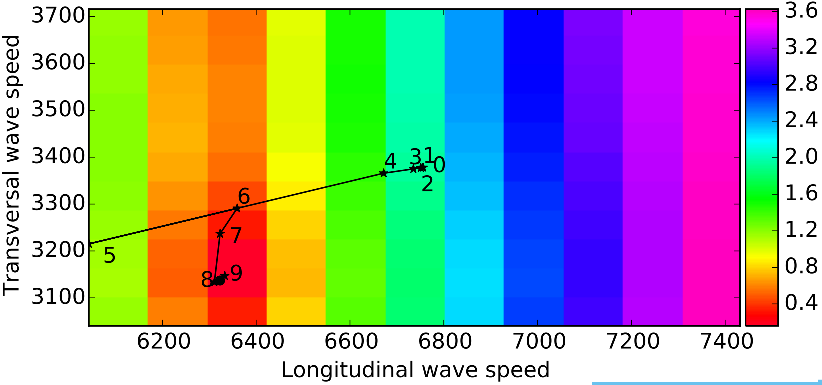
Problem set-up



Automatic calibration of wave speeds

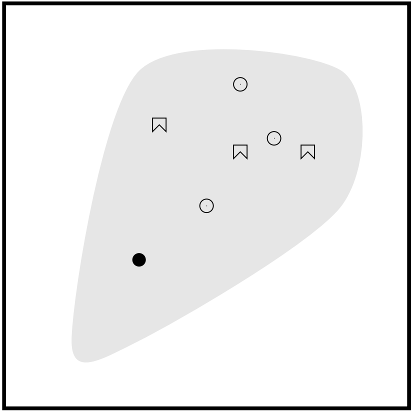
Results

$$\min_{c_l, c_t} \frac{1}{2} \left\| q_i^*(R,t) - q_i(R,t) \right\|^2$$



Inverse scattering

Problem statement

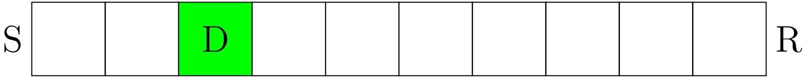


- Defect/scatterer
- Receiver
- Source

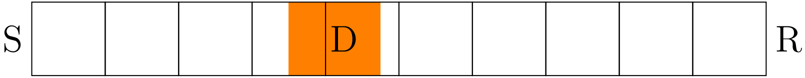
Inverse scattering

Inverse scattering: pitfalls and outlook

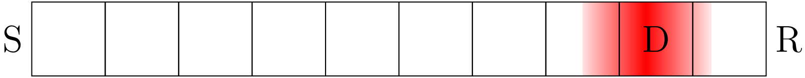
- Bad stability of the problem



- Mesh independent optimisation



- Function space for the control



Summary and conclusion

- Goal: Automated non-destructive testing and evaluation
- Solve adjoint PDE's to compute gradients
- Computationally demanding problem
- FEniCS and dolfin-adjoint make the job a lot easier
- Example: Automatic determination of wave speeds
- Outlook: Inverse scattering problems

Gradient based methods to solve inverse problems
have great potential in
non-destructive testing and evaluation
(but require some hard math)

Automated non-destructive testing and evaluation

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Simulation of wave propagation problems for

Automated Identification of Micro Defects with Discontinuous Properties

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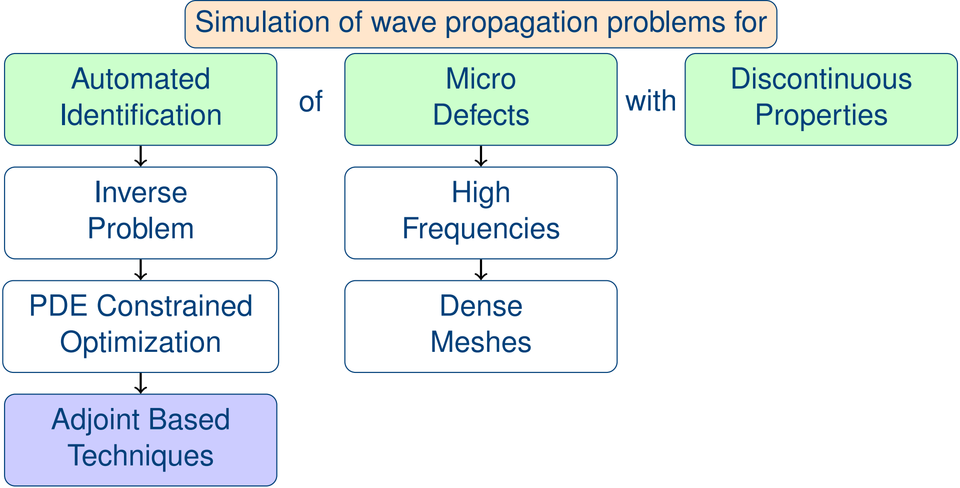
Automated Identification
↓
Inverse Problem
↓
PDE Constrained Optimization
↓
Adjoint Based Techniques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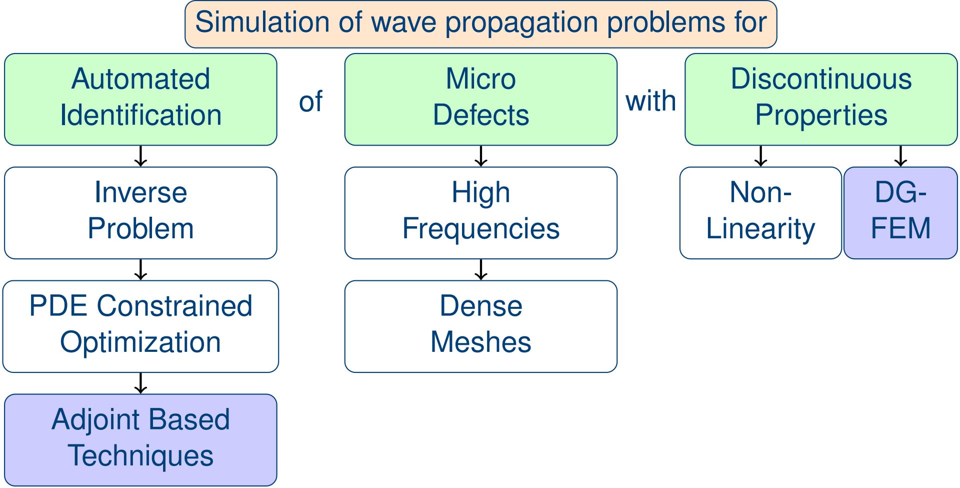
Adjoint Based Techniques
High Frequencies
Dense Meshes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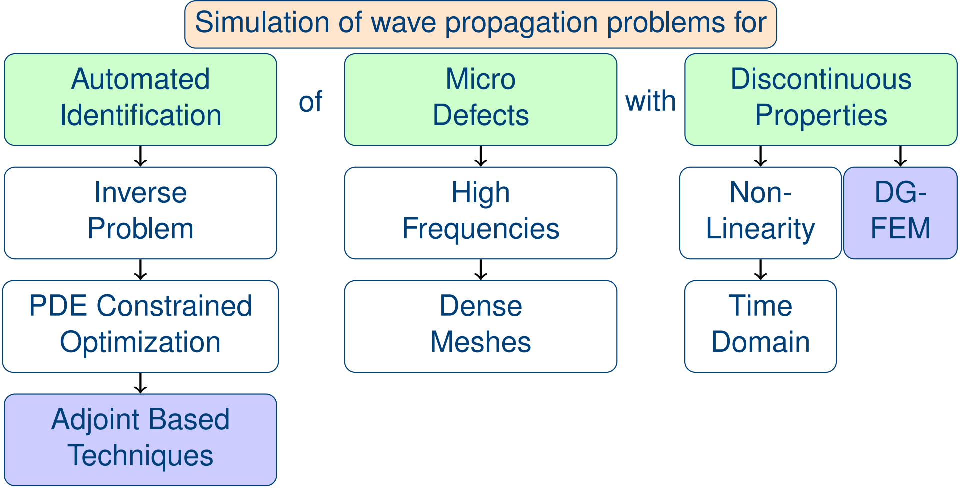
Non-Linearity
DG-FEM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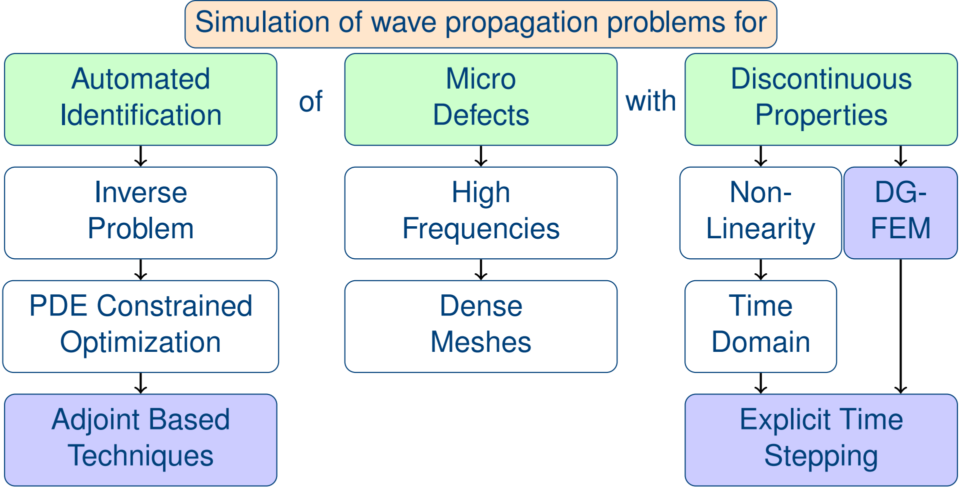

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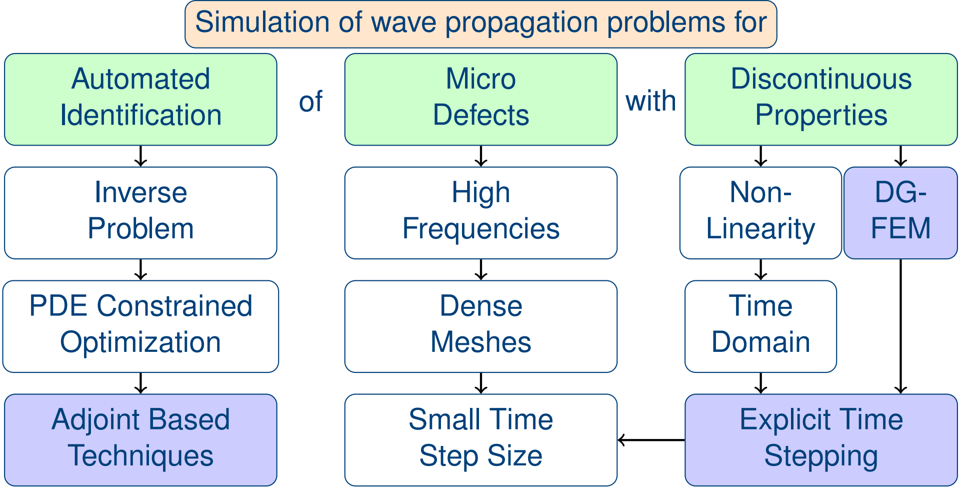
Explicit Time Stepping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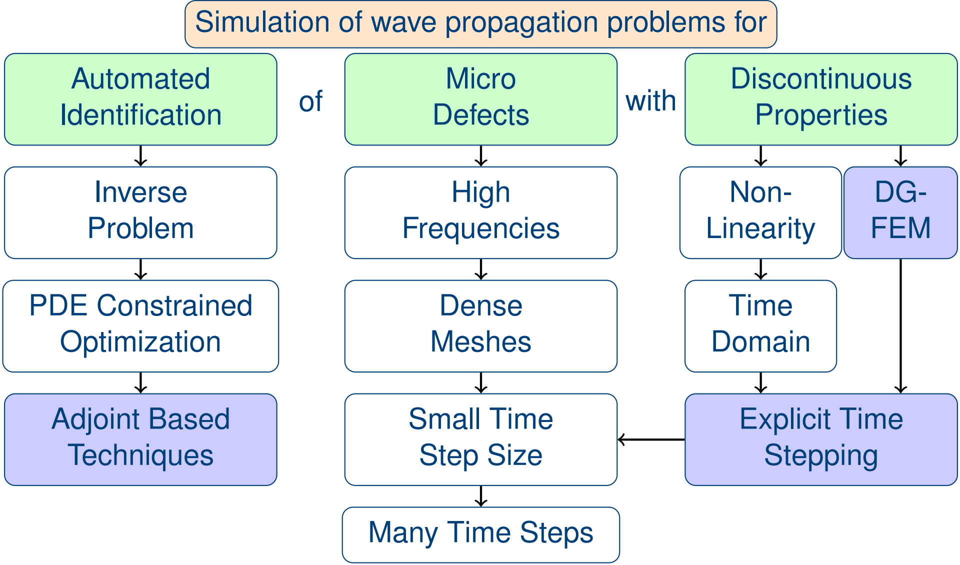
Small Time Step Size
Explicit Time Stepping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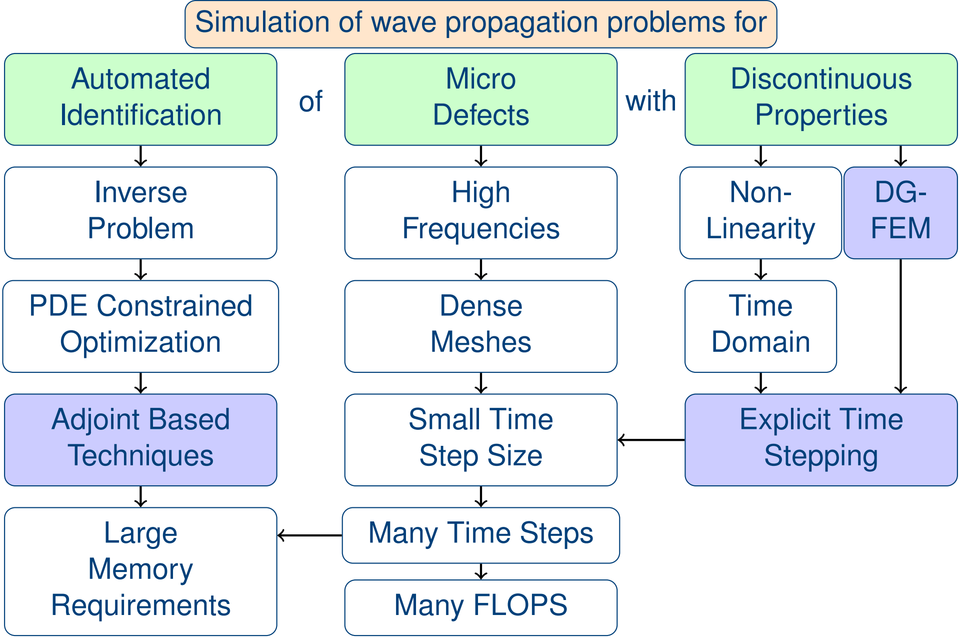
Small Time Step Size
Explicit Time Stepping
Many Time Steps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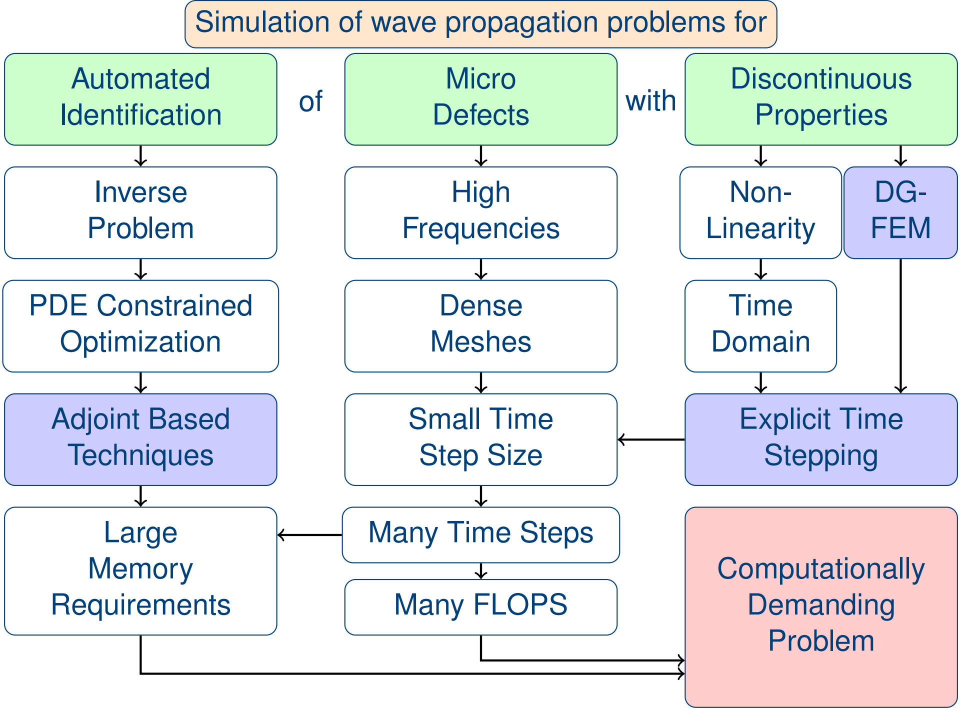
Large Memory Requirements
Many Time Steps
Many FLOPS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Computationally Demanding Problem