An Asset Liability Management model for a life insurance company's portfolio

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- Introduction to the problem
- Computation of Liability Duration
  - Maturity, Death and Surrender Payments
  - Mortality Model
  - Surrender and New Production Model
- Portfolio rebalancing
- Numerical results

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The Problem

Stochastic Asset Liability Management

We consider a Stochastic **ALM** problem with **multistep portfolio rebalancing**.

Policies

- The growth rate of policyholders' saving account is given by $\max(g, \beta R^P)$, where R^P is the **portfolio return** and g is the **minimum guaranteed rate of return**.
- New production is accepted.
- Policyholders are entitled to surrender the contract at any time before maturity.

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Liabilities and Asset Classes

Liabilities

Policies are gathered together (model points) according to:

- minimum guaranteed rate of return;
- age of policyholder;
- time to maturity.

Asset classes

The investable portfolio is composed of:

- bonds, divided into buckets of duration;
- cash;
- equity.

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Short Rate Model

Short rate model

We model the short rate r by the **G1++ model**, so that

$$r(t) = x(t) + f(t),$$

where f is a deterministic function and x satisfies the following SDE:

$$\begin{cases} dx_t = -a^x x_t dt + \sigma_t^x dW_t^x; \\ x(0) = 0. \end{cases}$$

Zero-coupon bond price

The price of a zero-coupon bond at time t with maturity at time T, $P(t, T)$, is given by:

$$P(t, T) = A(t, T; a^x, \sigma^x) \exp(-B(t, T; a^x, \sigma^x) x(t)).$$

Portfolio Rebalancing Strategy

At each time step:

1 - Simulations

- Simulate all the variables in the model;
- Compute Asset duration, Liability duration, portfolio return.

2 - Portfolio rebalancing

- Rebalance portfolio weights according to:
  - Matching between Asset Duration and Liability Duration;
  - Achievement of a target portfolio return.

Cash Flows

At period j for the model point i, we consider:

- Surrender payments, $\Gamma_{j,i}$;
- Death payments, $D_{j,i}$;
- Maturity payments, $M_{j,i}$.

We denote by $cf_{j,i}$ the expected cash flow at period j for the model point i:

$$cf_{j,i} = \begin{cases} \Gamma_{j,i} + D_{j,i} & \text{if } t_j < T_i, \\ M_{j,i} + D_{j,i} & \text{if } t_j = T_i, \\ 0 & \text{otherwise.} \end{cases}$$

Liability Duration

The Liability Duration at period k, according to Macaulay's formula, is given by:

$$L_k^D = \frac{\sum\limits_{j>k} j d_{k,j} cf_{j|k}}{\sum\limits_{j>k} d_{k,j} cf_{j|k}},$$

where

- $d_{k,j}$ is the price of a zero-coupon bond at period k with tenor j;
- $cf_{j|k}$ is the sum of expected cash flows of all model points at period j.

Maturity Payments, Death Payments and Surrender Payments

At time k for model point i we have:

$$M_{k,i} = n^M_{k,i} l^M_{k,i},$$
$$D_{k,i} = n^d_{k,i} l^d_{k,i},$$
$$\Gamma_{k,i} = n^s_{k,i} l^s_{k,i},$$

where $n^x_{k,i}$, for $x = \{M, d, s\}$, is the number of policies that reach maturity, die or surrender at period k, respectively, and $l^x_{k,i}$, for $x = \{M, d, s\}$, is the guaranteed payment in case of maturity, death or surrender, respectively.

Policyholder's saving account

- Each policyholder invests 1 when entering into the contract.
- Policyholder's saving account grows at a rate proportional to $\max(g, \beta R^P)$.

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At time k for model point i we have:

$$M_{k,i} = n^M_{k,i} I^M_{k,i},$$
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Surrender Model

For each period k and for each model point i, we define

$$\delta r_{k,i} = \max(R_k^I - \max(g, \beta R_k^P), 0),$$

where R_k^I is a benchmark rate of return at period k.
If $\delta r_{k,i}$ is in the threshold interval I_j^k, then $p_{ji}^{s,k}$ denotes the percentage of policies in the model point i that are surrendered at period k.

	$m.p.1$	$m.p.2$	$m.p.3$	$\dots$
I_1^k	$p_{11}^{s,k}$	$p_{12}^{s,k}$	$p_{13}^{s,k}$	$\dots$
I_2^k	$p_{21}^{s,k}$	$p_{22}^{s,k}$	$p_{23}^{s,k}$	$\dots$
I_3^k	$p_{31}^{s,k}$	$p_{32}^{s,k}$	$p_{33}^{s,k}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Least Squares Monte Carlo for Surrender Model

From now on we fix a model point i for ease of notation.

At each time k, for $j > k$, we have to compute

$$\Delta R(j|k) = E[\delta r_j|k] = E[(R_j^I - \hat{R}_j^P)^+|k], \quad \text{where } \hat{R}_j^P = \max(g, \beta R_j^P)^+.$$

We can write $\Delta R(j|k)$ as **linear combination of basis functions** $\{\psi^w\}_{w=1,\dots,W}$:

$$\Delta R(j|k) \simeq \sum_{w=1}^{W} b_{k,j}^w \psi^w(\hat{R}_k^P, R_k^I) = b_{k,j}^T \psi(\hat{R}_k^P, R_k^I).$$

We determine the expression for the regression coefficients $b_{k,j}$ using the Least Squares Method:

$$b_{k,j}^* = \underset{b_{k,j}}{\operatorname{argmin}}\, E_k\left[\left(\psi(\hat{R}_k^P, R_k^I)^T b_{k,j} - E_k[\delta r_j]\right)^2\right]$$

$$\Longrightarrow E_k\left[\psi(\hat{R}_k^P, R_k^I)\psi(\hat{R}_k^P, R_k^I)^T\right] b_{k,j} = E_k\left[\psi(\hat{R}_k^P, R_k^I)\delta r_j\right]$$

We solve the **system** $A_k^\psi b_{k,j} = d_{k,j}^\psi$, where we estimate A_k^ψ and $d_{k,j}^\psi$ using Monte Carlo techniques:

$$(A_k^\psi)_{uv} = \frac{1}{N_P}\sum_{n=1}^{N_P} \psi^u(\hat{R}_{k,n}^P, R_{k,n}^I)\psi^v(\hat{R}_{k,n}^P, R_{k,n}^I), \quad (d_{k,j}^\psi)_v = \frac{1}{N_P}\sum_{n=1}^{N_P} \psi^v(\hat{R}_{k,n}^P, R_{k,n}^I)(R_{j,n}^I - \hat{R}_{j,n}^P)^+.$$

New Production

We denote by $n_k^{k_0}$ the number of policies started at period k_0 and still alive at period k.

New production model

The number of policyholders who enter into the contract at period k is modelled by a Binomial distribution:

$$\begin{cases} n_0^0 = 100, \\ n_k^k = Bin\left(\sum_{j=0}^{k-1} n_{k-1}^j, p_k^p\right), \end{cases}$$

where p_k^p is the probability of new production at time k.

Death Model and Surrender Model

Death model

The number of policyholders who entered into the contract at time k_0 and die at period k is modelled by a Binomial distribution:

$$n_k^{k_0,d} = Bin(n_{k-1}^{k_0}, p^d),$$

where p^d is the probability of death, given by a specific life table.

Surrender model

The number of policyholders who entered into the contract at time k_0 and surrender at period k is modelled by a Binomial distribution:

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where p_k^s is the probability of surrender at period k.

Number of Alive Policies

The number of policyholders who entered into the contract at time k_0 and are still alive at time k is given by:

$$n_k^{k_0} = n_{k-1}^{k_0} - n_k^{k_0,d} - n_k^{k_0,s}.$$

Number of Alive Policies

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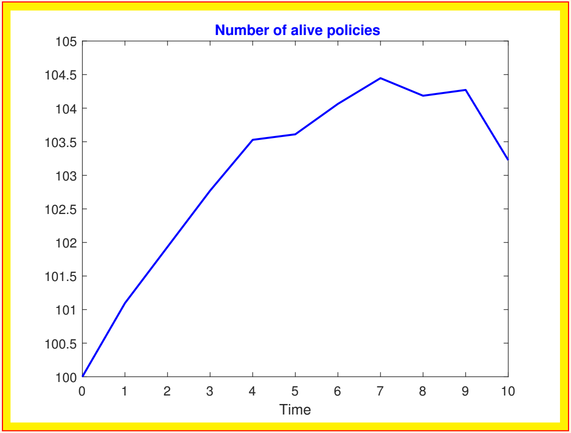

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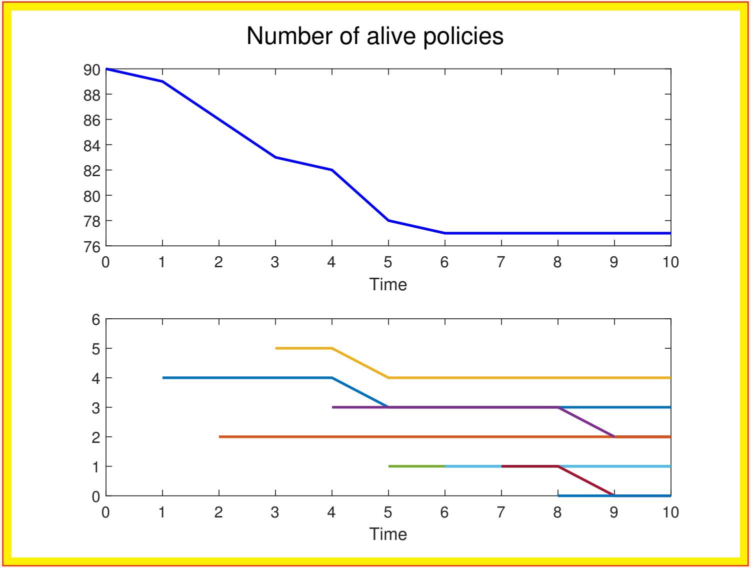


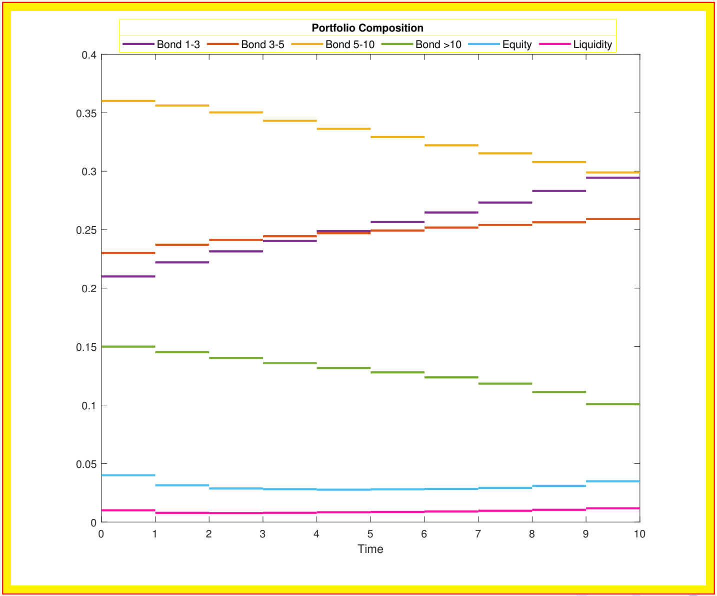
Portfolio Rebalancing

Optimization Problem

In order to rebalance the portfolio, at each time we have to solve a **nonlinearly constrained optimization problem**.

- **Objective function**: distance between Asset duration and Liability duration;
- **Constraint on portfolio performance**: portfolio return is near a benchmark return;
- **Typical constraints**, such as:
  - Lower and upper bounds for each asset class;
  - Budget constraint;
  - No short selling constraint;
  - Turnover constraints.

Numerical Results



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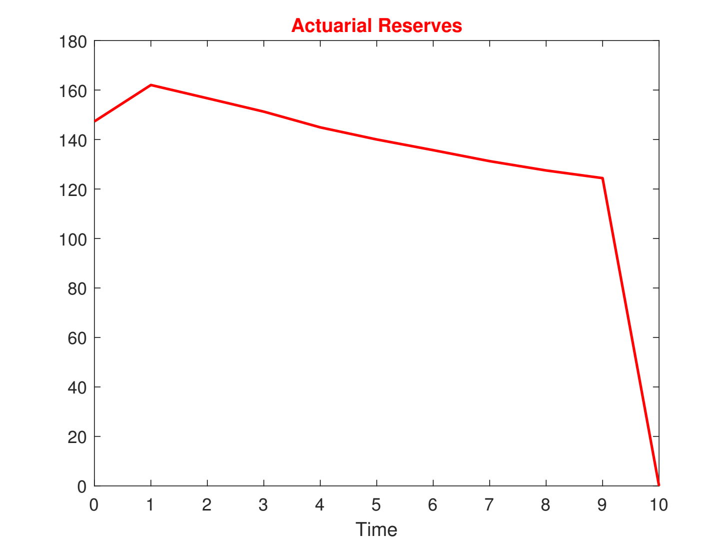

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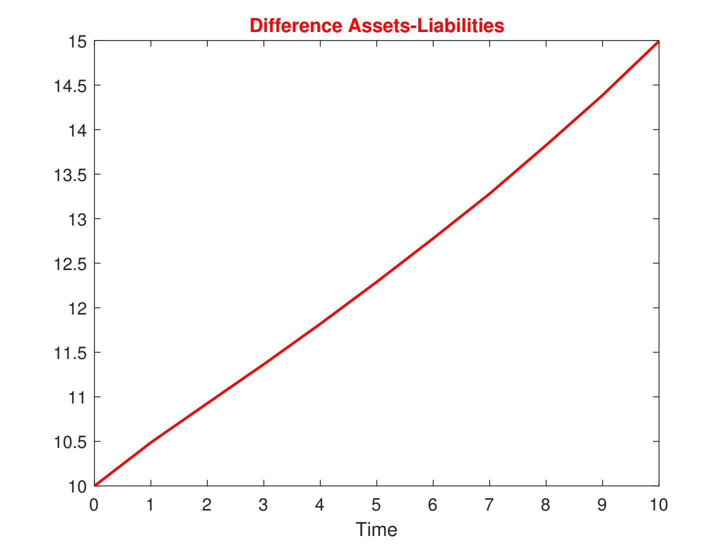

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