



*About source term models to include
vortex generator effects in CFD codes*

- 1. Introduction
- 2. VG modelling
- 3. Factors that influence performance BAY model
- 4. Optimization of source term
- 5. Conclusions

Vortex Generators (VGs)...

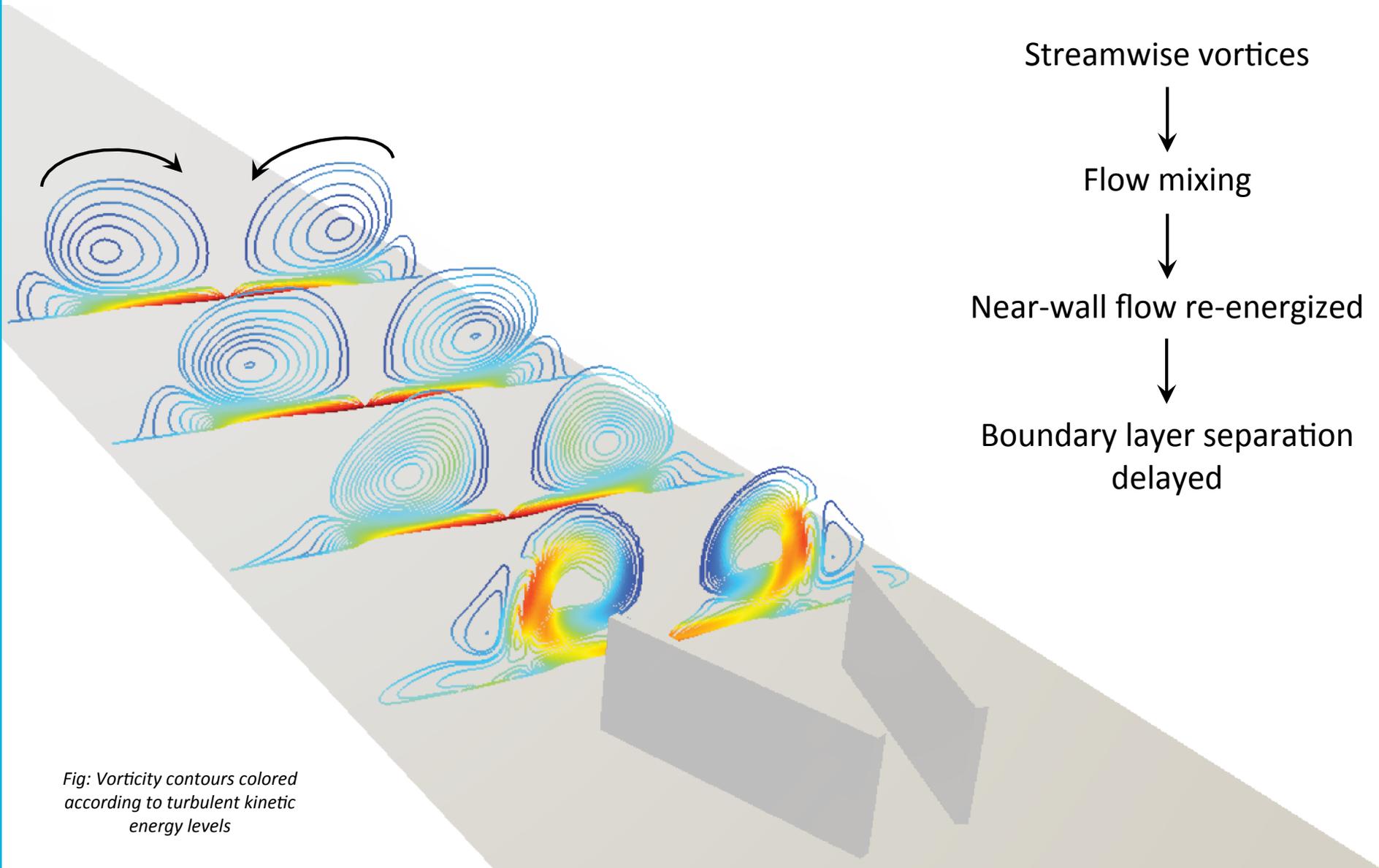


Fig: Vorticity contours colored according to turbulent kinetic energy levels

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Many areas of application...

Airplane wings, engine inlets, cars

... including wind turbines

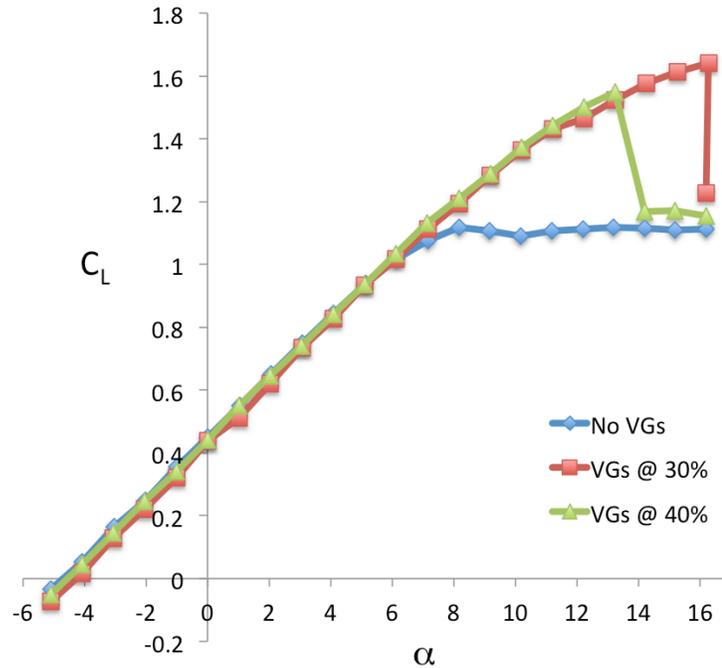
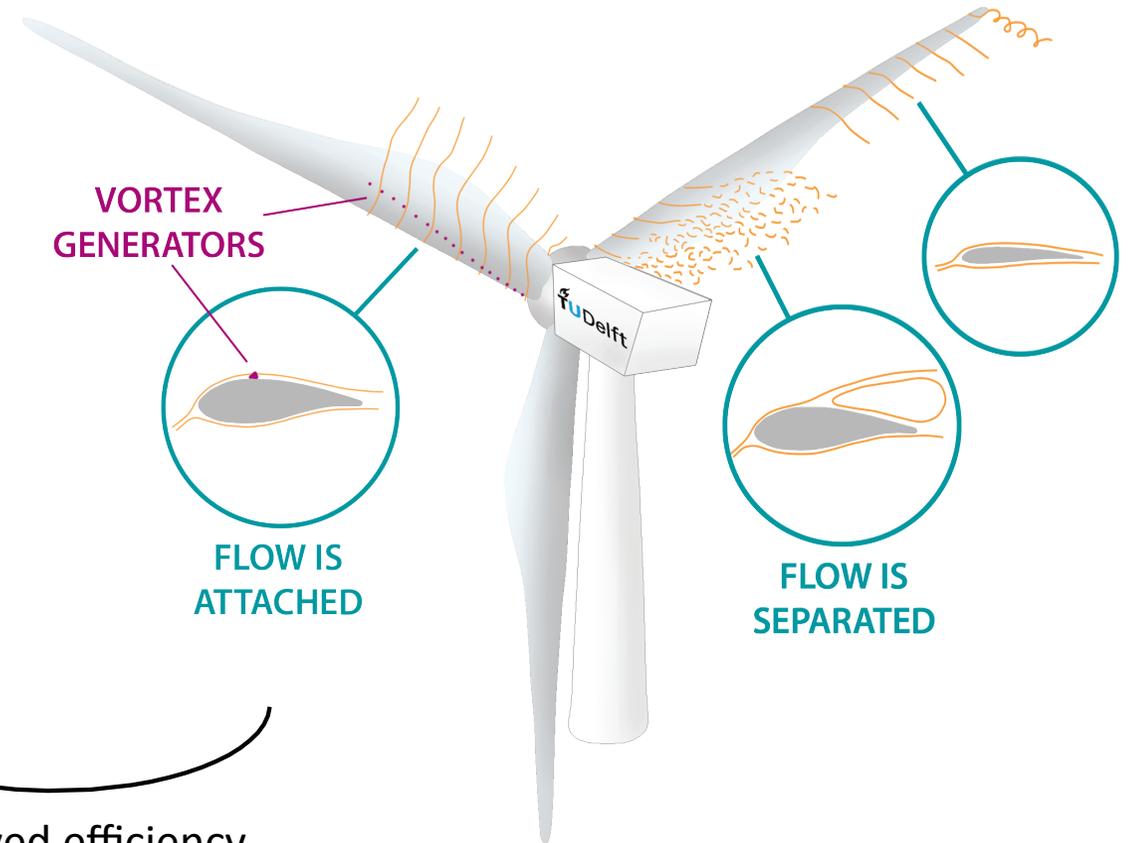


Fig: Experimental lift coefficient polar for NTUA18% airfoil^[1]



Improved efficiency

How to include VG effects in CFD simulations?

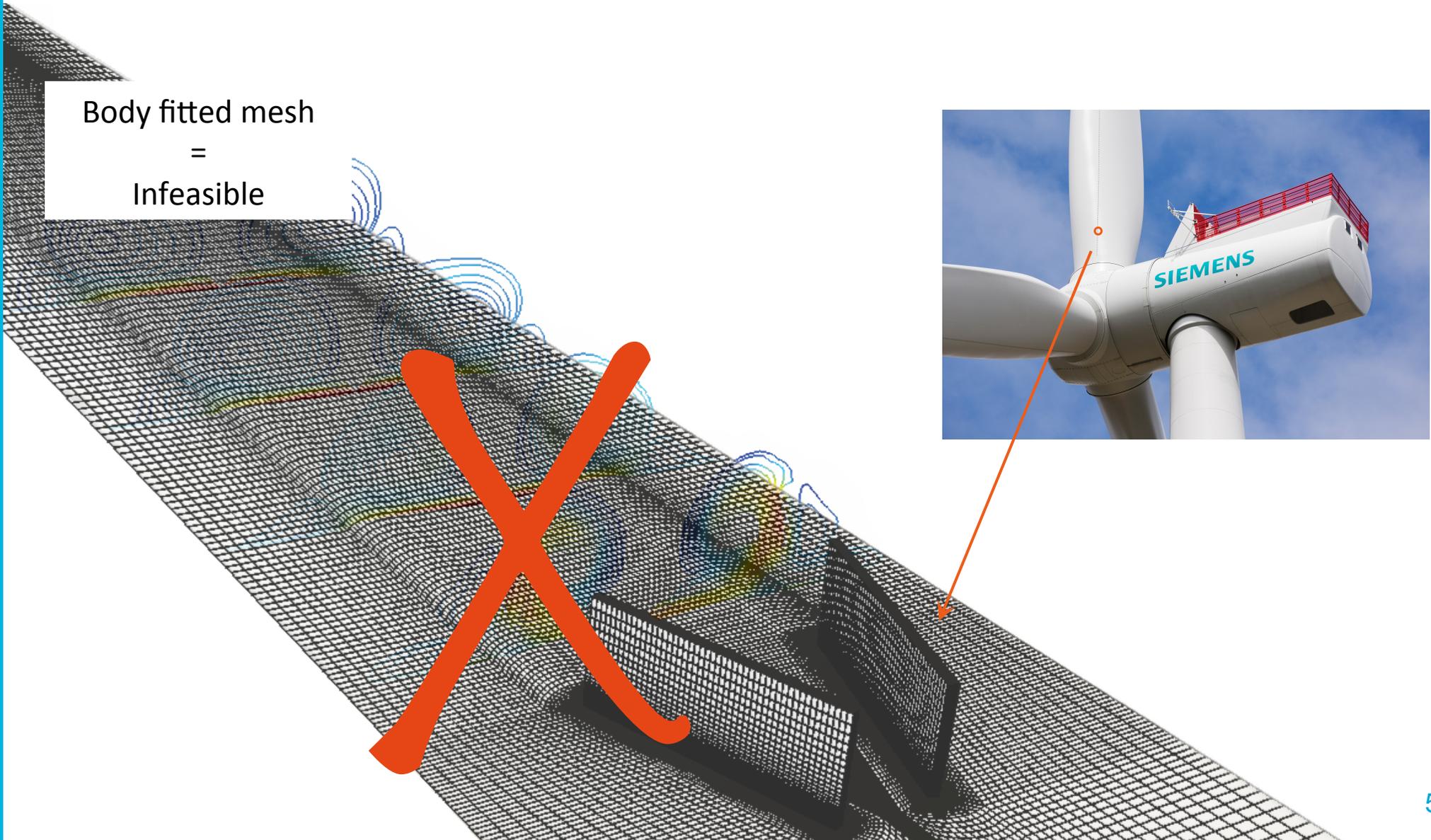
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How to include VG effects in CFD simulations?

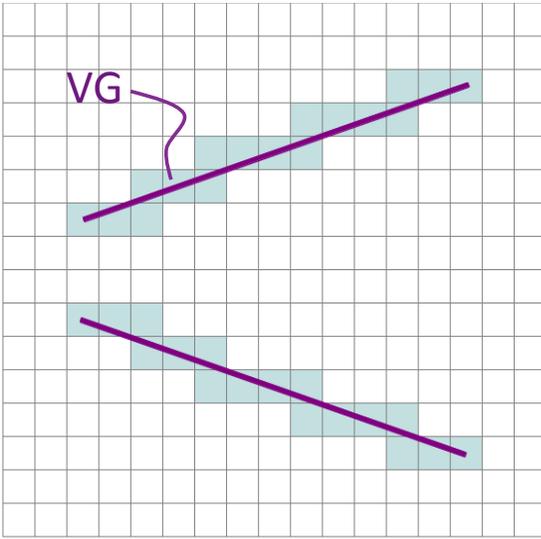
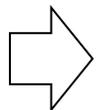
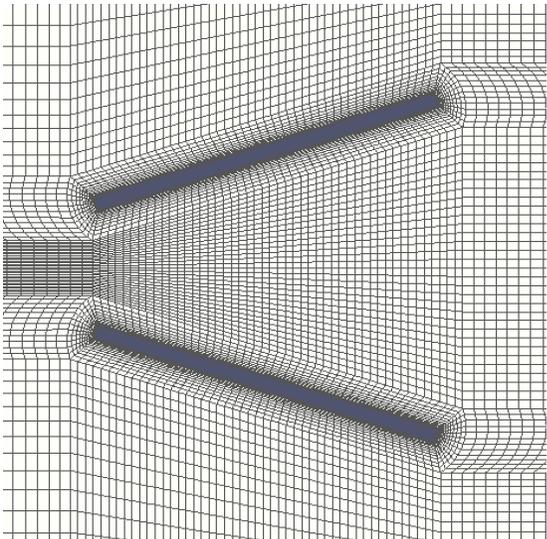
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Body fitted mesh
=
Infeasible



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Model VG effect instead of geometry



Adapt governing equations instead of mesh: $(\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p - \nabla \cdot (2\nu D(\mathbf{u})) = 0$

$$\nabla \cdot \mathbf{u} = 0$$

BAY model [2]

- Local body force triggers formation of vortex
- Formulation based on thin airfoil theory:

$$\mathbf{f}_i = \frac{V_i}{\sum V_i} cS\rho (\mathbf{u}_i \cdot \hat{\mathbf{n}})(\mathbf{u}_i \times \hat{\mathbf{b}}) \left(\frac{\mathbf{u}_i}{|\mathbf{u}_i|} \cdot \hat{\mathbf{t}} \right)$$

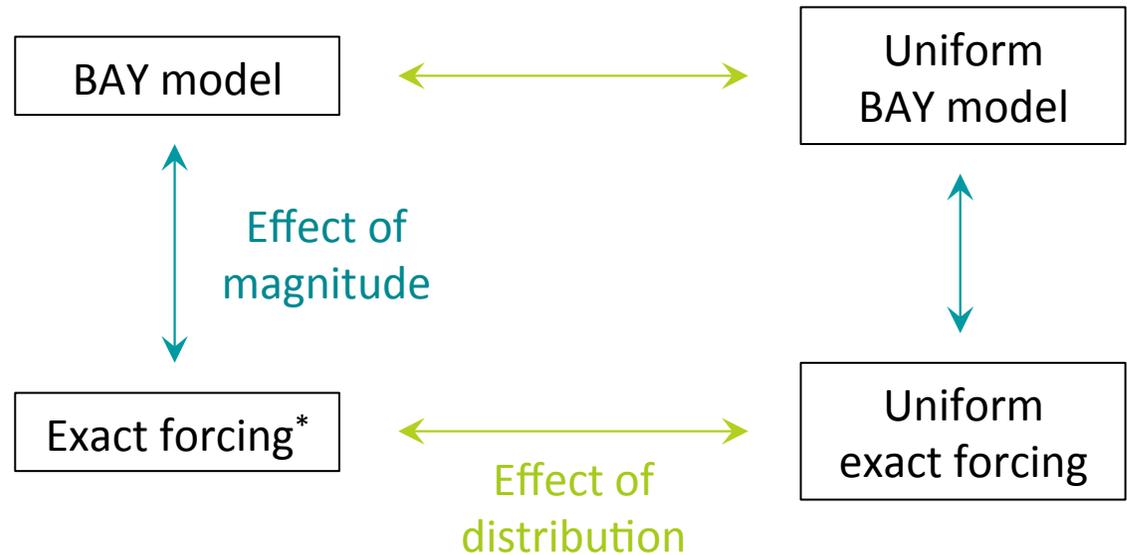
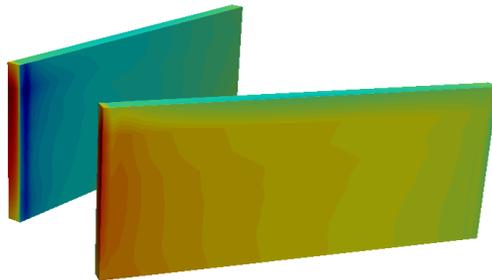
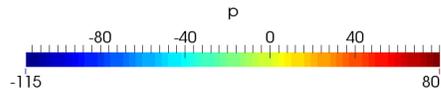
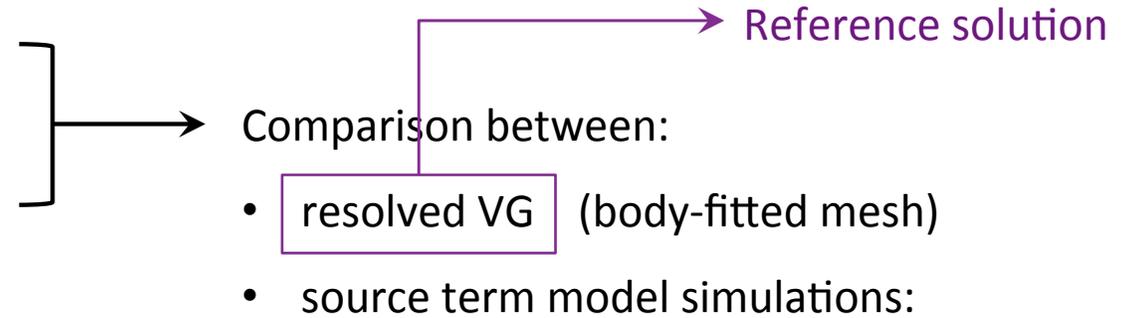
[2] Bender, E.E., Anderson, B.H. and Yagle, P.J. *Vortex generator modeling for Navier-Stokes codes*. FEDSSM99-6919, 1999

What factors influence success of BAY model?

1. Mesh resolution

2. Total forcing

3. Source term distribution



(* Extracted from simulation with resolved VG)

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 - test cases
 - mesh resolution
 - total forcing & distribution
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Test case A: Flat plate flow (zero pressure gradient)

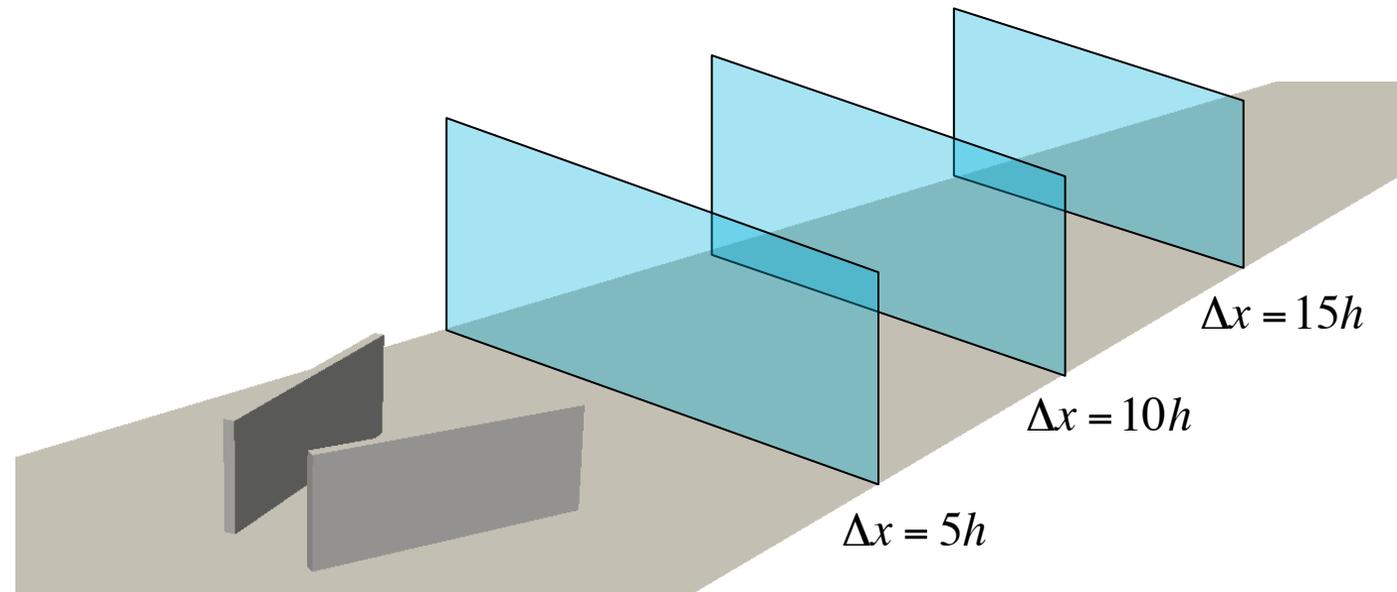
→ **Steady, incompressible flow,
RANS with k- ω SST turbulence model**

Flat plate with counter-rotating common down rectangular VG pair

- Symmetry b.c.
- $h = \delta/3$ and $h = \delta$
- $U_\infty = 15$ m/s
- $Re_x = 1.2$ million
- $\delta = 15$ mm

Mesh

- $\Delta_{bf} \approx 0.08h$
- $\Delta_{BAY} \approx 0.4h, 0.2h, 0.1h$
- $y^+ \approx 1$



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Test case B: Airfoil section (adverse pressure gradient)

→ **Steady, incompressible flow,
RANS with k- ω SST turbulence model**

Half a counter-rotating common up rectangular VG pair @ 30% of chord

- Symmetry b.c.
- $h = \delta$
- $U_\infty = 24$ m/s
- $Re_c = 0.87$ million
- $\delta = 6$ mm

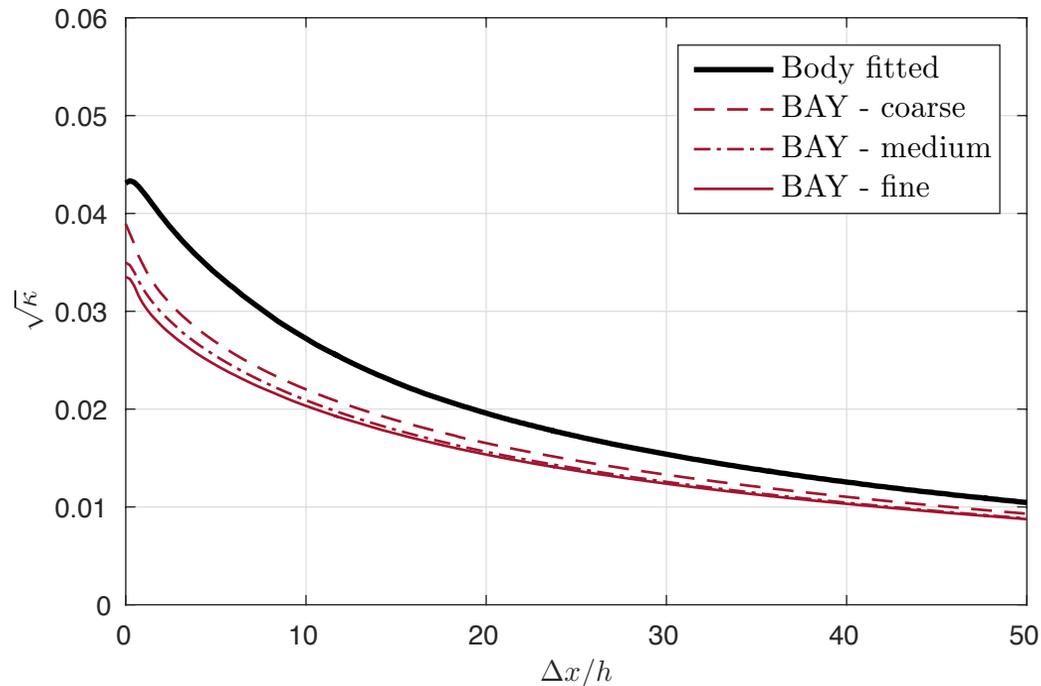
Mesh

- $\Delta_{bf} \approx 0.07h$
- $\Delta_{BAY} \approx 0.4h, 0.2h, 0.1h$
- $y^+ \approx 1$

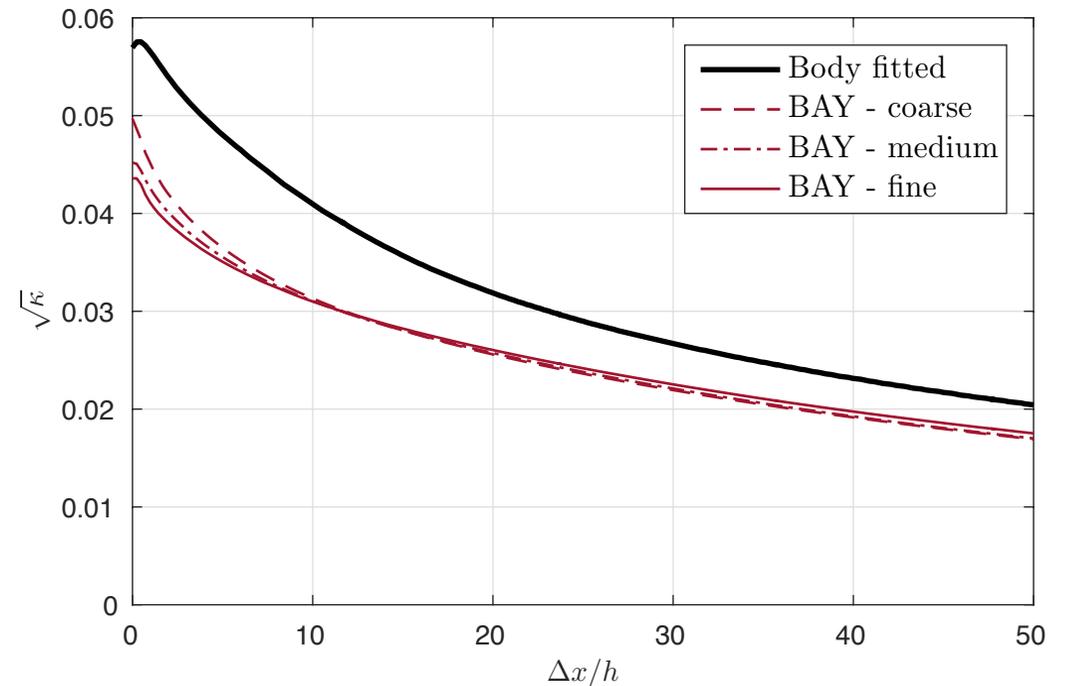


Mesh resolution: effect on kinetic energy

→ Model error: BAY model underestimates flow mixing



(a) Flat plate + low VG pair



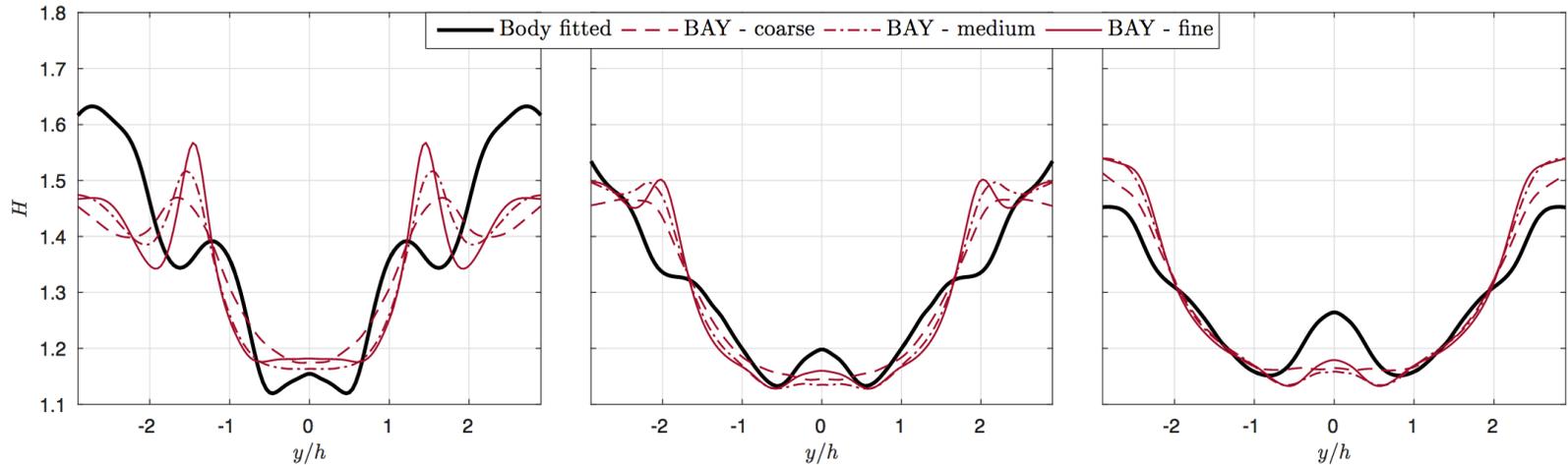
(b) Flat plate + high VG pair

Fig: Cross stream kinetic energy evolution downstream of VG pair.

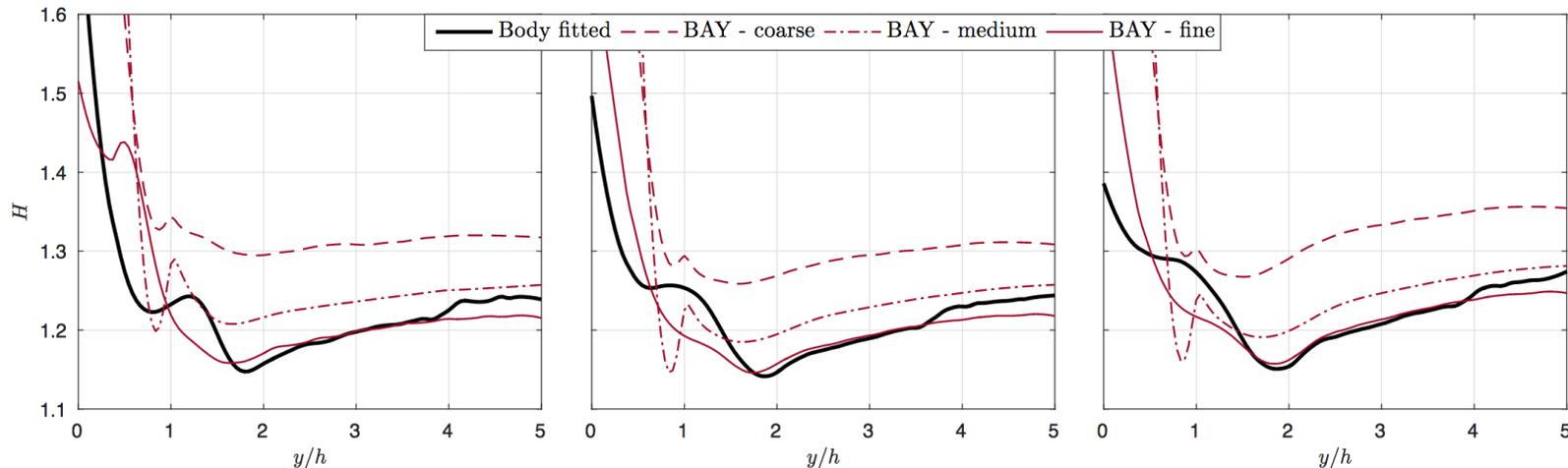
Mesh resolution: shape factor profiles

→ Unreliable prediction of boundary layer state

$$H = \frac{\delta^*}{\theta} = \frac{\int_0^\delta \left(1 - \frac{U}{U_\infty}\right) dz}{\int_0^\delta \frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty}\right) dz}$$



(a) Flat plate + high VG pair



(b) Airfoil section

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Large effect mesh resolution on separation location

- Use of BAY model introduces an additional uncertainty w.r.t. flow separation
- fine mesh required

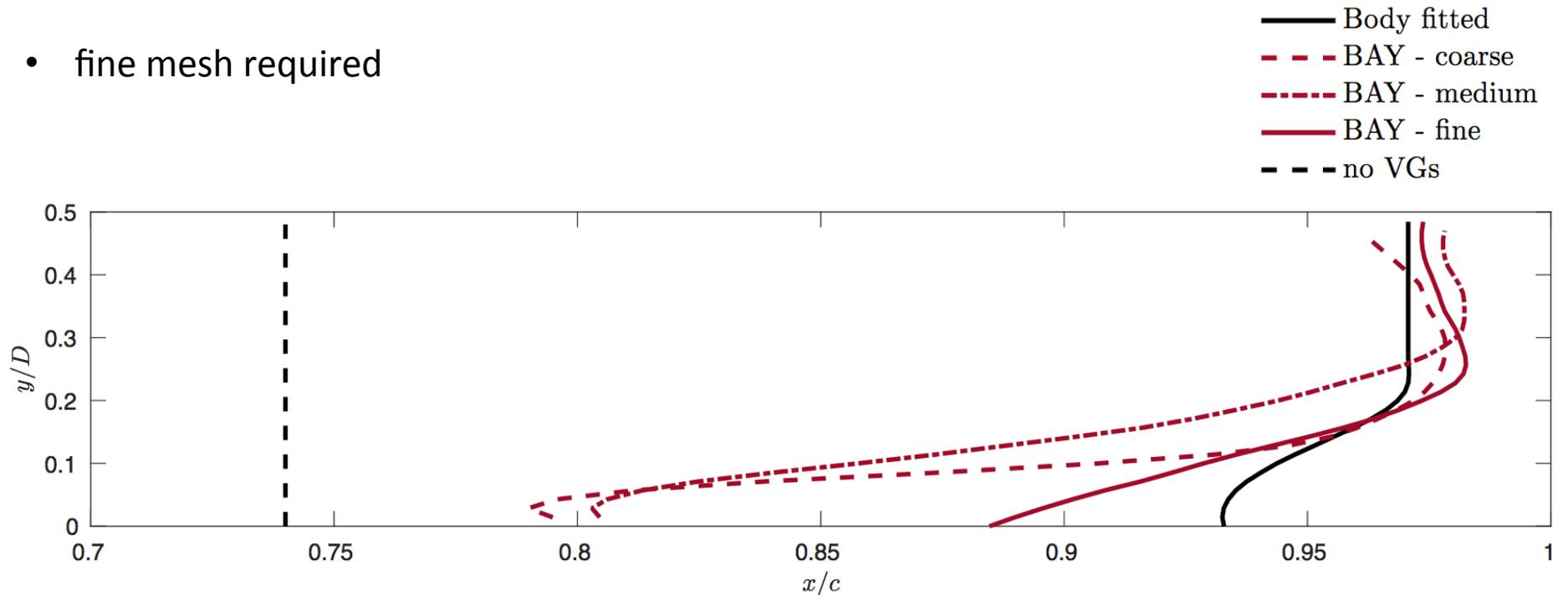
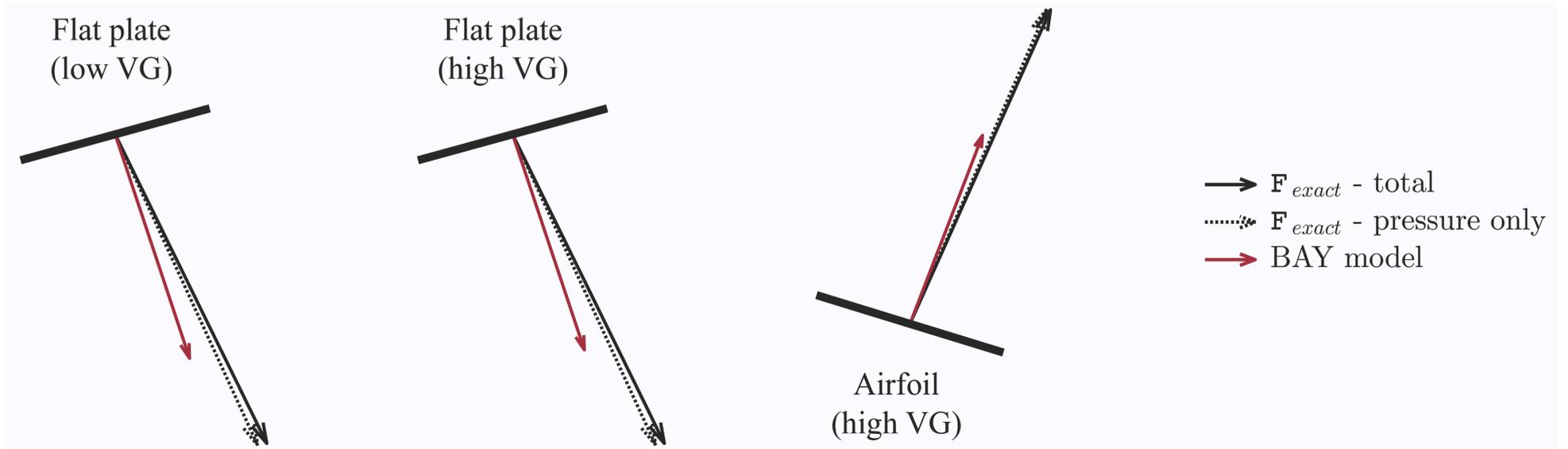


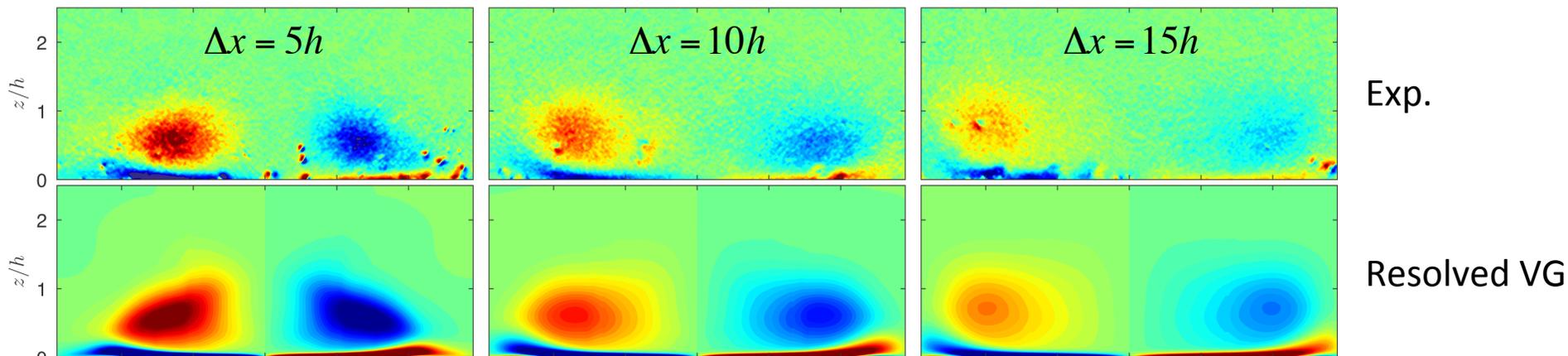
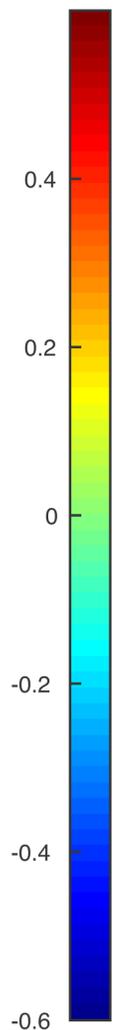
Fig: Locations of sign reversal in the streamwise component of the wall shear stress, indicating lines of boundary layer separation, BAY model with different mesh resolutions.

Effect of source term properties: Total forcing & distribution



- (Strong) underestimation magnitude total VG force
- Error in orientation

$$\omega_x \cdot h / U_\infty$$



- Effect of total forcing > distribution
- Large improvement possible with calibrated VG force:
 - ✓ Magnitude \approx vortex strength
 - ✓ Direction \approx vortex shape

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Effect of source term properties: Total forcing & distribution

- Large spread in separation locations
- Even F_{exact} yields poor result

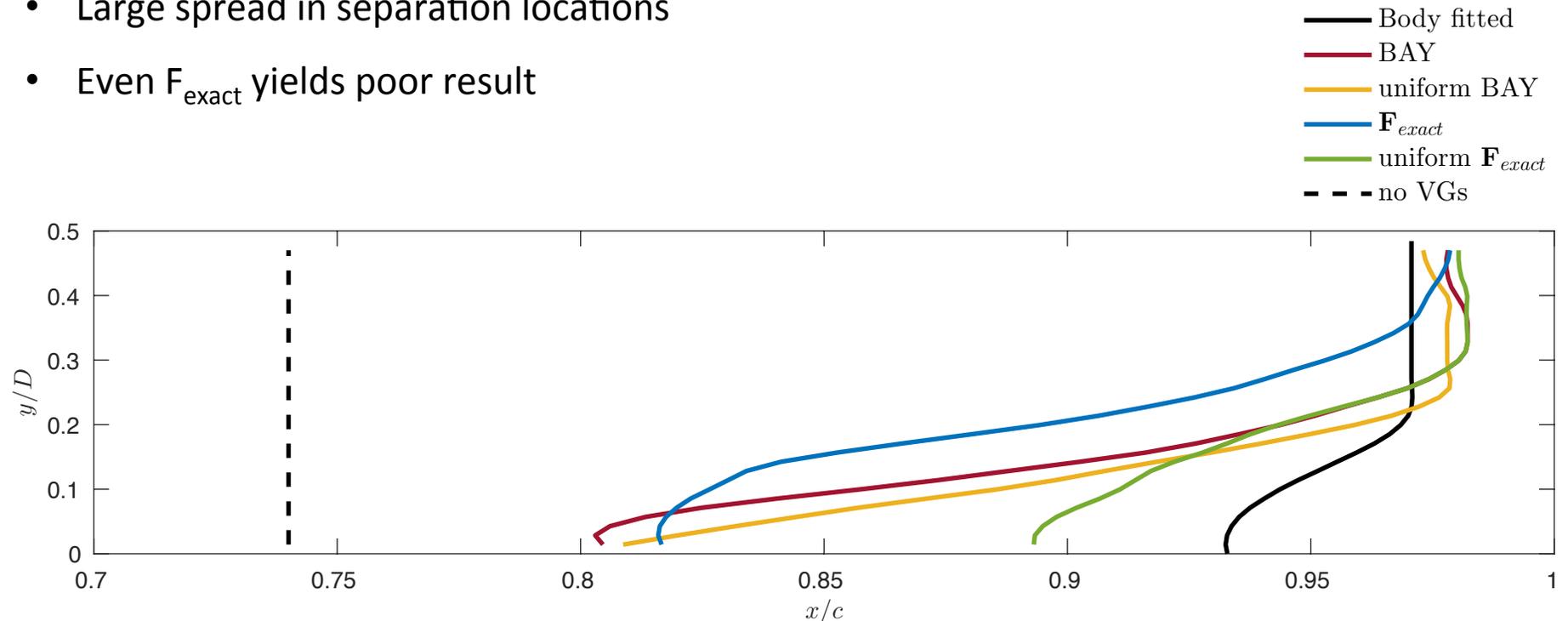


Fig: Locations of sign reversal in the streamwise component of the wall shear stress, indicating lines of boundary layer separation, for different source term models.

Conclusions w.r.t. BAY model concept

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Successful representation of flow field, but

- **Performance** Strong dependence on mesh resolution
- **Accuracy** Additional error in prediction of flow separation

Errors due to

- Low mesh resolution
- Discrete application forcing term
- Approximation of VG force



Drop BAY model assumption

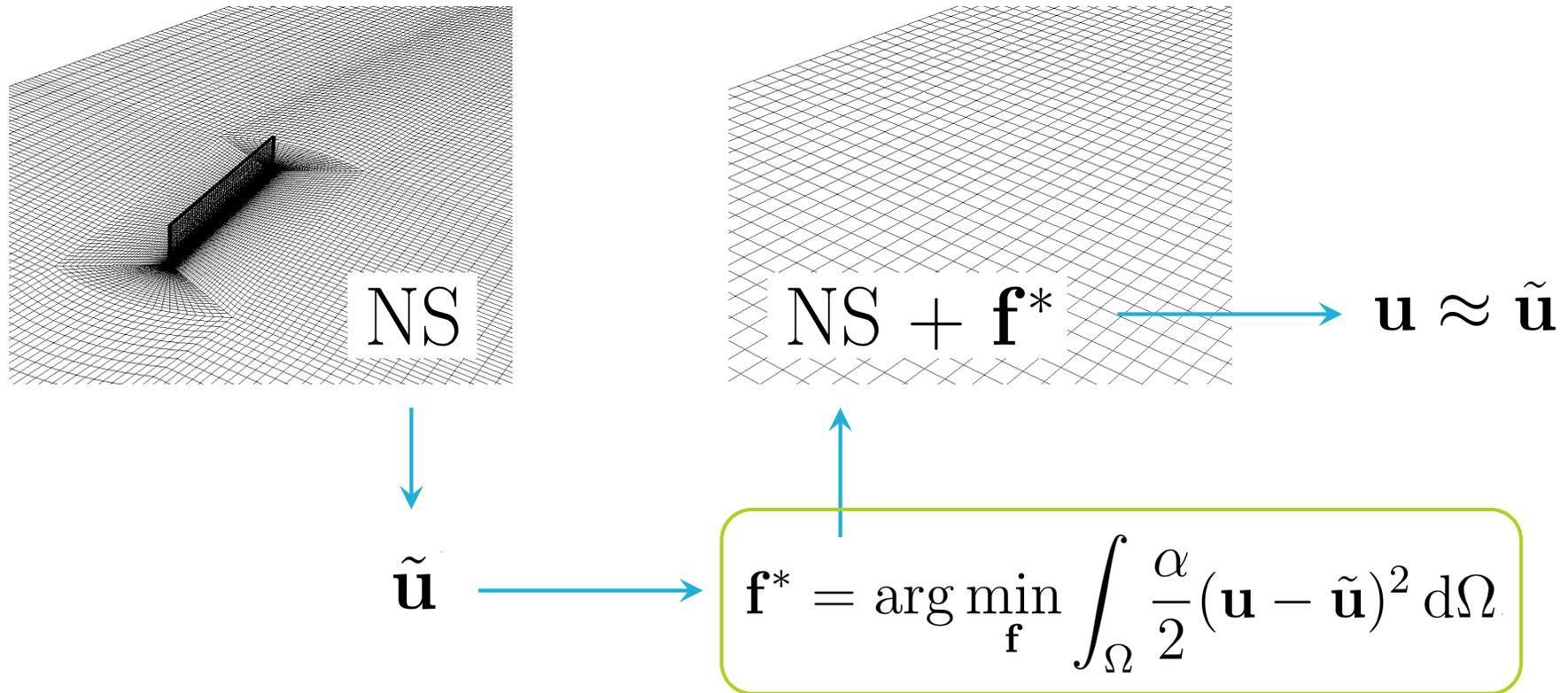


Ideal source term?

→ Can we do better?

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Find optimal source term \mathbf{f}^* to reproduce flow field on given mesh



Gradient optimization

$\tilde{\mathbf{u}}$ = high-fidelity result mapped on coarse mesh

Optimization using continuous adjoint approach

Objective function: $J(\mathbf{u}) = \int_{\Omega} \frac{\alpha}{2} (\mathbf{u} - \tilde{\mathbf{u}})^2 d\Omega$

Constraints (NS eq): $\mathbf{R}(\phi, \mathbf{f}) = 0 + \text{b.c.}$

Lagrangian: $\mathcal{L}(\phi, \mathbf{f}, \lambda) = J + \int_{\Omega} \lambda \cdot \mathbf{R}(\phi, \mathbf{f}) d\Omega$

Optimality conditions:

$$\nabla_{\phi} \mathcal{L} = \nabla_{\mathbf{f}} \mathcal{L} = \nabla_{\lambda} \mathcal{L} = 0$$

Adjoint system
(frozen turbulence assumption)

NS equations

$$\begin{aligned} (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p - \nabla \cdot (2\nu D(\mathbf{u})) + \mathbf{f} &= 0 \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

$$\nabla \mathbf{v} \cdot \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{v} + 2\nu \nabla \cdot D(\mathbf{v}) - \alpha(\mathbf{u} - \tilde{\mathbf{u}}) = \nabla q$$

Exact gradient of J

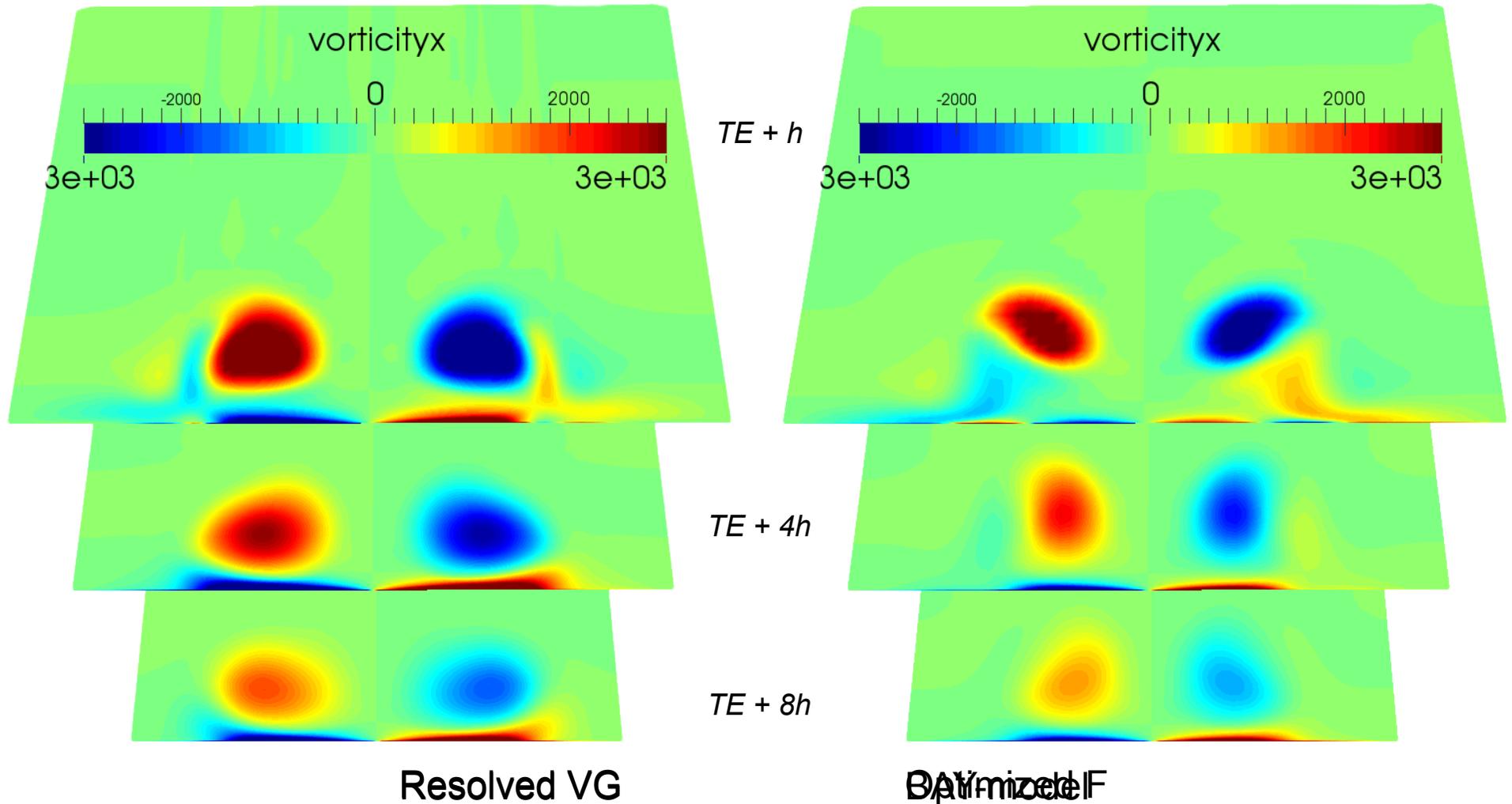
$$\nabla_{\mathbf{f}} \mathcal{L} = \nabla_{\mathbf{f}} J = \int_{\Omega} \mathbf{v} d\Omega$$

\mathbf{f}^* from gradient optimization

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Visual improvement of obtained flow field

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Large improvement in shape factor possible with optimized source term

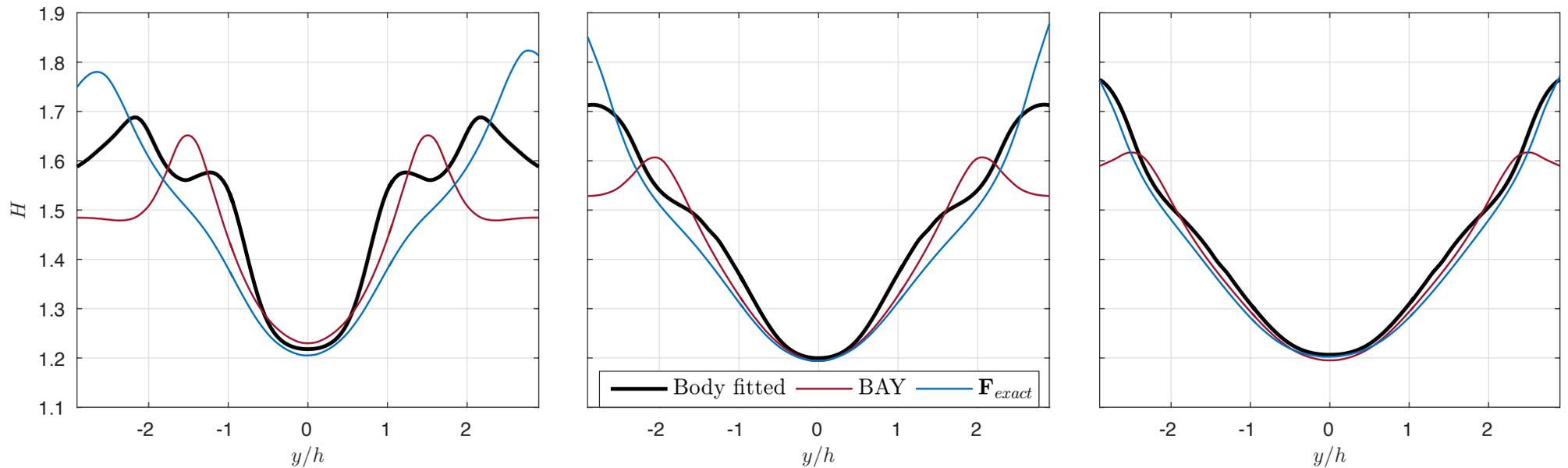


Fig: Shape factor profiles at 5h, 10h and 15h behind the VG trailing edge for different source term formulations, flat plate flow with low VG pair.

Large improvement in shape factor possible with optimized source term

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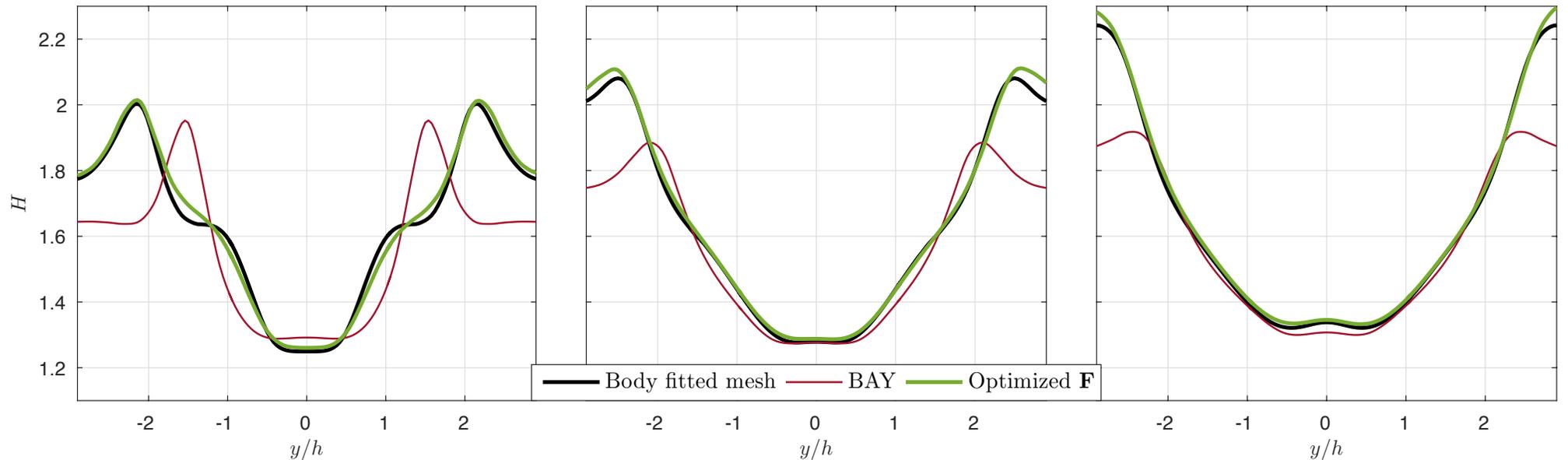
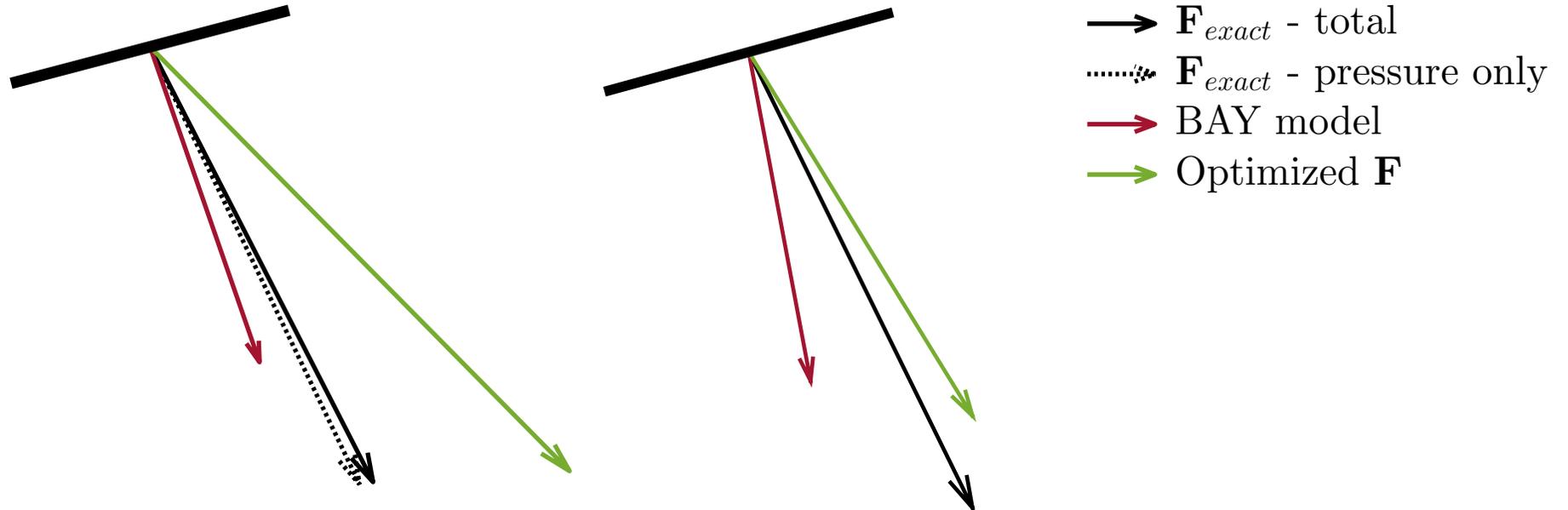


Fig: Shape factor profiles at 5h, 10h and 15h behind the VG trailing edge for different source term formulations, flat plate flow with low VG pair (different inflow velocity).

Interesting observation w.r.t. total forcing

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Resultant optimized forcing vector is tilted in streamwise direction

Conclusions

BAY model

- Performance strongly depends on mesh resolution
- Errors w.r.t. vortex strength and shape → unreliable separation prediction

Improvement for BAY model is desired...

... and possible!

Conclusions

Improvement for BAY model is possible!

- Better estimate for total VG force
 - Also tangential component is important
- Highly accurate result possible with optimized source term
 - Low mesh resolution not as big a problem as expected
 - Limited number of cells where source term should be applied

Proof of source term modeling concept

Optimization tool = Generic methodology to identify dominant patterns in optimal source term distributions for a given mesh



Aid to construct improved VG model

References:

- [1] Manolesos, M., Voutsinas, S., *Experimental investigation of the flow past passive vortex generators on an airfoil experiencing three-dimensional separation*, J. Wing Eng. Ind. Aerodyn., 142, p. 130-148, 2015
- [2] Bender, E.E., Anderson, B.H. and Yagle, P.J. *Vortex generator modeling for Navier-Stokes codes*. FEDSSM99-6919, 1999

