A Stochastic Asset Liability Management Model for Life Insurance Companies

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Outline

The Model

Stochastic Asset Liability Management

We build a scenario-based stochastic **ALM** model with **dynamic reinvestment strategy**.

Portfolio rebalancing

- Matching between asset duration and liability duration;
- Achievement of a target portfolio return;
- Subject to real world constraints.

Asset Portfolio

- Bonds, divided into buckets of duration
- Equity
- Cash

Liability Portfolio

- With-profit life policies

Liability Portfolio

With-profit life policies

- Single or periodic premiums
- Saving account growth rate: $\max(g, \beta R^P)$
- Surrender option
- New production

Model points

Policies are gathered together according to:

- minimum guaranteed rate of return;
- maturity;
- age of policyholder.

Liability Portfolio

With-profit life policies

- Single or periodic premiums
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Cash Flows

Premium payments	$\Pi_{k,i} = n_{k-1,i} l^{\Pi}_{k,i}$
New production payments	$P_{k,i} = n^{P}_{k,i} l^{P}_{k,i}$
Death payments	$D_{k,i} = \boldsymbol{n}^{D}_{k,i} \cdot \boldsymbol{l}^{D}_{k,i}$
Surrender payments	$\Gamma_{k,i} = \boldsymbol{n}^{S}_{k,i} \cdot \boldsymbol{l}^{S}_{k,i}$
Maturity payments	$M_{k,i} = \boldsymbol{n}^{M}_{k,i} \cdot \boldsymbol{l}^{M}_{k,i}$

Cash outflows

$$cf_{k,i} = \begin{cases} \Gamma_{k,i} + D_{k,i} & \text{if } t_k < T_i, \\ M_{k,i} + D_{k,i} & \text{if } t_k = T_i, \\ 0 & \text{otherwise} \end{cases}$$

$$cf_k = \sum_{i=1}^{N_M} cf_{k,i}$$

Mortality, Surrender and New Production Model

Mortality Model

Number of policyholders who entered into the contract at time s and die at period k:

$$_s n_{k,i}^{D,\mathcal{M}} \sim Bin({}_s n_{k-1,i}^{\mathcal{M}}, p_{k,i}^{D,\mathcal{M}}),$$

$$_s n_{k,i}^{D,\mathcal{F}} \sim Bin({}_s n_{k-1,i}^{\mathcal{F}}, p_{k,i}^{D,\mathcal{F}}),$$

where $p_{k,i}^{D,\mathcal{M}}$ and $p_{k,i}^{D,\mathcal{F}}$ are given by specific life tables, depending only on age.

Surrender Model

Number of policyholders who entered into the contract at time s and surrender at period k:

$$_s n_{k,i}^{S} \sim Bin({}_s n_{k-1,i}, p_{k,i}^{S}).$$

New Production Model

Number of policyholders who entered into the contract at time k:

$$n_{k,i}^{P} \sim Bin(n_{k-1,i}, p_{k,i}^{P}).$$

Surrender and New Production Probabilities

For each model point m_i, we define

$$\delta r^S_{k,i} = (R^I_k - \max(g_{k,i}, \beta_{k,i} R^P_k))^+ \quad \text{and} \quad \delta r^P_{k,i} = (\max(g_{k,i}, \beta_{k,i} R^P_k) - R^I_k)^+,$$

where R^I_k is a benchmark rate of return at period k.

- If $\delta r^S_{k,i}$ is in the threshold interval I^q, then the surrender probability at period k is given by $p^S_{k,i} = p^S_{qk}$;
- If $\delta r^P_{k,i}$ is in the threshold interval I^q, then the new prod probability at period k is given by $p^P_{k,i} = p^P_{qk}$.

		Period				
		0	1	2	$\cdots$	$T-1$
Intervals	I^1	p^S_{10}, p^P_{10}	p^S_{11}, p^P_{11}	p^S_{12}, p^P_{12}	$\cdots$	p^S_{1T-1}, p^P_{1T-1}
	$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	I^Q	p^S_{Q0}, p^P_{Q0}	p^S_{Q1}, p^P_{Q1}	p^S_{Q2}, p^P_{Q2}	$\cdots$	p^S_{QT-1}, p^P_{QT-1}

Portfolio Rebalancing Strategy

Nonlinearly constrained optimization problem

- **Objective function**: distance between asset duration and liability duration;
- **Constraint on portfolio performance**: portfolio return is near a benchmark return;
- **Typical constraints on portfolio composition**.

Minimize $(A^D(\boldsymbol{\alpha}_k) - L_k^D)^+, \quad \boldsymbol{\alpha}_k = (\alpha_k^{B1}, \alpha_k^{B2}, \alpha_k^{B3}, \alpha_k^{B4}, \alpha_k^{E}, \alpha_k^{C});$

Subject to

$$\begin{cases} \beta^L R_{k+1}^I \leq R_{k+1}^P \leq \beta^U R_{k+1}^I, \quad \text{with constant } \beta^L, \beta^U, \\ \sum_{i \in I_\alpha} \alpha_k^i = 1 \text{ (budget constraint)} \\ \alpha_k^i \geq 0, \quad \forall i \in I_\alpha \text{ (no short selling constraint)} \\ \sum_{n=1}^4 \alpha_k^{Bn} \geq 0.70, \quad \alpha_k^E \leq 0.20 \text{ (investement policy constraints)} \\ \left|\alpha_k^i - \alpha_{k-1}^i\right| \leq 0.05, \quad \forall i \in I_\alpha \text{ (turnover constraint)} \\ \sum_{i \in I_\alpha} \left|\alpha_k^i - \alpha_{k-1}^i\right| \leq 0.30 \text{ (turnover constraint)} \end{cases}$$

Liability Duration

Macaulay's Formula

$$L_k^D = \frac{\sum\limits_{j>k} j d_{j|k} cf_{j|k}}{\sum\limits_{j>k} d_{j|k} cf_{j|k}}$$

- $d_{j|k}$: price at period k of a zero-coupon bond with tenor j ($G1++$ model)
- $cf_{j|k}$: expected cash outflows at period j evaluated at period k

At each time k, for each model point i and for $j > k$, we have to compute:

- $E[\max(g_{j,i}, \beta_{j,i} R_j^P)|\mathcal{F}_k]$
- $E[\delta r_{j,i}^S|\mathcal{F}_k] = E[(R_j^I - \max(g_{j,i}, \beta_{j,i} R_j^P))^+|\mathcal{F}_k]$
- $E[\delta r_{j,i}^P|\mathcal{F}_k] = E[(\max(g_{j,i}, \beta_{j,i} R_j^P) - R_j^I)^+|\mathcal{F}_k]$

$\Rightarrow$ Least Squares Monte Carlo method

Numerical Results
Assumptions

- All contracts have the same value, say €10 000, in the moment they are signed.
- All policies expire at the same future date, say at time $T = 10$ years.
- At time 0 policies are equally distributed between male and female policyholders (**gender equality**).
- Portfolio is rebalanced at each time step (**one year**).
- Policyholders pay a **single premium** at the beginning of the contract.
- The participation rate is the same for all model points and is constant over time ($\beta = 95\%$).
- $L_0 = 88.7\% A_0$.

Initial scenario

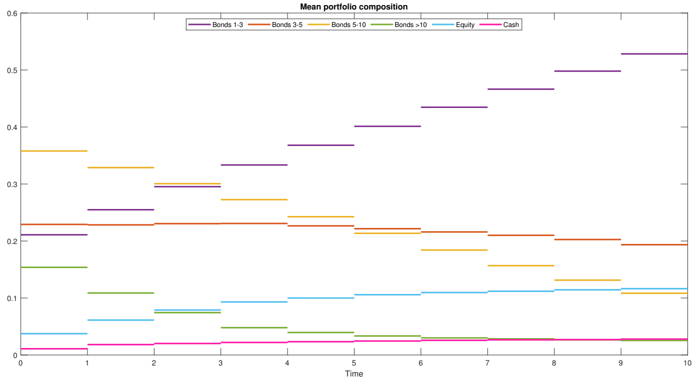
Asset class		Weight
$B1$	bonds, maturity 1-3	21.09%
$B2$	bonds, maturity 3-5	22.91%
$B3$	bonds, maturity 5-10	35.79%
$B4$	bonds, maturity >10	15.38%
E	equity	3.74%
C	cash	1.09%

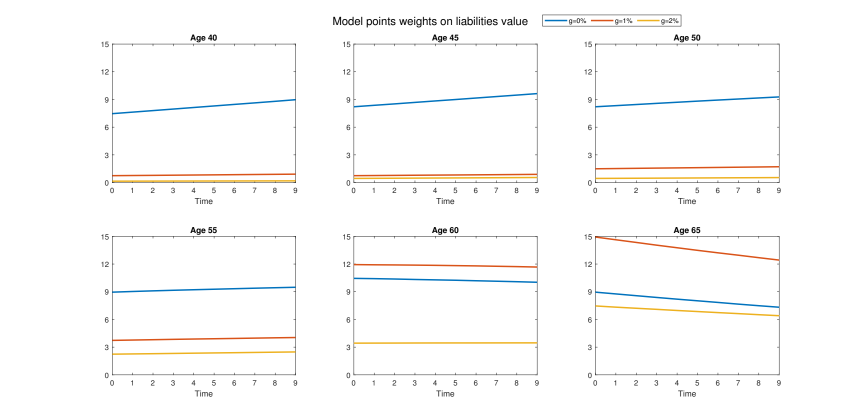
Table: Portfolio composition

	Minimum guarantee		
Age	0%	1%	2%
[40, 44]	50	5	1
[45, 49]	55	5	3
[50, 54]	55	10	3
[55, 59]	60	25	15
[60, 64]	70	80	23
[65, 69]	60	100	50

Table: Number of policies for each model point

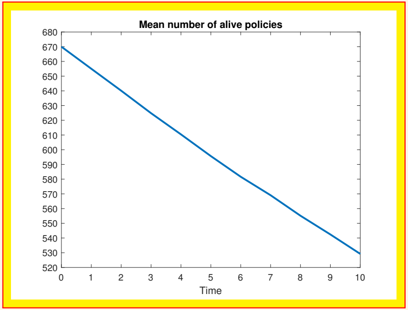
Numerical Results
Portfolio Composition Rebalancing

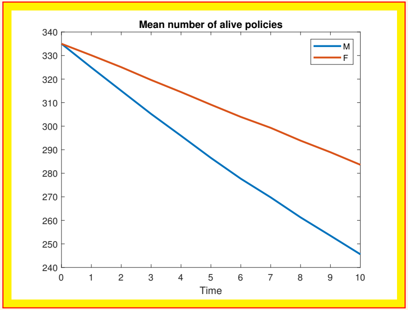




Model points weights on liabilities value
g=0%
g=1%
g=2%
Age 40
Age 45
Age 50
Age 55
Age 60
Age 65
Time

Numerical Results
Number of alive policies





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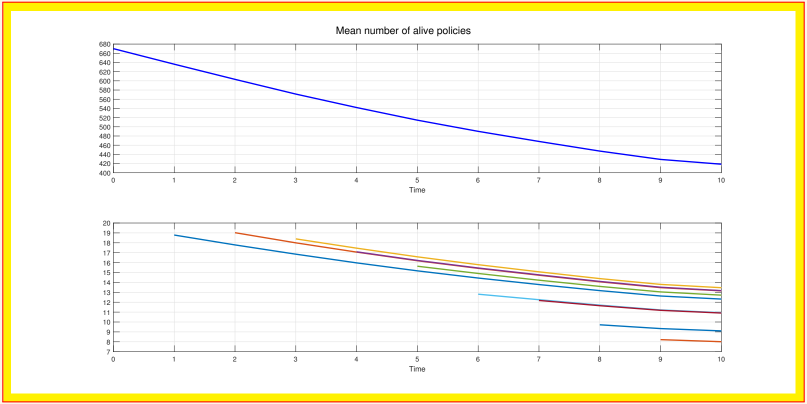

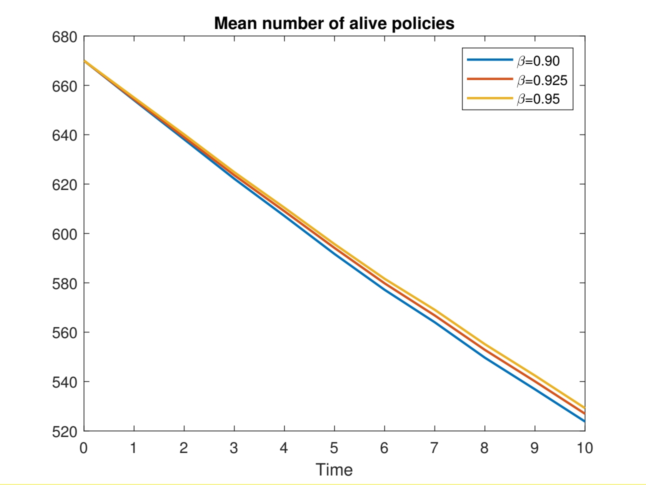

Mean number of alive policies
680
660
640
620
600
580
560
540
520
0
1
2
3
4
5
6
7
8
9
10
Time
β=0.90
β=0.925
β=0.95

Participation Rate Sensitivity