Selection in explorable heaps

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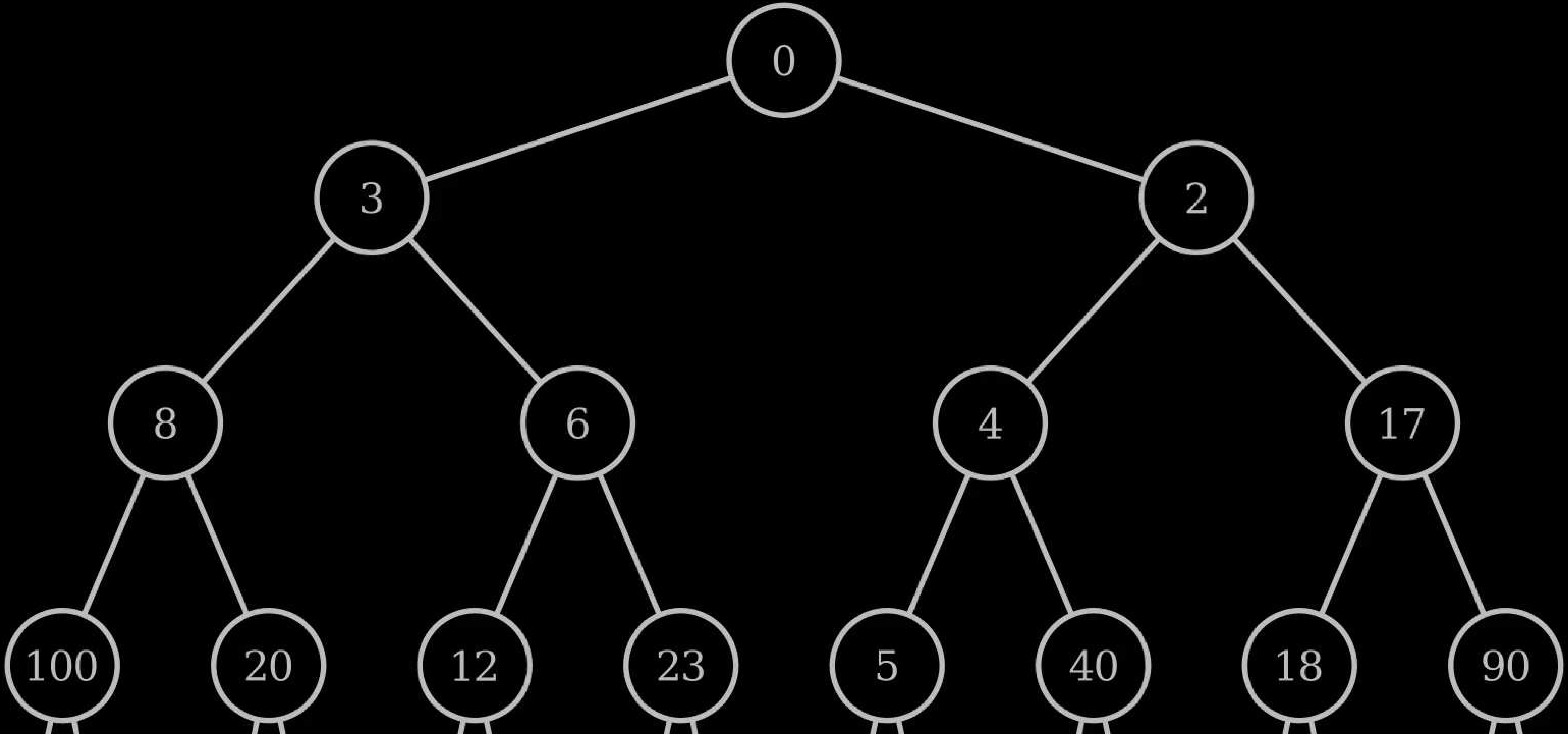
Selection in explorable heaps

finding the
nth smallest element

binary tree
with increasing values

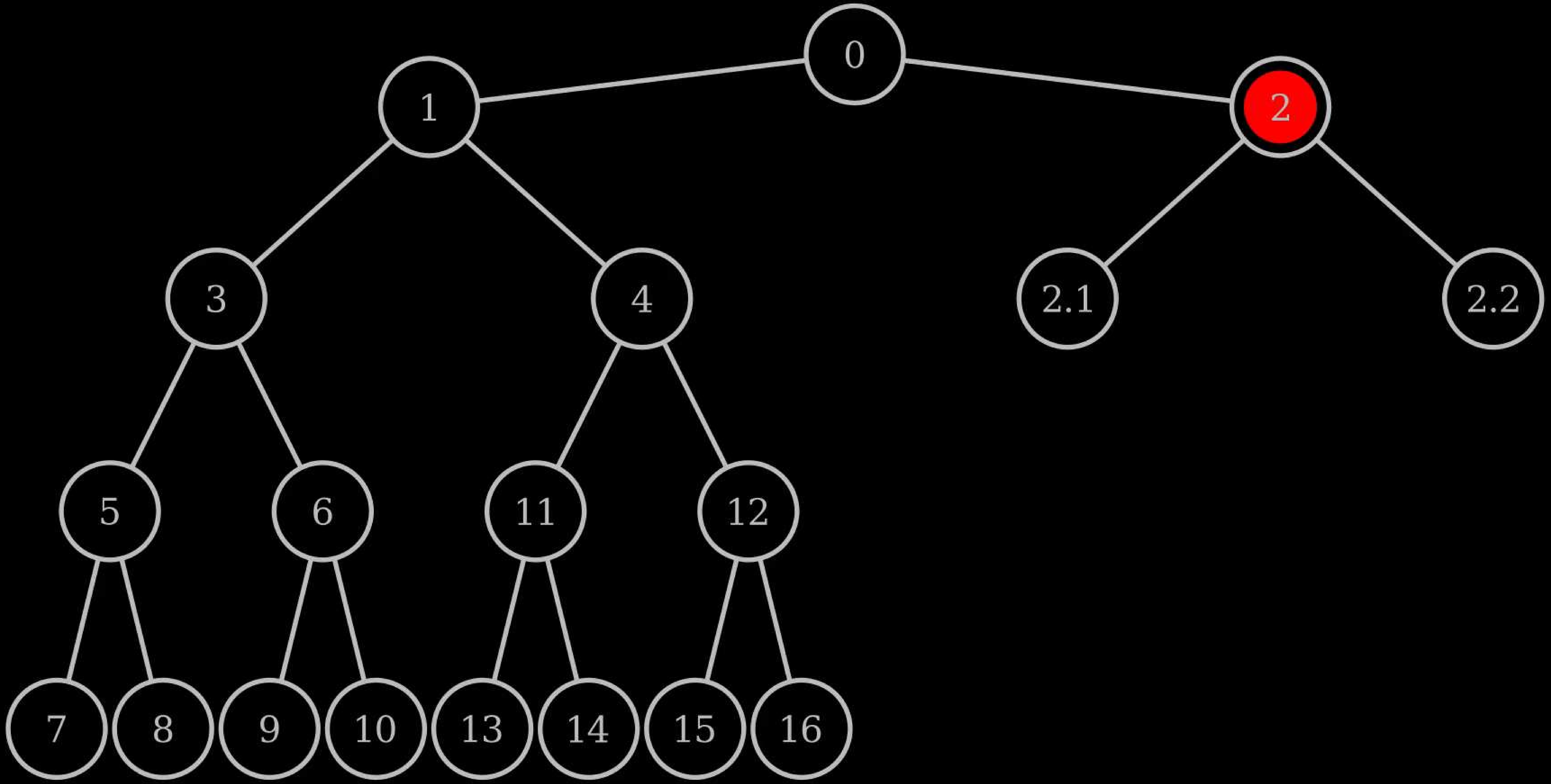


- Branch & Bound algorithm for solving MIP’s
- B&B-tree is a heap
- Node selection: Which subproblem to solve next?
- Travel is expensive

Best First Search

$O(n^2)$

Depth First Search



Prior work

- $O(n \cdot 2^{\sqrt{2\log(n)}})$ [KSW86]
- $O(n \cdot \log(n)^3)$ (our result)

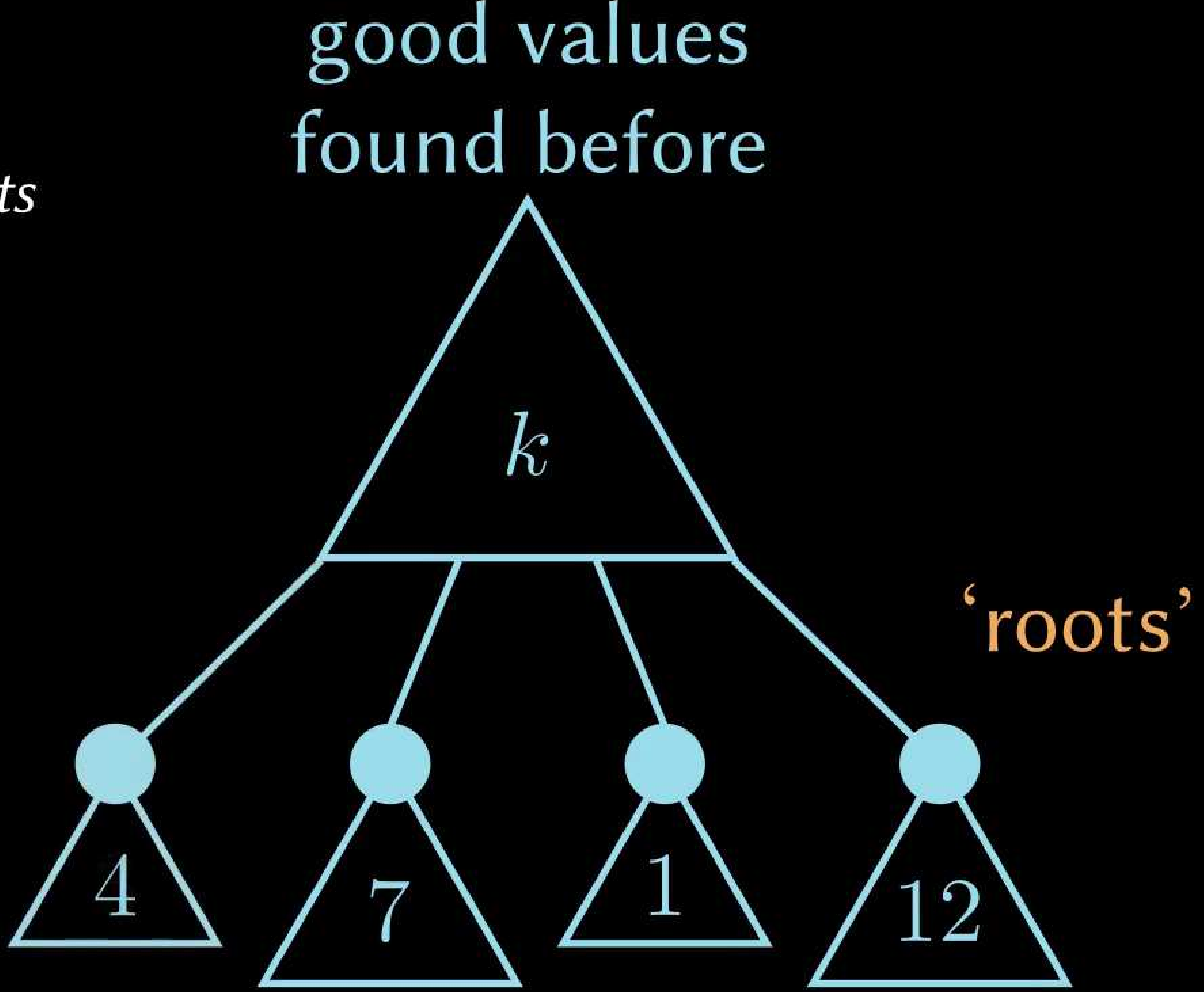
[KSW86] Richard M Karp, Michael E Saks, and Avi Wigderson. On a search problem related to branch-and-bound procedures. In *FOCS*, pages 19–28, 1986.

The algorithm

- Call the n smallest values the *good* values
- Can test if value is good in $O(n)$ time using DFS
- For set of m values: can test which are good using binary search in $O(n\log(m))$

The algorithm

- Find n smallest nodes, assuming that the $k \geq \frac{1}{2}n$ smallest are given
- Children of the k smallest nodes are called *roots*
- Take a random root:
  - Find n' smallest nodes under root by recursive call for $n' = 1, 2, 4, \ldots$
  - When a found node is not good, find highest good value under root



The algorithm

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  - Find n' smallest nodes under root by recursive call for $n' = 1, 2, 4, \ldots$
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$\log(n)$

$n \log(n)$

The algorithm

$\mathsf{ALG}(n, k)$

$\mathsf{ALG}(n_1^{(1)}, k_1^{(1)}), \ldots, \mathsf{ALG}(n_{r_1}^{(1)}, k_{r_1}^{(1)})$

$\mathsf{ALG}(n_1^{(2)}, k_1^{(2)}), \ldots, \mathsf{ALG}(n_{r_2}^{(2)}, k_{r_2}^{(2)})$

$\vdots \quad \vdots$

$$n \log(n)^2 \geq \sum_{i=1}^{r_1} n_i^{(1)} \log(n)^2 \geq \sum_{i=1}^{r_2} n_i^{(2)} \log(n)^2$$

Comparison

	Time	Space
[KSW86]	$n \cdot 2^{\sqrt{2\log(n))}}$	$\sqrt{\log(n)}$
New algorithm	$n\log(n)^3$	$\log(n)$

- Lower bound $\Omega(\frac{n\log(n)}{\log\log(n)})$ when in $O(\log(n))$ space
- In full space: only trivial lower bound known ($\Omega(n)$)

Open problems

- Closing the gap between $\Omega(n)$ and $n\log(n)^3$ in full space setting
- Is randomness needed?

Questions?