Regulatory and economic capital requirements related to credit risk

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Outline

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Definitions

To estimate **economic capital**, models split the loss distribution into two segments:

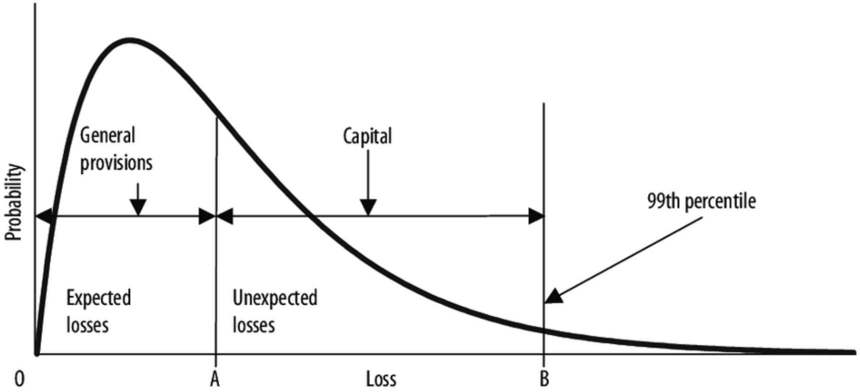
- **Expected Loss** ("likely"): estimated as the mean of the losses.
- **Unexpected Loss** ("unlikely"): estimated by setting an extremely high threshold (unlikely probability)

The difference between unexpected and expected loss serves as an estimate for economic capital.

Moreover, it is assumed that:

- reserves and provisions held on the balance sheet of the bank should adequately cover and compensate for "expected" loss caused by normal operating conditions
- economic capital should cover the "unexpected" portion of the loss distribution

Loss distribution



Outline

Value at risk

Banks develop **economic capital models for credit risk** that allow them to establish profitability objectives based on the capital assigned to each area or product, making management decisions and overcoming the limitations of the Basel models of regulatory capital.

A certain amount of capital reserve is required as a cushion for potential large losses in the portfolio. The most popular risk measure for this purpose is the **Value at Risk (VaR)**, which is defined as follows:

If α is some confidence level, the VaR_α is simply the α-quantile of the loss distribution of L. Thus,

$$VaR_\alpha = \text{ínf}\left\{x : P(L \leq x) \geq \alpha\right\} \tag{1}$$

We choose $\alpha = 99,9$ as confidence level to be consistent with the regulatory capital premises, and one year time horizon.

Portfolio loss

We consider a **credit portfolio** consisting of n $(n \to \infty)$ homogeneous obligors or counterparties with exposures $E_i, i = 1, \dots, n$.
We assume that obligor i defaults if its standardized log asset value X_i is less than some default threshold y_i (related to debt level) after a fixed time horizon.
The **event of default** can be modelled as a Bernoulli random variable $D_i = 1_{\{X_i < y_i\}}$ with known default probability $PD_i = P(X_i < y_i)$ as follows:

$$\begin{cases} P(D_i = 1) = PD_i \\ P(D_i = 0) = 1 - PD_i \end{cases} \tag{2}$$

If we assume that some fraction (R_i) can be recovered in case of default of obligor i, it follows that the loss L_i due to obligor i default is simply $(1 - R_i) \cdot E_i \cdot D_i$, so that the **portfolio loss** is given by

$$L = \sum_{i=1}^{n} L_i = \sum_{i=1}^{n} E_i \cdot D_i \cdot (1 - R_i) \tag{3}$$

The term $(1 - R_i)$ is known as severity or Loss Given Default **(LGD)**, already used in regulatory capital.

Vasicek model for default rates

In the **Vasicek model**, the dependence structure among counterparties in the portfolio is simplified by the introduction of a common factor that affects all counterparties.
Thus, it is assumed that the standardized asset log-return X_i of obligor i can be decomposed into:

- a **systematic factor** Y (related to the economy),
- an **idiosyncratic factor** ϵ_i,

such that:

$$X_i = \sqrt{\rho} \cdot Y + \sqrt{1-\rho} \cdot \epsilon_i, \tag{4}$$

where

- Y and all ϵ_i are independent standard normal random variables
- ρ is the (average) asset correlation of the portfolio

It is important to note that, conditional on the realisation of the systematic factor Y, the asset values and the defaults are independent.

Vasicek model for default rates

Under this approach, the counterparty ability to pay is represented by a random variable with distribution $N(0,1)$. The default of a counterparty $(D_i = 1)$ takes place when its capacity to pay falls below the threshold K_i (level of debt), so that:

$$PD_i = P(X_i < K_i) \tag{5}$$

and, if we assume that all obligors have the same default threshold $K_i = K$, then

$$K = \Phi^{-1}(PD_i) \tag{6}$$

Conditioning this expression to the value of the systematic factor $Y = y$ and clearing the idiosyncratic factor, the default rate distribution (DR) of a counterparty i is given by:

$$P(D_i = 1|Y = y) = \Phi\left(\frac{1}{\sqrt{1-\rho}}\left(\Phi^{-1}(PD_i) + \sqrt{\rho}\cdot y\right)\right) \tag{7}$$

Vasicek model for default rates

Equation (7) defines a Bernoulli random variable with parameter PD_i. The default is a dichotomous variable with values 0 or 1. The frequency of $D_i = 1$ is given by the parameter PD_i.

If we assume an homogeneous portfolio, that is:

- with n $(n \to \infty)$ borrowers with similar PD and
- the same sensitivity to the common systematic risk factor Y

then the cumulative distribution function of the default rate (DR) of the portfolio converges to

$$P(DR \leq \alpha) = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\alpha) - \Phi^{-1}(PD)}{\sqrt{\rho}}\right) \tag{8}$$

Intra-portfolio correlation estimation

The asset **correlation** ρ should be interpreted as a parameter that determines the shape of default rate distribution. It provides an indication how volatile the default rate fluctuates over time.

One technique to **estimate the correlation parameter** ρ is the **method of moments**, which uses the first (mean) and the second (variance) moments of the default rate. The theoretical mean and variance of the Vasicek distribution (Eq. (12)) are given by

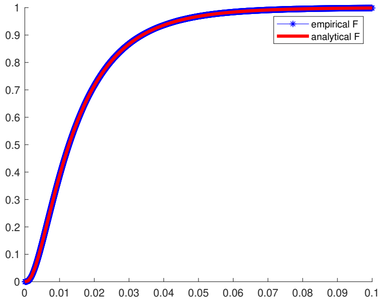
$$E[DR] = PD, \tag{9}$$

$$Var(DR) = \Phi_2(\Phi^{-1}(PD), \Phi^{-1}(PD), \rho) - PD^2, \tag{10}$$

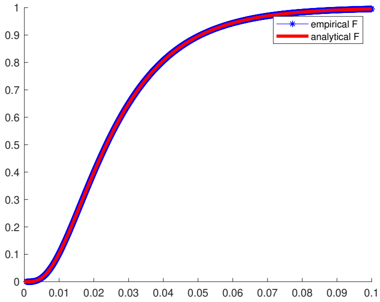
where Φ_2 is a bivariate normal distribution.
Given default rate data of credit portfolio one can create homogeneous groups and calculate the sample mean $\hat{\mu}^2$ and sample variance $\hat{s}^2$ as estimates. The method of moments obtains the estimate by solving for $\hat{\rho}$ the equation

$$\hat{\mu}^2 + \hat{s}^2 = \Phi_2(\Phi^{-1}(PD), \Phi^{-1}(PD), \hat{\rho}) \tag{11}$$

CDF of the default rates



Mortgages



Corporates

Stochastic LGD

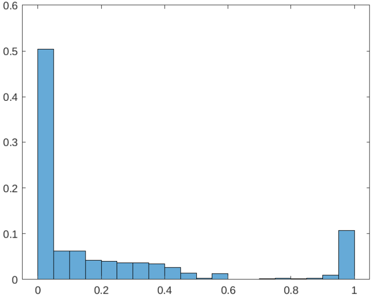
Vasicek's model can be extended by assuming that the **LGD is stochastic** rather than deterministic.

The most widespread alternative for modeling random severity is the choice of the **beta distribution**. In this case, the historical data are also required as the mean of the beta distribution. The article

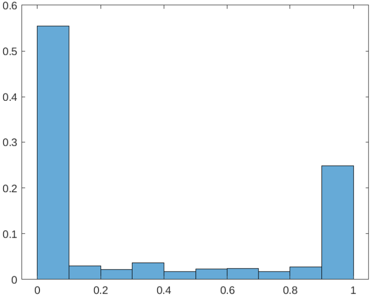
Farinelli, S. and Shkolnikov, M.(2012). Two models of stochastic loss given default. The Journal of Credit Risk, 8 (2), pp. 3–20.

provides two models for stochastic LGD.
However, as we have not enough data, we just performed a MLE with the historical data.

Histograms LGDs

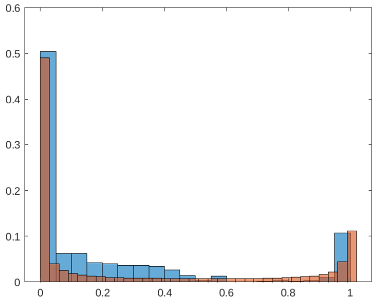


Mortgages

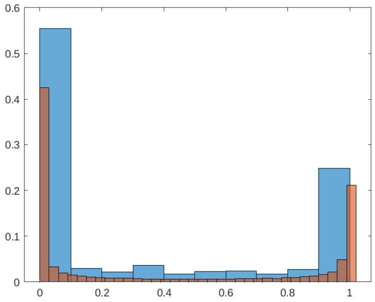


Corporates

Histograms LGDs: beta regression



Mortgages α =0.1045, $\beta = 0.2288$



Corporates α =0.1006, β =0.1472

Correlation between portfolios

One of the most important effects that can be considered in economic capital and that is not addressed in regulatory capital is diversification between portfolios, sectors, etc.

In order to correlate the default rates and LGR in each sector we just used a Gaussian copula between default rates (normal distribution) and LGD (beta distribution). Moreover, correlation between default rates in mortgages and corporates has been included.

We have analysed the results of economic capital for different correlation parameters.

Numerical results illustrate the expected qualitative behaviour of models and methods.

LGD	EC Mortgages (%)	EC Corporate (%)
Deterministic	1.98	3.56
Stochastic (0.5, 0.0, 0.0)	7.89	10.22
Stochastic (-0.5, 0.0, 0.0)	7.83	10.29
Stochastic (0.5, 0.9, 0.9)	10.53	12.16
Stochastic (0.5, -0.9, -0.9)	1.39	4.03
Stochastic (-0.5, -0.9, -0.9)	1.39	4.00
Stochastic (0.5, 0.9, -0.9)	10.40	4.03

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Thanks for your attention